CATALOGUE OF STARS WITHIN TWO DEGREES OF THE NORTH POLE

DEDUCED FROM

PHOTOGRAPHIC MEASURES MADE AT VASSAR COLLEGE OBSERVATORY

BY

CAROLINE E. FURNESS, Ph. D.,

Assistant in the Observatory.

WASHINGTON, D. C.:
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PUBLICATION No. 45.

PUBLICATIONS OF THE VASSAR COLLEGE OBSERVATORY, No. 2.

MARY W. WHITNEY, *Director.*

PREFACE.

The investigation involved in this memoir is an extension of the stellar catalogue presented in Publication No. 1 of the Vassar College Observatory. Publication No. 1 gives a catalogue of stars within one degree of the north pole. Publication No. 2 extends the catalogue to two degrees from the pole. Both catalogues are based upon photographs taken by Professor Donner of Helsingfors, Finland, and secured through the courtesy of Professor Jacoby of Columbia University. The direction of the measuring and reduction of the plates has been in the hands of Caroline E. Furness, Ph. D., assistant in the observatory. The observations for magnitude, photographic and visual, made after the manner described in the text, were carried on by the director. The computing corps has consisted mainly of graduates of Vassar College, of whom Miss M. E. Tarbox and Miss B. Tompkins have rendered especial service.

The major part of the expense of the reduction has been met by a grant from the Carnegie Institution of Washington, D. C.

MARY W. WHITNEY,
Director of Observatory.

VASSAR COLLEGE, *June*, 1905.

CATALOGUE OF STARS WITHIN TWO DEGREES OF THE NORTH POLE.

I.

Introductory.

The first publication of the Vassar College Observatory treats of the measurement and reduction of a set of four photographic plates covering the region of the sky within one degree of the north pole. These photographs were taken at the Helsingfors Observatory, Finland, and form part of a series of twelve negatives, which together include all the stars of less than two degrees polar distance. The four plates first treated have their centers at declination 90°, the remaining eight have their centers lying on the circle of 89° declination at intervals of 45° right ascension. Because of the high declination this procedure results in a peculiar kind of overlapping, the character of which can easily be seen by referring to Publication 1, Sec. I, in which is given a description of the plates and also an illustrative figure. The present paper treats first of the measurement and reduction of the 89° plates with a preliminary catalogue of the stars found thereon; second, of the inter-adjustment of the several plates and their combination with the former catalogue, a problem which presents considerable interest on account of the peculiarity of the overlapping above referred to; and third, of the formation of the final catalogue of all the stars found on the twelve plates.

The method of performing the measurement and reduction, which form the first part of this paper, is in general the same as that employed in Publication 1, Secs. II–VIII. Deviations from it are occasionally necessary because of the fact that the north pole is no longer at the center of each plate but is on the edge. These variations will be mentioned in their appropriate places. A detailed description of the plates is given in Publication 1, Sec. I.

The meteorological and other data connected with the exposure of the plates were sent from the Helsingfors observatory, and are found in the accompanying table.

TABLE I.

Plate 1895.	Observer.	Barom.	Thermom. Attached R.	Thermom. External R.	Length of Exposure.	End of Exposure.	α	δ
		mm	°	°	m s	h m s	h m	° ′
Sept. 16	D.	757.2	+ 8.4	+ 7.8	6 0	22 48 56	22 24	89 0
no. 2					3 0	52 50		
					20	53 51		
Sept. 16	D.	757.2	+ 8.2	+ 7.5	6 0	23 32 50	22 24	91 0
no. 4					3 0	36 40	(10 24)	(89 0)
					20	37 46		
Sept. 16	Dr.	756.85	+ 8.2	+ 7.4	6 0	0 10 33	1 24	91 0
no. 5					3 0	14 13	(13 24)	(89 0)
					20	15 9		
Sept. 16	Dr.	756.2	+ 8.0	+ 6.9	6 0	0 43 13	1 24	89 0
no. 7					3 0	46 47		
					20	47 53		
Sept. 18	D.	751.2	+ 9.0	+ 8.4	6 0	20 22 6	19 24	89 0
no. 1					3 0	25 45		
					20	26 40		
Sept. 18	D.	751.3	+ 8.5	+ 8.0	6 0	20 57 47	19 24	91 0
no. 3					3 0	21 1 19	(7 24)	(89 0)
					20	2 52		
Sept. 21	Dr.	764.4	+ 2.7	+ 2.2	6 0	3 2 2	4 24	91 0
no. 13					3 0	5 34	(16 24)	(89 0)
					20	6 26		
Sept. 21	Dr.	764.6	+ 2.5	+ 2.0	6 0	3 33 56	4 24	89 0
no. 15					3 0	37 37		
					20	38 21		

D. Donner. Dr. Dreyer.

II.

Measurement and Reduction; Preliminary Catalogue.

After some trial it seemed advisable to use both vertical and horizontal threads in measuring the rectangular coördinates of the star images. Thus both the X and Y coördinates were determined in the same position of the plate. It was also decided to set upon both corners of the reseau square at once, thus reducing greatly the number of micrometer readings. It was not possible to set upon the lines as is customary, since for the most part they were too indistinct. Otherwise the method of measuring and recording is the same as that described in Publication 1, Sec. II and III. Six plates were measured by Dr. Furness and two by Miss Elise Whitney. The coördinates of the standard stars were measured by both observers. Corrections were made for errors of the screw, for the reseau and for error of runs. Table II which follows contains the corrected coördinates of the two sets of images measured on each 89° plate. It also contains the coördinates of certain stars on the 90° plates which have a polar distance greater than 1° and hence are not included in the catalogue of sixty-five stars found in Publication 1.

TABLE II.—Corrected Coördinates. Plate Sept. 16. No. 2.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
1	+ 22.5042	22.5141	+ 8.6316	9.0339
2	+ 30.0238	30.0335	− 2.4325	2.0292
4	+ 25.4530	25.4563	+ 10.4520	10.8574
5	+ 4.9571	4.9746	+ 49.7152	50.1230
6	+ 45.1850	....	− 25.0711	....
7	+ 36.8030	36.8089	− 7.7324	7.3244
8	+ 24.5354	24.5426	+ 14.9632	15.3665
9	+ 39.1598	39.1670	− 5.8690	5.4639
10	+ 36.8814	36.8852	− 0.5848	0.1778
11	+ 59.8598	59.8558	− 33.5996	33.1894
14	+ 54.1760	54.1820	− 16.0368	15.6325
15	+ 52.0083	52.0198	− 11.7727	11.3655
17	+ 56.5390	56.5450	− 16.5920	16.1941
16	+ 39.3317	39.3378	+ 5.8675	6.2714
21	+ 57.6026	57.6084	− 6.8952	6.4916
25	+ 35.0026	35.0150	+ 26.8502	27.2554
28	+ 42.3644	42.3640	+ 21.3662	21.7711
31	+ 41.3172	....	+ 26.3678	....
36	+ 47.3690	47.3803	+ 30.4554	30.8550
37	+ 43.7152	....	+ 33.5268	....
38	+ 30.8982	30.9088	+ 41.7176	42.1278
41	+ 30.9062	....	+ 43.0572	....
43	+ 22.8843	22.8964	+ 48.0392	48.4402
47	+ 18.3613	18.3682	+ 51.5093	51.9124
49	+ 44.5790	44.5908	+ 38.8710	39.2720
54	+ 20.8442	20.8578	+ 51.6943	52.1014
66	+ 47.3022	47.3188	+ 52.7854	53.1926
279	− 13.2251	13.2156	+ 59.4532	59.8588
288	− 20.9115	20.9030	+ 55.5455	55.9446
299	− 6.4938	6.4834	+ 59.1956	59.6078
301	− 47.5537	47.5430	+ 43.2271	43.6246
310	− 60.8507	60.8387	+ 31.3026	31.7118
313	− 56.3284	56.3236	+ 32.1838	32.5928
316	− 26.6741	26.6658	+ 46.5486	46.9538
322	− 17.8706	17.8664	+ 49.6828	50.0854
323	− 37.3684	37.3606	+ 36.4808	36.8860
329	− 33.0095	32.9992	+ 35.9271	36.3216
332	− 61.6729	61.6684	+ 9.7966	10.1978
335	− 20.0548	20.0472	+ 42.3364	42.7400
340	− 44.1132	44.1033	+ 15.8367	16.2406
341	− 51.8892	51.8833	+ 6.8956	7.3054
346	− 59.3544	59.3483	− 3.7846	3.3791
348	− 15.0460	....	+ 44.5691	....
351	− 52.2999	52.2926	− 1.5974	1.1926
353	− 42.2948	42.2790	+ 6.9677	7.3679
354	− 53.7446	53.7368	− 9.2464	8.8398
356	− 42.9593	42.9496	+ 1.6158	2.0192
359	− 49.2374	49.2268	− 14.0824	13.6704
360	− 47.5996	47.5934	− 11.7993	11.3948
362	− 14.0508	....	+ 38.7499	....
363	− 51.6166	51.5968	− 34.6618	34.2558
364	− 43.5092	43.4996	− 20.5871	20.1838
365	− 31.6286	31.6260	+ 2.0280	2.4233

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. NO. 2.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
367	− 7.4286	7.4232	+ 48.2824	48.6882
368	− 32.9756	32.9674	− 5.8866	5.4898
369	− 41.1678	....	− 25.7891	....
370	− 36.2698	36.2724	− 21.8838	21.4838
371	− 46.0214	46.0164	− 46.2241	45.8213
372	− 11.2664	11.2544	+ 36.6443	37.0512
373	− 9.5852	9.5712	+ 40.0630	40.4734
374	− 37.2110	37.2057	− 36.2352	35.8334
375	− 32.4648	32.4548	− 24.1612	23.7588
376	− 19.4548	19.4472	+ 9.8949	10.2994
377	− 33.1640	33.1582	− 32.2045	31.8019
378	− 26.4021	26.3917	− 13.9716	13.5644
379	− 27.6518	27.6494	− 21.6146	21.2066
380	− 27.2348	27.2249	− 22.4504	22.0510
381	− 6.3649	6.3549	+ 41.9317	42.3380
382	− 28.8660	28.8524	− 42.1272	41.7269
384	− 14.8352	14.8260	− 6.6000	6.1928
385	− 14.3036	14.2924	− 18.7038	18.2986
386	− 13.8332	13.8263	− 38.9474	38.5443
387	− 4.6228	4.6130	− 4.9148	4.5099
388	− 0.9641	0.9616	− 33.2718	32.8700
389	+ 0.4974	0.5054	− 18.8895	18.4842
390	+ 7.7294	7.7384	− 12.9100	12.5106
391	+ 14.4020	14.4038	− 55.6617	55.2488
392	+ 8.2250	8.2339	+ 13.4193	13.8238
393	+ 24.1046	24.1158	− 54.7655	54.3636
394	+ 15.7591	....	− 6.3174	....
395	+ 18.5363	18.5406	− 9.4986	9.0896
396	+ 28.1404	....	− 43.5508	....
397	+ 21.7908	21.8004	− 18.2050	17.8017
399	+ 22.9884	22.9979	− 21.4205	21.0214
400	+ 24.4803	24.4827	− 20.4642	20.0499
401	+ 27.8302	27.8368	− 25.3326	24.9340
402	+ 32.8565	32.8708	− 40.2595	39.8598
403	+ 31.2851	31.2884	− 32.4516	32.0473
404	+ 33.5662	33.5715	− 38.9350	38.5366
405	+ 20.7844	20.7979	− 0.1327	+ 0.2692
406	+ 23.8272	23.8357	− 5.2712	4.8672
407	+ 44.0546	44.0514	− 53.5390	53.1322
408	+ 32.8814	32.8875	− 22.8953	22.4922
	PLATE SEPT. 16. NO. 4.			
89	+ 20.0138	20.0212	− 57.5936	57.1912
93	+ 38.4492	38.4636	− 53.6324	53.2422
116	+ 27.9613	27.9752	− 45.5351	45.1426
120	+ 59.7208	59.7366	− 24.5162	24.1226
121	+ 46.3142	46.3256	− 31.1205	30.7208
123	+ 10.7472	10.7616	− 51.5105	51.1199
124	+ 47.4716	47.4813	− 27.2418	26.8464
127	+ 55.1755	55.1946	− 11.4184	11.0274

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. No. 4.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
129	+ 40.5392	40.5600	− 16.5830	16.1830
130	+ 52.3250	52.3430	− 0.5329	0.1362
131	+ 40.5012	40.5206	− 12.0400	11.6430
132	+ 39.5212	39.5444	− 9.4180	9.0242
134	+ 47.1353	47.1630	+ 1.8191	2.2101
135	+ 14.8138	14.8306	− 38.4014	38.0028
137	+ 36.5506	36.5704	− 7.3010	6.9066
139	+ 42.0048	42.0286	+ 7.5750	7.9723
140	+ 52.4154	52.4344	+ 39.0463	39.4450
141	+ 42.3112	42.3331	+ 31.0856	31.4790
142	+ 52.2780	52.3152	+ 55.3312	55.7320
143	+ 36.8953	36.9097	+ 23.6449	24.0462
144	+ 8.1517	8.1672	− 38.9622	38.5672
145	+ 31.2004	31.2214	+ 15.2400	15.6390
147	+ 23.4288	23.4522	+ 9.6520	10.0511
148	+ 32.8702	32.8952	+ 37.2309	37.6293
149	+ 30.2770	30.3072	+ 40.6604	41.0514
150	+ 31.1375	31.1664	+ 45.8823	46.2856
151	+ 23.4812	23.5096	+ 30.2636	30.6638
152	+ 12.3056	12.3267	− 9.1412	8.7514
153	+ 11.2216	11.2490	− 9.4492	9.0522
154	+ 17.6041	17.6286	+ 20.1184	20.5166
155	+ 13.8266	13.8468	+ 10.6351	11.0409
156	+ 17.7557	17.7802	+ 36.4378	36.8379
157	+ 11.6720	11.6922	+ 5.5137	5.9161
158	+ 15.3039	15.3284	+ 25.0844	25.4834
159	+ 2.9484	2.9683	− 29.4972	29.1078
160	+ 8.5910	8.6118	+ 23.4860	23.8848
161	+ 5.6495	5.6785	+ 34.8962	35.2966
162	+ 1.3627	1.3816	− 25.1980	24.8015
163	+ 0.1386	0.1642	− 1.7114	1.3072
164	− 1.6419	1.6175	+ 15.1254	15.5241
165	− 7.9618	7.9370	+ 30.9841	31.3856
166	− 7.3144	7.2901	+ 22.9012	23.2990
167	− 5.3829	5.3640	− 3.4694	3.0558
168	− 4.6009	4.5830	− 20.5547	20.1530
169	− 15.0623	15.0376	+ 45.5413	45.9434
170	− 16.8011	16.7713	+ 37.0670	37.4608
171	− 16.1483	16.1290	+ 26.0874	26.4932
172	− 21.3263	21.3130	+ 54.3436	54.7497
173	− 18.9486	18.9218	+ 29.7772	30.1784
174	− 22.3850	22.3556	+ 39.6722	40.0712
175	− 9.6585	9.6390	− 22.1334	21.7324
176	− 25.6282	25.6009	+ 39.8818	40.2844
177	− 7.7029	7.6846	− 32.7038	32.3050
178	− 28.0044	27.9821	+ 33.1789	33.5863
179	− 22.7457	22.7230	+ 10.6820	11.0818
180	− 31.8640	31.8412	+ 28.3120	28.7158
181	− 26.6648	26.6394	+ 12.7756	13.1782
182	− 18.1331	18.1194	− 17.4504	17.0482
183	− 20.8176	20.7966	− 11.8031	11.4036
184	− 45.0289	45.0062	+ 31.8883	32.2965
185	− 43.2571	43.2322	+ 27.3988	27.8042

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. No. 4.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
186	− 43.1466	43.1276	+ 22.0902	22.5004
187	− 26.2756	26.2550	− 11.0088	10.6038
188	− 36.1745	36.1524	+ 1.2454	1.6472
189	− 19.8036	19.7884	− 30.0690	29.6664
190	− 24.2328	24.2176	− 23.5059	23.1088
191	− 24.3488	24.3381	− 23.4440	23.0400
192	− 48.6698	48.6417	+ 11.4004	11.8090
193	− 37.1948	37.1751	− 8.5631	8.1603
194	− 56.1344	56.1100	+ 14.0466	14.4529
195	− 45.8113	45.8020	+ 0.3671	0.7694
196	− 24.0697	24.0554	− 28.8842	28.4810
197	− 33.6702	33.6444	− 16.7361	16.3350
198	− 52.3598	52.3456	+ 5.2572	5.6604
200	− 9.7721	9.7562	− 47.6468	47.2546
201	− 45.0629	45.0348	− 5.0638	4.6482
202	24.7140	24.6911	− 29.7332	29.3290
203	− 55.3954	55.3748	+ 4.4682	4.8758
205	− 51.1048	51.0760	− 2.7549	2.3567
206	− 41.7992	41.7800	− 14.1060	13.7046
207	48.7084	48.7019	− 6.8740	6.4631
209	36.3514	36.3418	− 21.1246	20.7277
212	− 57.0977	57.0758	− 2.8944	2.4924
214	− 44.5312	44.5115	− 16.8587	16.4452
215	− 49.1033	....	− 12.5281	....
217	− 43.2274	43.2094	− 21.2108	20.8032
223	− 23.0712	23.0588	− 42.5434	42.1440
227	− 31.6998	31.6954	− 39.6024	39.2039
228	− 38.4066	38.3922	− 35.5399	35.1317
232	− 35.6786	35.6623	− 38.4318	38.0246
242	− 61.1917	61.1763	− 37.9043	37.4980
250	− 19.6051	19.5977	− 55.3850	54.9862
266	− 45.4986	45.4876	− 58.2164	45.4876

PLATE SEPT. 16. No. 5.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
135	+ 26.7558	26.7516	− 54.5635	54.1822
144	+ 21.6597	21.6578	− 50.2385	49.8612
157	+ 55.6760	55.6784	− 21.3587	20.9729
162	+ 26.6294	26.6291	− 35.7189	35.3404
164	+ 53.0986	53.0924	− 5.1289	4.7462
165	+ 59.8813	59.8836	+ 10.5360	10.9246
166	+ 54.6062	54.6082	+ 4.3718	4.7572
168	+ 25.7154	25.7152	− 28.2040	27.8194
170	+ 57.9583	....	+ 21.0966	....
171	+ 50.6331	50.6434	+ 12.8728	13.2510
173	+ 51.2720	51.2772	+ 17.4648	17.8446
174	+ 55.8559	55.8573	+ 26.8820	27.2644
175	+ 21.0328	21.0322	− 25.7388	25.3554
176	+ 53.7234	53.7326	+ 29.3336	29.7126
177	+ 14.9106	14.9110	− 34.5847	34.1964
178	+ 47.2913	47.2978	+ 26.2985	26.6829

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. NO. 5.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
179	+ 35.0468	35.0509	+ 6.6922	7.0735
180	+ 41.1150	41.1244	+ 25.5954	25.9829
181	+ 33.7663	33.7720	+ 10.9415	11.3276
182	+ 18.3661	18.3664	− 16.4283	16.0482
183	+ 20.4788	20.4836	− 10.5366	10.1581
184	+ 34.3608	34.3665	+ 37.4514	37.8355
185	+ 32.4260	32.4363	+ 33.0199	33.4040
186	+ 28.7424	28.7520	+ 29.1982	29.5804
188	+ 18.8832	18.8858	+ 9.5229	9.9434
189	+ 8.2481	8.2502	− 24.1440	23.7570
190	+ 9.7681	9.7668	− 16.3797	15.9972
191	+ 9.7278	9.7326	− 16.2482	15.8652
192	+ 17.2750	17.2806	+ 25.5698	25.9543
193	+ 11.2165	11.2178	+ 3.3548	3.7393
194	+ 13.8799	13.8880	+ 32.7247	33.1090
195	+ 11.4654	11.4651	+ 15.7647	16.1493
196	+ 6.0784	6.0756	− 20.2831	19.9076
197	+ 7.9210	7.9124	− 4.9060	4.5170
198	+ 10.3110	10.3186	+ 23.8627	24.2458
199	+ 12.5130	12.5214	+ 46.6111	46.9963
200	+ 2.8608	2.8634	− 43.6525	43.2742
201	+ 8.1462	8.1528	+ 11.4028	11.7870
203	+ 7.6135	7.6204	+ 25.4456	25.8382
204	+ 5.8880	5.8909	+ 15.5457	15.9225
205	+ 5.5182	5.5243	+ 17.3032	17.6953
206	+ 4.0408	4.0408	+ 2.7072	3.0928
207	+ 4.2980	4.2969	+ 12.7074	13.0830
208	+ 3.9370	3.9371	+ 17.4214	17.8111
209	+ 2.9022	2.9068	− 6.1123	5.7328
211	+ 2.0978	2.1114	+ 54.7632	55.1491
212	+ 1.1975	1.2020	+ 21.4567	21.8416
213	+ 1.0074	....	+ 23.4798	....
214	+ 0.1630	0.1656	+ 2.7016	3.0905
215	+ 0.0119	0.0132	+ 9.0000	9.3844
216	− 2.8608	2.8590	+ 26.8028	27.1970
217	− 1.9877	1.9924	− 1.2900	0.9002
218	− 3.3936	3.3870	+ 12.8901	13.2766
219	− 9.6018	9.5964	+ 42.7336	43.1197
220	− 6.8624	6.8597	+ 3.4432	3.8268
221	− 13.8028	13.7938	+ 47.7784	48.1682
222	− 12.2459	12.2393	+ 34.0550	34.4434
223	− 2.9115	2.9106	− 30.6307	30.2470
224	− 16.0674	16.0604	+ 42.7092	43.0972
225	− 16.1004	16.0944	+ 33.0818	33.4751
226	− 17.2252	17.2164	+ 36.6852	37.0786
227	− 6.9007	6.9020	− 22.4408	22.0464
228	− 8.7520	8.7566	− 14.8183	14.4368
229	− 19.9530	19.9442	+ 32.1120	32.5003
230	− 23.9423	23.9272	+ 45.1835	45.5741
231	− 17.3456	17.3464	+ 17.6028	17.9824
232	− 8.8836	8.8824	− 18.7888	18.4002
233	− 20.2191	20.2132	+ 19.9558	20.3418
234	− 23.6500	23.6504	+ 6.6566	7.0386

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. No. 5.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
235	− 24.7765	24.7732	+ 7.2684	7.6566
236	− 34.0148	34.0087	+ 28.5240	28.8991
237	− 40.3477	40.3434	+ 41.9668	42.3606
239	− 38.4691	38.4667	+ 26.8326	27.2240
240	− 49.5655	49.5622	+ 49.0366	49.4282
242	− 26.5008	26.5004	− 0.3366	+ 0.0500
243	− 8.9306	8.9336	− 37.4982	37.1019
245	− 49.1558	49.1480	+ 38.4076	38.7950
247	− 44.3446	44.3414	+ 24.9922	25.3840
250	− 9.5672	9.5655	− 42.1424	41.7528
252	− 17.1884	17.1854	− 32.0056	31.6212
258	− 39.3802	39.3864	− 9.5008	9.1174
259	− 40.2112	40.2118	− 8.5523	8.1614
260	− 39.7155	39.7148	− 9.5476	9.1522
261	− 62.1189	62.1191	+ 15.5574	15.9549
264	− 63.0124	63.0076	+ 14.7594	15.1563
265	− 29.9711	29.9706	− 23.0472	22.6572
266	− 29.8373	29.8354	− 25.7821	25.3952
267	− 63.5475	63.5486	+ 7.1351	7.5294
272	− 48.4295	48.4278	− 10.5402	10.1500
279	− 8.2589	8.2614	− 50.5065	50.1166
288	− 16.4512	16.4504	− 47.8091	47.4214
291	− 45.6256	45.6290	− 32.1440	31.7512
297	− 61.9838	61.9792	− 25.7888	25.4018
298	− 3.6802	3.6827	− 55.4448	55.0524
301	− 43.9786	43.9742	− 37.6165	37.2293
310	− 61.7982	61.8082	− 36.6036	36.2200
313	− 58.0005	57.9872	− 39.1876	38.7896
316	− 26.8860	26.8920	− 50.0654	49.6881
323	− 41.5800	41.5818	− 49.5986	49.2098
329	− 38.8906	38.8964	− 53.0706	52.6830
335	− 25.2078	25.2099	− 57.7330	57.3410
340	− 60.9745	60.9769	− 59.3676	58.9762
348	− 20.0948	20.0914	− 59.7127	59.3154

PLATE SEPT 16. No. 7.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
1	− 18.8382	18.8406	+ 8.2834	8.6841
2	− 21.3760	21.3782	− 4.8512	4.4541
4	− 15.4711	15.4752	+ 7.4778	7.8793
5	− 2.1054	2.1118	+ 49.6990	50.1058
6	− 26.7084	26.7077	− 31.5658	31.1638
7	− 20.3379	20.3436	− 13.3920	12.9996
8	− 12.9176	12.9240	+ 11.3128	11.7145
9	− 17.3550	17.3548	− 13.7498	13.3542
10	− 15.2167	15.2137	− 8.4016	8.0029
11	− 22.4018	22.4015	− 47.9596	47.5570
12	− 18.5761	18.5773	− 32.4268	32.0170
13	− 11.4222	11.4290	− 0.4362	0.0341
14	− 13.9665	13.9663	− 31.5594	31.1644
15	− 12.4708	12.4746	− 27.0086	26.6114

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. No. 7.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
16	− 8.9241	8.9246	− 5.5923	5.1962
17	− 12.6956	12.6942	− 33.6197	33.2224
18	− 3.2656	3.2708	+ 29.1366	29.5410
19	− 15.0176	15.0194	− 53.1275	52.7328
20	− 4.6934	4.6943	− 20.6882	20.2938
21	− 5.0730	5.0729	− 27.5328	27.1324
22	− 6.6429	6.6430	− 59.2532	58.8592
24	− 0.9890	....	− 50.8681	....
25	+ 2.8989	2.8952	+ 12.2810	12.6780
27	+ 4.9404	4.9390	− 42.4090	42.0210
28	+ 4.1926	4.1922	+ 3.2042	3.6070
29	+ 6.5276	6.5251	+ 3.8932	4.2920
30	+ 11.7878	11.7912	− 48.8930	48.4960
31	+ 7.0133	7.0068	+ 7.4725	7.8720
32	+ 9.8024	9.8019	− 4.4569	4.0558
33	+ 10.8676	10.8610	− 6.0791	5.6813
34	+ 14.6702	14.6672	− 18.4018	18.0048
35	+ 21.0820	21.0846	− 44.3623	43.9679
36	+ 14.1650	14.1656	+ 6.0661	6.4653
37	+ 13.7698	13.7674	+ 10.8306	11.2271
38	+ 10.5315	10.5329	+ 25.6824	26.0826
39	+ 26.6402	26.6407	− 33.9540	33.5556
40	+ 21.9104	21.9058	− 13.3110	12.9034
41	+ 11.4886	11.4910	+ 26.6248	27.0264
42	+ 26.3417	26.3429	− 23.6627	23.2659
43	+ 9.3550	9.3516	+ 35.8224	36.2236
44	+ 36.5882	36.5868	− 52.1140	51.7162
45	+ 31.3968	31.3988	− 32.9504	32.5514
46	+ 21.0973	21.0940	+ 1.1181	1.5157
47	+ 8.6159	8.6108	+ 41.4743	41.8779
48	+ 38.6330	38.6318	− 46.5314	46.1325
49	+ 18.1619	18.1600	+ 13.9836	14.3871
50	+ 36.0627	36.0664	− 35.3811	34.9783
51	+ 37.7356	37.7322	− 35.4526	35.0422
52	+ 39.9622	39.9624	− 39.7066	39.3070
53	+ 36.7123	36.7128	− 29.3812	28.9838
54	+ 10.5039	10.4954	+ 39.8440	40.2439
55	+ 47.6131	47.6176	− 51.7351	51.3380
56	+ 28.5177	28.5155	− 2.5882	2.1860
57	+ 38.7108	38.7101	− 26.3965	25.9936
58	+ 49.7256	49.7384	− 48.7187	48.3232
59	+ 44.0340	44.0409	− 31.7508	31.3498
61	+ 36.0345	36.0254	+ 3.1652	3.5638
63	+ 34.3243	34.3185	+ 12.5400	12.9323
64	+ 56.2824	56.2924	− 20.5128	20.1082
66	+ 29.9394	29.9434	+ 21.8698	22.2693
67	+ 27.6829	27.6844	+ 26.1448	26.5447
69	+ 52.5511	52.5468	− 5.1760	4.7790
71	+ 43.3890	43.3918	+ 8.7686	9.1718
73	+ 51.6375	51.6294	+ 1.0824	1.4908
74	+ 40.9542	40.9546	+ 16.1880	16.5858
75	+ 36.9434	36.9390	+ 21.6216	22.0178
77	+ 53.5276	53.5324	+ 3.4193	3.8242

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. No. 7.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
82	+ 48.7732	48.7626	+ 14.7736	15.1700
83	+ 43.6940	43.6895	+ 19.9696	20.3683
85	+ 41.1560	41.1580	+ 24.1227	24.5296
89	+ 17.0550	17.0475	+ 48.1854	48.5850
93	+ 32.8566	32.8509	+ 37.9152	38.3134
116	+ 31.1983	31.1974	+ 51.0494	51.4577
121	+ 54.3714	54.3654	+ 48.2094	48.6175
123	+ 14.8274	14.8201	+ 59.0274	59.4326
124	+ 57.9290	57.9363	+ 50.1244	50.5359
367	− 11.8589	11.8678	+ 57.4660	57.8654
372	− 22.8159	22.8262	+ 51.9792	52.3684
373	− 19.2090	19.2096	+ 53.1984	53.5974
376	− 47.5470	47.5509	+ 38.8990	39.2919
381	− 15.6172	15.6208	+ 52.2308	52.6334
384	− 55.9852	55.9911	+ 23.9852	24.3880
387	− 47.5811	47.5843	+ 17.9438	18.3387
389	− 53.8706	53.8734	+ 4.4520	4.8468
390	− 44.5343	44.5404	+ 3.5482	3.9442
392	− 25.5188	25.5237	+ 21.7813	22.1790
393	− 62.6214	62.6224	− 37.5849	37.1984
395	− 34.4904	34.4977	− 1.6952	1.2965
397	− 38.3572	38.3583	− 10.1432	9.7478
398	− 52.2252	52.2310	− 35.4467	35.0542
399	− 39.7932	39.7940	− 13.2632	12.8746
401	− 39.1568	39.1581	− 19.4652	19.0668
402	− 46.1764	46.1778	− 33.5604	33.1652
403	− 41.7556	41.7606	− 26.9296	26.5356
404	− 44.7425	44.7493	− 33.1133	32.7206
405	− 26.2632	26.2652	+ 3.3128	3.7080
406	− 27.7569	27.7614	− 2.4660	2.0701
407	− 47.6794	47.6793	− 50.8394	50.4420
408	− 33.8526	33.8520	− 21.3134	20.9142

PLATE SEPT. 18. No. 1.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
1	+ 51.0872	51.1030	+ 40.5292	40.9070
5	+ 9.5996	9.6109	+ 57.1426	57.5359
8	+ 48.0394	48.0442	+ 46.4413	46.8172
223	− 30.0206	30.0146	+ 56.4376	56.8430
242	− 60.2491	60.2434	+ 32.7548	33.1493
243	− 23.1498	....	+ 50.4430	....
250	− 18.4918	18.4804	+ 49.8200	50.2154
252	− 28.6027	28.6031	+ 42.1728	42.5649
258	− 51.0432	51.0442	+ 19.9080	20.2952
259	− 51.9852	51.9828	+ 19.0690	19.4660
260	− 50.9896	50.9879	+ 19.5738	19.9694
265	− 37.5197	37.5196	+ 29.3592	29.7554
266	− 34.7936	34.7864	+ 29.5057	29.8985
268	− 25.4572	25.4549	+ 37.9708	38.3658
272	− 49.9660	49.9654	+ 10.8552	11.2450
279	− 10.1321	10.1244	+ 51.1632	51.5469

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 18. NO. 1.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
284	− 54.3229	54.3116	− 14.6022	14.2004
285	− 63.6306	63.6376	− 29.4106	29.0234
288	− 12.8038	12.7966	+ 42.9618	43.3531
291	− 28.3828	28.3795	+ 13.7432	14.1365
293	− 52.5138	52.5145	− 36.5061	36.1161
294	− 51.0504	51.0515	− 34.1392	33.7450
296	− 38.2610	38.2628	− 9.6030	9.2142
297	− 34.6711	34.6761	− 2.6394	2.2464
298	− 5.1970	5.1914	+ 55.7506	56.1454
300	− 37.8594	37.8602	− 16.6988	16.3142
301	− 22.9039	22.9024	+ 15.4068	15.7964
302	− 49.8380	49.8464	− 52.2218	51.8203
303	− 46.0750	....	− 46.2396	....
304	− 42.0172	42.0218	− 36.6658	36.2776
305	− 30.7772	30.7791	− 9.7484	9.3574
306	− 22.6808	22.6746	+ 7.4892	7.8786
307	− 30.3597	30.3546	− 13.0988	12.7064
308	− 43.8658	43.8719	− 49.3140	48.9098
310	− 23.8613	23.8608	− 2.4284	2.0405
312	− 30.5503	30.5562	− 26.4560	26.0570
313	− 21.2926	21.2892	+ 1.3952	1.7848
314	− 37.1314	37.1310	− 54.8958	54.5018
316	− 10.5086	10.5000	+ 32.5351	32.9174
317	− 21.6372	21.6358	− 22.0288	21.6435
319	− 14.9017	14.9029	+ 6.5057	6.8916
320	− 22.9626	22.9612	− 34.2410	33.8527
321	− 19.4962	19.4952	− 19.6396	19.2490
322	− 6.5015	6.4948	+ 40.9698	41.3604
323	− 10.9414	10.9342	+ 17.8469	18.2325
324	− 18.2934	18.2940	− 24.2840	23.8955
325	− 20.9050	20.9096	− 46.8950	46.5020
326	− 14.6248	14.6308	− 21.1973	20.8064
327	− 13.8732	13.8727	− 21.2400	20.8526
328	− 15.6512	15.6571	− 45.4306	45.0364
329	− 7.4679	7.4636	+ 20.5430	20.9254
330	− 14.9298	14.9387	− 51.8518	51.4716
331	− 8.4984	8.4966	− 0.8190	0.4332
332	− 9.2332	9.2340	− 18.2050	17.8158
333	− 8.3110	8.3158	− 51.0107	50.6305
335	− 2.8522	2.8444	+ 34.2293	34.6188
336	− 2.3555	2.3524	− 19.8532	19.4660
337	− 2.0458	2.0452	+ 9.4458	9.8364
338	− 2.3271	2.3272	+ 43.2223	43.6126
339	− 0.6746	0.6784	− 26.3619	25.9752
340	− 1.0963	1.0967	− 1.5152	1.1286
341	− 0.2682	0.2712	− 13.3354	12.9450
342	+ 0.9234	0.9251	− 31.8092	31.4134
343	+ 1.2356	1.2322	− 30.7173	30.3240
344	+ 1.7266	1.7258	− 27.3312	26.9478
345	+ 2.5608	2.5505	− 37.2018	36.8228
346	+ 2.0130	2.0112	− 26.1620	25.7762
347	+ 4.7908	4.7876	− 36.6382	36.2578
348	− 0.8932	0.8853	+ 39.3526	39.7414

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 18. No. 1.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
349	+ 8.3126	8.3054	− 53.4792	53.0992
350	+ 4.9104	4.9130	− 16.6212	16.2378
351	+ 5.4543	5.4543	− 19.6230	19.2341
352	+ 11.0476	11.0348	− 58.9000	58.5229
353	+ 6.4678	6.4686	− 6.4914	6.1092
354	+ 9.8448	9.8377	− 26.0492	25.6680
355	+ 5.8626	5.8652	+ 6.8264	7.2091
356	+ 9.7807	9.7833	− 10.7416	10.3586
357	+ 13.2288	13.2251	− 18.6714	18.2877
358	+ 17.8920	17.8818	− 32.8733	32.4938
359	+ 16.4505	16.4530	− 26.2858	25.8954
360	+ 15.9956	15.9936	− 23.5141	23.1271
361	+ 17.0162	17.0098	− 26.9918	26.6080
362	+ 3.9288	3.9368	+ 35.9517	36.3440
363	+ 29.3452	29.3384	− 42.4928	42.1164
364	+ 25.1078	25.1043	− 26.8246	26.4424
365	+ 17.4982	17.5019	− 2.4393	2.0475
366	+ 24.4670	24.4668	− 21.4414	21.0492
367	+ 1.8588	1.8687	+ 47.3690	47.7530
368	+ 22.1443	22.1452	− 8.9832	8.5940
369	+ 30.4380	30.4397	− 28.8414	28.4700
370	+ 31.1376	31.1408	− 22.6260	22.2436
371	+ 41.4708	41.4684	− 46.7224	46.3396
372	+ 7.3854	7.3914	+ 36.4326	36.8194
373	+ 6.1466	6.1573	+ 40.0349	40.4200
374	+ 40.6238	40.6241	− 33.4214	33.0522
375	+ 35.4356	35.4385	− 21.5406	21.1621
376	+ 20.5329	20.5384	+ 11.7396	12.1256
377	+ 40.6403	40.6390	− 27.7157	27.3368
378	+ 32.5208	32.5185	− 10.0472	9.6641
379	+ 37.0346	37.0390	− 16.3330	15.9512
380	+ 37.9253	37.9204	− 16.6271	16.2392
381	+ 7.1038	7.1138	+ 43.6384	44.0226
382	+ 50.7064	50.7018	− 31.6866	31.3042
384	+ 35.4650	35.4714	+ 3.3488	3.7245
386	+ 59.0654	59.0662	− 18.7985	18.4284
387	+ 41.4974	41.4982	+ 11.7744	12.1547
389	+ 54.9990	55.0096	+ 5.5125	5.8880
390	+ 55.8886	55.8968	+ 14.8505	15.2324
392	+ 37.5956	37.6036	+ 33.8180	34.1901
405	+ 56.0694	56.0801	+ 33.1324	33.5062

PLATE SEPT. 18. No. 3.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
36	+ 53.7680	53.7700	− 47.3518	46.9740
43	+ 24.0156	24.0220	− 52.2470	51.8680
47	+ 18.3622	18.3693	− 52.9883	52.6072
49	+ 45.8382	45.8407	− 43.3820	42.9966
66	+ 37.9100	37.9170	− 31.6184	31.2382
67	+ 33.6438	....	− 33.8895	....
71	+ 50.9756	50.9780	− 18.1444	17.7510

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 18. NO. 3.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
74	+ 43.5646	43.5730	− 20.5953	20.2066
75	+ 38.1358	38.1334	− 24.6232	24.2432
77	+ 56.3040	56.3044	− 7.9818	7.6008
82	+ 44.9632	44.9632	− 12.7786	12.3911
83	+ 39.7750	39.7745	− 17.8694	17.4836
85	+ 35.6212	35.6215	− 20.4152	20.0276
89	+ 11.6288	11.6320	− 44.5928	44.2054
93	+ 21.8586	21.8586	− 28.7564	28.3716
94	+ 50.8440	50.8403	+ 9.2836	9.6662
95	+ 42.1875	....	+ 5.0942	....
97	+ 40.1845	40.1816	+ 7.2949	7.6864
99	+ 47.3499	47.3421	+ 25.6618	26.0482
101	+ 34.8448	34.8470	+ 7.0399	7.4246
104	+ 41.7614	41.7604	+ 24.1720	24.5645
107	+ 30.9932	30.9914	+ 17.5190	17.9064
108	+ 22.0186	22.0194	− 1.3573	0.9711
109	− 32.6808	32.6780	+ 24.8001	25.1831
110	+ 20.6572	20.6602	− 4.3772	3.9908
111	+ 26.9640	26.9612	+ 13.2430	13.6296
112	+ 29.3972	22.3976	+ 2.3678	2.7519
113	+ 17.2979	17.2964	− 9.6272	9.2413
114	+ 32.9640	32.9586	+ 33.7378	34.1240
115	+ 21.1027	21.1021	+ 3.0355	3.4260
116	+ 8.7176	8.7204	− 30.4433	30.0604
117	+ 38.2216	38.2146	+ 54.7075	55.0848
118	+ 25.2220	25.2220	+ 17.8542	18.2396
120	+ 16.3014	16.3004	+ 6.8684	7.2503
121	+ 11.4958	11.4944	− 7.2698	6.8860
122	+ 23.2184	....	+ 51.1056	....
123	+ 0.7891	0.7930	− 46.8404	46.4623
124	+ 9.5702	9.5715	− 3.7118	3.3284
126	+ 13.0984	13.0987	+ 54.7851	55.1582
127	+ 3.8297	3.8266	+ 12.9184	13.2996
128	+ 1.8744	1.8710	+ 51.4697	51.8578
129	− 2.8693	2.8722	− 1.0812	0.6960
130	− 5.8804	5.8816	+ 18.6018	18.9832
131	− 6.0978	6.0982	+ 2.1085	2.4892
132	− 8.6471	8.6488	+ 3.2554	3.6373
133	− 10.7948	10.7935	+ 13.6937	14.0742
134	− 11.2076	11.2074	+ 16.5973	16.9771
135	− 5.6161	5.6103	− 34.7042	34.3239
136	− 19.2006	19.2014	+ 49.4440	49.8269
137	− 12.2412	12.2436	+ 2.6584	3.0370
138	− 19.5392	19.5421	+ 43.9902	44.3681
139	− 18.9068	18.9104	+ 17.0270	17.4104
140	− 33.8163	33.8198	+ 46.6371	47.0219
141	− 35.3182	35.3288	+ 33.8618	34.2428
142	− 45.4193	45.4308	+ 58.0519	58.4400
144	− 9.9295	9.9312	− 39.8144	39.4292
145	− 31.9748	31.9720	+ 14.8111	15.1937
147	− 33.5076	33.5124	+ 5.3642	5.7444
149	− 50.5990	50.6094	+ 32.1298	32.5118
151	− 48.0587	48.0586	+ 19.9658	20.3558

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 18. No. 3.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
155	− 40.9930	40.9914	− 0.7339	0.3510
157	− 38.9002	38.8962	− 5.8742	5.5020
158	− 50.1736	50.1708	+ 10.5246	10.8965
160	− 53.7892	53.7900	+ 4.6542	5.0289
161	− 63.9351	63.9383	+ 10.6533	11.0265
162	− 24.4620	24.4576	− 34.8828	34.4960
166	− 64.6186	64.6207	− 7.0142	6.6360
168	− 31.9726	31.9666	− 35.8104	35.4306
175	− 34.4343	34.4276	− 40.5101	40.1215
177	− 25.5656	25.5630	− 46.5978	46.2207
191	− 43.8686	43.8704	− 51.8283	51.4575
193	− 63.4980	63.4975	− 50.3934	50.0212
200	− 16.4606	16.4565	− 58.6358	58.2533
PLATE SEPT. 21. No. 13.				
168	+ 40.7374	40.7348	− 53.7891	53.3973
175	+ 38.9916	38.9881	− 48.8108	48.4178
177	+ 28.4841	28.4814	− 51.1155	50.7282
182	+ 43.3709	....	− 40.1566	....
189	+ 30.7084	30.7044	− 38.9426	38.5406
190	+ 37.1034	37.1006	− 34.2826	33.8896
191	+ 37.1588	37.1610	− 34.1538	33.7638
193	+ 51.5833	51.5807	− 20.7932	20.3988
196	+ 31.7348	31.7344	− 34.6408	34.2501
197	+ 43.5440	....	− 24.6095	....
200	+ 13.4832	13.4803	− 49.5846	49.1836
201	+ 54.8188	54.8152	− 12.8133	12.4152
202	+ 30.8570	30.8530	− 34.0240	33.6324
204	+ 55.9687	55.9704	− 8.2422	7.8414
205	+ 56.9068	56.9124	− 6.7060	6.3079
206	+ 45.8828	45.8769	− 16.3910	16.0088
207	+ 52.8840	52.8901	− 9.2350	8.8371
209	+ 39.0602	39.0594	− 22.0860	21.6898
212	+ 56.5514	56.5614	− 0.7134	0.3222
214	+ 43.0410	43.0404	− 13.7654	13.3668
215	+ 47.2154	47.2158	− 9.0376	8.6328
217	+ 38.7446	38.7534	− 15.2246	14.8259
218	+ 47.3702	47.3759	− 3.8754	3.4844
220	+ 38.3900	38.3946	− 8.4361	8.0478
222	+ 55.2666	55.2746	+ 17.6612	18.0562
223	+ 18.1114	18.1112	− 36.1064	35.7076
224	+ 58.3590	58.3621	+ 26.6048	26.9924
225	+ 51.7870	51.7894	+ 19.5716	19.9634
226	+ 53.4114	53.4194	+ 22.9743	23.3646
227	+ 20.7594	20.7592	− 27.3799	26.9782
228	+ 24.5880	24.5885	− 20.5430	20.1420
229	+ 48.3008	48.3108	+ 21.4760	21.8694
230	+ 54.2602	54.2734	+ 33.7686	34.1561
231	+ 40.3388	40.3428	+ 9.0746	9.4721
232	+ 21.7892	21.7896	− 23.3670	22.9704

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 21. No. 13.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
233	+ 39.8346	39.8352	+ 12.7486	13.1442
234	+ 28.2674	28.2734	+ 5.3245	5.7250
235	+ 27.8595	27.8651	+ 6.5510	6.9488
236	+ 35.5368	35.5349	+ 28.4148	28.8020
237	+ 40.0476	....	+ 42.5897	....
238	+ 15.6550	15.6560	− 21.3372	20.9386
239	+ 31.1263	31.1348	+ 30.2021	30.5939
240	+ 38.0998	38.1021	+ 54.0202	54.4162
241	+ 13.1294	....	− 25.5616	....
242	+ 21.4224	21.4242	+ 2.1507	2.5496
243	+ 9.0324	9.0322	− 37.0384	36.6390
244	+ 29.6303	29.6314	+ 36.1991	36.5955
245	+ 31.1772	31.1835	+ 45.9528	46.3468
246	+ 32.9518	....	+ 58.4804	....
247	+ 25.5688	25.5728	+ 32.8476	33.2450
248	+ 19.8862	19.8868	+ 12.4562	12.8517
249	+ 7.3351	7.3365	− 29.8677	29.4716
250	+ 5.4082	5.4014	− 40.0166	39.6151
251	+ 12.7602	12.7616	+ 13.0493	13.4470
252	+ 6.7113	6.7116	− 27.4006	26.9994
253	+ 9.7004	9.7003	− 2.8544	2.4574
254	+ 10.3536	10.3515	+ 3.0103	3.4022
255	+ 13.7434	13.7516	+ 57.0316	57.4286
256	+ 11.8238	11.8328	+ 45.8258	46.2284
257	+ 7.1101	7.1069	+ 3.1040	3.4990
258	+ 5.7474	5.7471	+ 4.1912	4.5930
259	+ 5.7866	5.7870	+ 5.4528	5.8512
260	+ 5.4692	5.4696	+ 4.3884	4.7892
261	+ 6.1120	6.1192	+ 38.0188	38.4196
262	+ 5.4951	5.4956	+ 38.1748	38.5787
263	+ 4.1348	4.1348	− 7.3450	6.9469
264	+ 4.9204	4.9231	+ 38.0351	38.4331
265	+ 3.4340	3.4336	− 12.1436	11.7435
266	+ 1.6732	1.6721	− 14.2368	13.8379
267	− 0.6613	0.6570	+ 32.8179	33.2158
268	+ 1.4978	1.4981	− 26.8364	26.4336
269	+ 1.4012	1.4008	− 29.1205	28.7240
270	− 0.7465	0.7380	+ 9.9042	10.3046
271	− 2.8635	2.8542	+ 39.9532	40.3578
272	− 1.5971	1.5959	+ 9.5829	9.9842
273	− 4.3708	4.3662	+ 48.4881	48.8896
274	− 7.1324	7.1270	+ 55.4752	55.8764
275	− 4.5055	4.5042	+ 9.5006	9.9014
276	− 10.7655	10.7579	+ 42.6626	43.0610
277	− 14.4347	14.4326	+ 48.8226	49.2233
278	− 13.3209	13.3174	+ 40.7230	41.1262
279	+ 0.6634	0.6638	− 47.0355	46.6386
280	− 9.2320	9.2300	+ 11.5144	11.9123
281	− 12.6342	12.6300	+ 22.7702	23.1741
282	− 14.9968	14.9909	+ 33.6508	34.0485
283	− 19.6806	19.6749	− 50.8963	51.2977
284	− 17.2238	17.2180	+ 30.1359	30.5386
285	− 21.7048	21.6974	+ 47.0609	47.4643

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 21. No. 13.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
286	− 19.9802	19.9776	+ 26.3237	26.7266
287	− 24.0476	24.0458	+ 38.0308	38.4405
288	− 3.4948	3.4944	− 39.4892	39.0878
289	− 21.0664	21.0670	+ 15.0732	15.4759
290	− 12.1276	12.1282	− 13.8026	13.4023
291	− 14.2172	14.2183	− 8.1602	7.7560
292	− 20.0848	20.0841	+ 7.7796	8.1820
293	− 34.4772	34.4756	+ 43.7651	44.1642
294	− 33.7440	33.7424	+ 41.0812	41.4858
296	− 24.5412	24.5391	+ 15.0042	15.4105
297	− 21.8918	21.8906	+ 7.6224	8.0220
298	+ 0.6682	0.6634	− 53.7726	53.3730
299	− 28.7886	28.7879	+ 20.3933	20.7888
300	− 30.0033	30.0002	+ 19.5522	19.9561
301	− 16.7379	16.7384	− 13.2837	12.8852
302	− 47.7825	47.7780	+ 52.5324	52.9375
304	− 41.7628	41.7556	+ 36.2212	36.6236
305	− 29.7553	29.7484	+ 9.6338	10.0360
306	− 22.6778	22.6772	− 8.0568	7.6539
307	− 32.4820	32.4829	+ 11.6061	12.0104
308	− 49.7404	49.7368	+ 46.1766	46.5822
309	− 12.6198	12.6196	− 29.9176	29.5158
310	− 29.1226	29.1210	− 0.4202	0.0150
311	− 20.8604	20.8602	− 16.5035	16.0968
312	− 42.1142	42.1146	+ 20.8529	21.2565
313	− 28.0818	28.0825	− 4.9084	4.5063
314	− 58.4192	58.4237	+ 45.0736	45.4869
316	− 12.6862	12.6866	− 34.0390	33.6423
317	− 44.9726	44.9746	+ 11.3278	11.7346
318	− 18.7774	18.7799	− 27.4978	27.0932
319	− 28.7107	28.7124	− 13.0676	12.6602
320	− 52.9871	52.9861	+ 20.6294	21.0372
321	− 44.6824	44.6820	+ 8.1312	8.5404
322	− 9.2537	9.2540	− 42.7338	42.3347
323	− 23.1222	23.1225	− 23.7017	23.2989
324	− 48.8966	48.8964	+ 10.4186	10.8299
325	− 63.6568	63.6488	+ 27.7688	28.1861
326	− 49.6936	49.6926	+ 5.1221	5.5292
328	− 23.5301	23.5298	− 28.0803	27.6776
331	− 38.4375	38.4342	− 12.7509	12.3452
332	− 50.6451	50.6364	− 0.3450	+ 0.0553
335	− 16.6698	16.6744	− 40.8040	40.3968
338	− 10.4554	10.4573	− 47.3340	46.9338
340	− 43.9979	44.0001	− 17.6751	17.2715
341	− 53.2004	53.1942	− 10.2203	9.8192
346	− 64.1320	64.1355	− 3.1328	2.7294
348	− 14.2658	14.2683	− 45.7265	45.3252
351	− 61.7017	61.7040	− 10.1001	9.6952
353	− 52.7975	52.7991	− 19.8146	19.4103
356	− 58.1662	58.1680	− 19.3300	18.9148
362	− 20.0388	20.0394	− 46.9327	46.5338
367	− 10.2786	10.2862	− 53.2129	52.8191
372	− 22.0499	22.0550	− 49.7942	49.3851

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 21. No. 13.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
373	− 18.5731	18.5734	− 51.3606	50.9547
376	− 49.0776	49.0795	− 42.5412	42.1423
381	− 16.5942	16.5942	− 54.5088	54.1068

PLATE SEPT. 21. No. 15.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
1	− 49.1170	49.1191	+ 35.4701	35.8748
2	− 59.9066	59.9109	+ 27.5611	27.9565
5	− 8.6929	8.6945	+ 54.4890	54.8872
8	− 42.7124	42.7136	+ 33.6728	34.0684
9	− 63.0086	63.0112	+ 18.2944	18.6988
16	− 51.2698	51.2691	+ 18.5555	18.9614
18	− 23.5121	....	+ 40.1790	....
21	− 63.3538	63.3520	− 0.1557	+ 0.2364
23	− 53.6494	53.6536	+ 7.0812	7.4777
25	− 30.4464	30.4490	+ 23.6348	24.0334
26	− 21.2140	21.2206	+ 34.1179	34.5168
28	− 35.6628	35.6659	+ 16.1020	16.5046
31	− 30.6979	30.6994	+ 17.3088	17.7102
32	− 36.7512	36.7572	+ 6.6622	7.0572
33	− 37.0818	37.0789	+ 4.7561	5.1470
34	− 42.6627	42.6648	− 6.8674	6.4706
35	− 55.6002	55.6032	− 30.2710	29.8818
36	− 26.4013	26.4030	+ 11.4205	11.8160
37	− 23.4538	23.4543	+ 15.1789	15.5748
38	− 15.7433	15.7436	+ 28.2739	28.6723
39	+ 44.4588	44.4578	− 26.4164	26.0208
40	− 33.8928	33.8945	− 8.0510	7.6585
41	− 14.3978	14.3991	+ 28.3186	28.7134
42	− 37.6736	37.6768	− 18.6581	18.2623
43	− 9.7116	9.7121	+ 36.5194	36.9146
44	− 49.4924	49.4966	− 46.4935	46.0944
45	− 40.2691	40.2777	− 28.9062	28.5072
46	− 24.6770	24.6780	+ 3.0814	3.4746
47	− 6.4160	6.4169	+ 41.1723	41.5684
48	− 44.1898	44.1938	− 43.7860	43.3940
49	− 18.0964	18.0976	+ 14.5129	14.9122
50	− 38.5074	38.5084	− 33.8610	33.4614
51	− 37.3286	37.3262	− 35.0322	34.6425
52	− 38.5900	38.5977	− 39.6750	39.2830
53	− 33.9503	33.9548	− 29.9017	29.5046
54	− 6.1389	6.1372	+ 38.6876	39.0828
55	− 41.1358	41.1360	− 53.6956	53.3095
56	− 21.7540	21.7546	− 4.6776	4.2759
57	− 30.4551	30.4583	− 29.0637	28.6680
58	− 37.5420	37.5494	− 52.9124	52.5271
59	− 30.1878	30.1906	− 36.6012	36.2058
60	− 9.6236	9.6228	+ 8.4382	8.8362
61	− 12.3364	12.3380	− 5.5584	5.1592
62	− 2.9752	2.9750	+ 29.5213	29.9268

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 21. No. 15.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
63	− 7.2200	7.2226	+ 2.4791	2.8759
64	− 13.5704	13.5726	− 36.6890	36.2909
65	− 11.6972	11.7000	− 32.5688	32.1684
66	− 4.0866	4.0891	+ 12.2972	12.6930
67	− 2.8382	2.8424	+ 16.9664	17.3634
68	− 10.3191	10.3258	− 58.8944	58.4960
69	− 5.8828	5.8844	− 22.9036	22.5063
70	− 7.4811	7.4816	− 52.3900	51.9980
71	− 3.1284	3.1273	− 6.4482	6.0518
72	− 2.7294	2.7320	− 23.4406	23.0490
73	− 2.2998	2.2985	− 17.6877	17.2916
74	+ 0.1230	0.1223	+ 0.6444	1.0429
75	+ 0.8736	0.8724	+ 7.3556	7.7538
76	+ 1.2746	1.2740	+ 15.4330	15.8304
77	+ 0.6750	0.6750	− 17.2618	16.8617
78	+ 1.3156	1.3122	− 26.8139	26.4158
79	+ 2.0024	2.0024	− 0.8563	0.4587
80	+ 2.2510	2.2504	+ 14.4468	14.8477
81	+ 3.5559	3.5571	− 34.9872	34.5908
82	+ 4.8934	4.8938	− 5.7010	5.3017
83	+ 4.7031	4.7037	+ 1.5616	1.9602
84	+ 8.5974	8.5976	− 51.7160	51.3169
85	+ 5.6674	5.6666	+ 6.3335	6.7294
86	+ 10.0395	10.0376	− 51.6111	51.2112
87	+ 8.3657	8.3654	− 24.5523	24.1544
88	+ 8.0155	8.0138	− 4.0010	3.6052
89	+ 4.3340	4.3340	+ 40.3596	40.7545
90	+ 17.4270	17.4296	− 54.7114	54.3157
91	+ 14.4830	14.4812	− 28.8633	28.4624
92	+ 19.8084	19.8060	− 47.2958	46.9017
93	+ 8.9528	8.9510	+ 22.0888	22.4833
94	+ 17.0264	17.0286	− 25.0428	24.6474
95	+ 19.8668	19.8653	− 15.8566	15.4611
96	+ 27.9746	27.9745	− 51.2648	50.8710
97	+ 22.8413	22.8398	− 15.8961	15.5044
98	+ 32.2745	32.2779	− 50.2005	49.8024
99	+ 31.4007	31.4002	− 33.6478	33.2534
100	+ 13.4293	13.4296	+ 26.2734	26.6724
101	+ 26.2896	26.2862	− 11.8042	11.4066
102	+ 17.9090	17.9106	+ 15.9640	16.3664
103	+ 19.7212	19.7222	+ 11.8668	12.2680
104	+ 34.1121	34.1136	− 28.5478	28.1546
105	+ 25.0428	25.0444	+ 6.9076	7.3050
106	+ 14.0944	14.0930	+ 33.7752	34.1686
107	+ 36.5792	36.5824	− 16.1350	15.7392
108	+ 28.8884	28.8864	+ 3.3057	3.7031
109	+ 40.7564	40.7552	− 22.3410	21.9416
110	+ 27.6042	27.6060	+ 6.3598	6.7566
111	+ 36.1974	36.2004	− 10.2716	9.8811
112	+ 31.3543	31.3542	+ 0.4787	0.8776
113	+ 26.0592	26.0592	+ 12.3864	12.7829
114	+ 47.1002	47.1013	− 28.6269	28.2309
115	+ 32.7246	32.7249	+ 0.9786	1.3762

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 21. No. 15.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
116	+ 16.6670	16.6638	+ 32.8543	33.2490
117	+ 58.8743	58.8712	− 46.7780	46.3766
118	+ 40.7610	40.7621	− 12.1479	11.7510
119	+ 30.9871	30.9854	+ 10.4751	10.8746
120	+ 38.8041	38.8028	+ 1.8770	2.2706
121	+ 31.7298	31.7296	+ 15.0299	15.4228
123	+ 10.0631	10.0712	+ 49.8336	50.2216
124	+ 35.6393	35.6422	+ 14.0080	14.4061
125	+ 17.6350	17.6383	+ 39.8874	40.2914
127	+ 51.7340	51.7353	+ 6.8802	7.2768
129	+ 46.0528	46.0547	+ 21.3106	21.7104
131	+ 50.5770	50.5774	+ 21.5090	21.9046
132	+ 53.1680	53.1650	+ 22.5837	22.9834
135	+ 23.3148	23.3168	+ 46.2439	46.6392
137	+ 55.1732	55.1741	+ 25.6275	26.0228
144	+ 22.5190	22.5162	+ 52.8748	53.2692
392	− 44.8432	44.8477	+ 49.9100	50.3014
405	− 57.9398	57.9406	+ 36.8664	37.2674
406	− 62.9724	62.9662	+ 33.6474	34.0470
PLATE SEPT. 16. No. 3.				
3	+ 9.3697	9.3726	− 18.7030	18.3611
28	+ 42.8042	42.8040	− 38.0598	37.7133
31	+ 41.7666	41.7622	− 30.3638	32.7178
36	+ 47.8024	47.8064	− 28.9814	28.6334
106	+ 26.9062	26.9136	+ 12.3054	12.6493
120	+ 59.6800	59.6915	+ 35.8446	36.1786
125	+ 20.9330	20.9354	+ 16.0600	16.4026
127	+ 55.1461	55.1614	+ 48.9532	49.2999
129	+ 40.5140	40.5286	+ 43.7813	44.1255
132	+ 39.4976	39.5076	+ 50.9374	51.2748
137	+ 36.5313	36.5389	+ 53.0639	53.4056
146	+ 2.9206	2.9324	+ 30.8645	31.2168
200	− 9.8013	9.7936	+ 12.7188	13.0734
202	− 24.7440	24.7374	+ 30.6388	30.9726
210	− 10.1188	10.1120	+ 11.3050	11.6540
212	− 57.1287	57.1174	+ 57.4817	57.8244
220	− 50.0330	50.0199	+ 39.0534	39.3976
242	− 61.2132	61.2088	+ 22.4656	22.8252
258	− 63.8062	63.8029	+ 6.8769	7.2290
259	− 65.0718	65.0702	+ 6.9504	7.3101
268	− 32.9440	32.9437	+ 1.5390	1.8940
310	− 60.4174	60.4167	− 28.1404	27.7928
313	− 55.8992	55.8966	− 27.2466	26.9017
353	− 41.8591	41.8638	− 52.4696	52.1254
356	− 42.5326	42.5285	− 57.8142	57.4752
405	+ 21.2274	21.2220	− 59.5734	59.2250

TABLE II (cont'd).—CORRECTED COÖRDINATES. PLATE SEPT. 16. No. 6.

Star	X Big Image.	X Middle Image.	Y Big Image.	Y Middle Image.
62	+ 18.2387	18.2291	− 25.2614	24.8705
75	+ 36.1242	36.1118	− 38.9160	38.5349
82	+ 47.9462	47.9373	− 45.7803	45.3842
85	+ 40.3350	40.3263	− 36.4198	36.0237
100	+ 32.4818	32.4566	− 16.5029	16.1056
106	+ 27.8703	27.8518	− 10.5468	10.1505
125	+ 26.3182	26.2981	− 3.6577	3.2554
129	+ 59.7970	59.7790	+ 2.0127	2.4150
159	+ 24.1144	24.1005	+ 19.5648	19.9598
202	+ 4.4300	4.4178	+ 38.9974	39.4057
209	+ 2.3258	2.3072	+ 53.3235	53.7210
228	− 9.3321	9.3440	+ 44.6164	45.0224
242	− 27.0748	27.0846	+ 59.1022	59.4960
259	− 40.7872	40.7987	+ 50.8894	51.2789
268	− 21.9770	21.9916	+ 24.3042	24.7014
310	− 62.3798	62.4005	+ 22.8461	23.2304
313	− 58.5738	58.5843	+ 20.2616	20.6525
334	− 7.5679	7.5869	+ 1.6091	1.9964
384	− 56.8066	56.8177	− 36.5169	36.1368
389	− 54.7074	54.7160	− 56.0514	55.6719
390	− 45.3710	45.3752	− 56.9631	56.5660
405	− 27.0994	27.1084	−. 57.2066	56.8152
PLATE SEPT. 18. No. 2.				
62	+ 23.9657	23.9806	+ 17.1389	17.5390
75	+ 37.5768	....	+ 35.0590	....
82	+ 44.3961	44.4070	+ 46.9015	47.3125
83	+ 39.2008	39.2162	+ 41.8208	42.2120
100	+ 15.1702	15.1674	+ 31.3474	31.7576
162	− 25.0286	25.0230	+ 24.8081	25.2132
210	− 16.2461	16.2364	− 0.1773	+ 0.2328
228	− 45.8393	....	− 10.6354	....
242	− 60.2746	60.2636	− 28.4270	28.0264
259	− 52.0144	52.0034	− 42.1140	41.7146
313	− 21.3360	21.3260	− 59.8208	59.4102
334	− 2.8218	2.8068	− 8.7423	8.3371
338	− 2.3421	2.3320	− 17.9819	17.5749
362	+ 3.9080	....	− 25.2496	....
384	+ 35.4454	35.4567	− 57.8818	57.4850
389	+ 54.9946	55.0040	− 55.7276	55.3149
405	+ 56.0518	56.0628	− 28.1037	27.6941
PLATE SEPT. 21. No. 14.				
2	− 59.7073	59.7107	− 32.5663	32.1804
3	− 17.3198	17.3265	− 9.9694	9.5829
4	− 46.9976	47.0042	− 27.5490	27.1664
18	− 23.3108	23.3180	− 19.9662	19.5798

TABLE II (concl'd).—CORRECTED COÖRDINATES. PLATE SEPT. 21. No. 14.

Star	X		Y	
	Big Image.	Middle Image.	Big Image.	Middle Image.
26	− 21.0058	21.0164	− 26.0338	25.6524
31	− 30.5139	30.5242	− 42.8440	42.4484
62	− 2.7642	2.7620	− 30.6336	30.2528
106	+ 14.2954	14.2904	− 26.3976	26.0116
112	+ 31.5544	31.5470	− 59.7102	59.3374
125	+ 17.8434	17.8404	− 20.2830	19.9001
129	+ 46.2562	46.2560	− 38.8734	38.4839
132	+ 53.3692	53.3657	− 37.6045	37.2248
137	+ 55.3697	55.3692	− 34.5576	34.1747
146	+ 41.3192	41.3098	− 12.5214	12.1264
159	+ 32.0012	31.9855	− 1.7626	1.3751
202	+ 30.7740	30.7736	+ 25.8800	26.2618
209	+ 38.9659	38.9681	+ 37.8184	38.1951
220	+ 38.3062	38.3002	+ 51.4702	51.8494
228	+ 24.4981	24.4904	+ 39.3548	39.7352
241	+ 13.0420	13.0418	+ 34.3242	34.7086
268	+ 1.3246	1.3122	+ 30.7547	31.1293
310	− 29.2275	29.2252	+ 59.4424	59.8372
313	− 28.1895	28.1880	+ 54.9412	55.3296
315	− 7.9484	7.9458	+ 18.3913	18.7778
351	− 61.8014	61.8019	+ 49.7312	50.1126
353	− 52.9036	52.9017	+ 40.0212	40.4080
356	− 58.2748	58.2734	+ 40.5111	40.8951
383	− 4.9366	4.9418	+ 2.2576	2.6451
387	− 63.4172	63.4166	+ 1.9674	2.3538
405	− 57.7422	57.7453	− 23.2644	22.8812
406	− 62.7648	62.7782	− 26.4875	26.1058

The right ascensions and polar distances of the stars were determined from the measured coördinates according to the method given in Publication 1, Sec. IV, with one modification. Certain terms in the right ascension equation which are negligible for the 90° plates where the pole is very near the center, must be included for the 89° plates, where the pole is a degree from the center. Furthermore, since there is an obvious error in the last line on p. 27, the form given in the first equation for α on p. 28 is incorrect. The correct form and the one which has been used in the present investigation is given below, and with it is repeated for convenience the equation for polar distance.

$$\begin{aligned} \alpha &= B - \tfrac{1}{2}\rho^2 \sin B'' \cos B'' \omega^2 \sin 1'' + \tfrac{1}{2}\xi\eta\omega^2 \sin 1'' + A\,, \\ \pi &= p\omega - \tfrac{1}{2}p\rho^2 \cos^2 B''\, \omega^2 \sin^2 1'' - \tfrac{1}{2}p\,(X^2 + Y^2)\,\omega^2 \sin^2 1'' \qquad (1a) \\ &\qquad + \tfrac{1}{6}p^3\omega^3 \sin^2 1''\,. \end{aligned}$$

Since the small terms in α are negligible in any case for the 90° plates, the equations which were used in the first paper, *i. e.*, equations (1), are correct as they stand.

The plate constants were determined according to the method of Secs. V to VII. The right ascensions and polar distances of the standard stars were taken from Elkin's Heliometer Triangulation of Stars in the Vicinity of the North Pole.* All of the stars in this work, twenty-four in number, were used with the exception of Polaris, the photographic images of which were not suitable for measurement. The companion of Polaris was obscured by the primary on every plate but one, namely, Sept. 21, no. 15. The following Table III, gives the polar coördinates of the standard stars together with their numbers in the final catalogue by means of which their rectangular coördinates may be taken from Table II.

* Transactions of Yale Observatory, Vol. I, part III.

TABLE III.—POSITIONS OF STANDARD STARS.

Elkin.	No. in Cat.	Right Ascension.	N. P. D.	Annual P. M. in R. A.	Annual P. M. in N. P. D.
		° ′ ″	″	″	″
α	2	3 38 22	4232.28		
β	14	13 9 27	5678.28	+ 2.310	+ 0.037
δ	47	41 26 32	1297.25		
ε	34	31 50 45	4875.66		
ζ	42	38 43 31	5337.53		
η	101	83 37 18	4585.10		
θ	104	84 57 29	5685.48		
ι	131	116 8 33	3728.37	− 1.8070	− 0.020
κ	161	153 39 40	5621.81		
λ	169	165 22 30	6306.18		
μ	184	183 35 19	6045.41	− 0.1320	− 0.076
ν	193	193 33 33	3718.88		
ξ	199	196 34 40	6297.54		
ο	239	227 49 25	5611.39		
π	261	243 28 12	5818.21		
ρ	266	246 46 54	2685.92		
σ	304	270 30 0	6296.23		
τ	305	270 45 47	4549.94		
υ	327	284 42 21	4961.27		
φ	340	293 55 50	3735.81	− 0.7560	+ 0.006
ψ	351	298 14 30	4843.32		
χ	386	330 15 7	6034.64		
ω	387	334 27 22	3953.79		

The apparent places of the standard stars referred to the equinox of 1888.0 were computed according to the method of Sec. VI. Proper motions were taken from the following sources: for β = Bradley 65 from Harvard Annals, Vol. XVIII, p. 283; for ι = Groom. 1119, and μ = Bradley 1672 from Green. Ten Year Catalogue 1890; for $\varphi = \lambda$ Ursae Majoris from the Berliner Jahrbuch. The plate constants were determined by a comparison with the apparent places computed by equations (1*a*), quoted above, using approximate values which could easily be found. Table IV which follows contains the constants for the eight 89° plates, and Table V contains their probable errors.

TABLE IV.—PLATE CONSTANTS.

Plate.	Image.	Standard stars.	ξ	η	A	ω
			mm.	*mm.*	° ′ ″	″
Sept. 16	Big	$\alpha, \beta, \delta, \phi, \psi, \chi, \omega$.	− 1.4046	+ 60.9458	67 30 44	59.8572
no. 2	Mid.		− 1.3959	+ 61.3492	67 30 45	.8579
Sept. 16	Big	$\iota, \kappa, \lambda, \mu, \nu, \rho$.	− 0.9446	− 58.8603	67 29 59	.8617
no. 4	Mid.		− 0.9310	− 58.4621	67 30 24	.8598
Sept. 16	Big	$\mu, \nu, \xi, o, \pi, \rho, \phi$.	+ 1.1355	− 57.8153	112 38 24	.8606
no. 5	Mid.		+ 1.1323	− 57.4296	112 38 43	.8596
Sept. 16	Big	$\alpha, \beta, \delta, \epsilon, \zeta, \omega$.	+ 1.3521	+ 62.1408	112 37 46	.8551
no. 7	Mid.		+ 1.3466	+ 62.5421	112 37 34	.8519
Sept. 18	Big	$\rho, \sigma, \tau, \upsilon, \phi, \psi, \chi, \omega$.	− 2.8353	+ 60.5764	22 27 48	.8579
no. 1	Mid.		− 2.8232	+ 60.9637	22 28 23	.8566
Sept. 18	Big	$\delta, \eta, \theta, \iota, \kappa, \nu$.	− 2.2755	− 60.3089	22 29 17	.8574
no. 3	Mid.		− 2.2685	− 59.9306	22 28 52	.8538
Sept. 21	Big	$\nu, o, \pi, \rho, \sigma, \tau, \upsilon, \phi, \psi$.	+ 2.5529	− 58.7921	155 30 1	.8608
no. 13	Mid.		+ 2.5464	− 58.3941	155 30 29	.8604
Sept. 21	Big	$\alpha, \delta, \epsilon, \zeta, \eta, \theta, \iota$.	+ 2.2749	+ 61.2484	155 26 49	.8557
no. 15	Mid.		+ 2.2736	+ 61.6462	155 26 50	.8539

The numbers in the last column of Table V are the probable errors of the single determinations of polar distance for each plate, and r cosec π will be the probable error of one determination of right ascension for any star of polar distance π.

TABLE V.—PROBABLE ERRORS OF PLATE CONSTANTS.

Plate.	Im.	Probable error of			r_0
		ξ or η	ω	A	
		mm.	″	″	″
Sept. 16 no. 2	Big	± .0015	± .0012	± 4	± .141
	Mid.	.0015	.0012	4	.139
Sept. 16 no. 4	Big	.0015	.0011	4	.125
	Mid.	.0018	.0013	5	.153
Sept. 16 no. 5	Big	.0019	.0014	5	.179
	Mid.	.0019	.0014	5	.179
Sept. 16 no. 7	Big	.0022	.0018	6	.154
	Mid.	.0025	.0020	7	.172
Sept. 18 no. 1	Big	.0024	.0018	6	.206
	Mid.	.0022	.0017	6	.184
Sept. 18 no. 3	Big	.0018	.0015	5	.190
	Mid.	.0017	.0014	5	.176
Sept. 21 no. 13	Big	.0013	.0010	3	.127
	Mid.	.0011	.0008	3	.102
Sept. 21 no. 15	Big	.0019	.0016	5	.194
	Mid.	.0018	.0014	5	.174

In determining the right ascensions and polar distances of the unknown stars the process employed was the reverse of that used for the standard stars and similar to that of Publication 1, Sec. VIII, in each detail with one exception in the case of annual aberration. Here a second step was necessary, since the formulas given on p. 32, Publication 1, have in the second member the quantities α_1 and π_1 which are supposed to be free from the effects of aberration, whereas these quantities as derived from the rectangular coördinates have been freed from the effects of refraction only. The following differential forms derived by Jacoby for a similar case give the necessary corrections. (Columbia Contributions, no. 21, p. 25.)

$$\alpha - \alpha_1 = \frac{2(\pi - \pi_1)(\alpha - \alpha_1)}{\pi},$$

$$\pi - \pi_1 = -\pi(\alpha - \alpha_1)^2 \sin^2 1''.$$

In combining the results from the eight plates into a single preliminary catalogue, each plate was given weight unity. The means of the separate results from the four 90° plates were treated as if they also were obtained from a single plate and were given weight unity in the final combination. The right ascensions and polar distances derived from each of the four 90° plates had previously been reduced to the mean standard of the four plates by the method described in Publication 1, Sec. X, pp. 68–72. Hereafter when reference is made to the 90° plate the mean result from the four plates is to be understood.

Table VI contains the preliminary catalogue. The first column contains the catalogue number, the second the number of plates upon which each star appears, the third the mean right ascension from all the plates, and the remaining nine columns the residuals in the sense plate minus mean. The corresponding values in polar distance are given on the opposite pages.

TABLE VI.—Preliminary Catalogue.

Star	Number Plates	Mean R. A. 1888.0			$\Delta\alpha$ sin π								
					90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		°	′	″	″	″	″	″	″	″	″	″	″
1	5	1	45	54	+.17	−.05			+.05	−.34			+.19
2	4	3	38	14	−.08	+.10			−.15				+.17
3	1	3	50	12	.00								
4	3	5	12	44	+.26	−.12			−.12				
5	5	5	41	53	+.13	−.04			+.31	−.42			+.02
6	2	5	46	31		−.11			+.11				
7	2	6	22	46		−.09			+.12				
8	5	6	36	31	+.16	−.03			+.08	−.52			+.33
9	3	8	33	24		−.32			+.09				+.23
10	2	9	10	4		−.32			+.29				
11	2	10	18	1		+.13			−.13				
12	1	10	32	54					.00				
13	1	10	49	4					.00				
14	2	13	9	31		−.08			+.08				
15	2	13	37	31		−.03			+.03				
16	3	13	45	15		−.12			+.02				+.08
17	2	14	6	30		−.03			.00				
18	3	14	9	49	−.01				−.34				+.36
19	1	14	23	59					.00				
20	1	18	15	23					.00				
21	3	18	20	59		−.21			−.16				+.34
22	1	18	43	33					.00				
23	1	19	18	48									.00
24	1	21	18	1					.00				
25	4	24	5	32	+.25	−.46			+.06				+.18
26	2	24	6	32	+.25								−.26
27	1	24	26	45					.00				
28	4	25	7	52	+.53	−.15			−.17				−.22
29	1	27	27	2					.00				
30	1	27	51	56					.00				
31	4	28	16	1	+.05	−.50			+.13				
32	2	29	38	50					−.23				+.23
33	2	30	21	33					−.10				+.10
34	2	31	50	43					−.07				+.07
35	2	32	59	51					−.19				+.19
36	5	35	16	5	+.17	+.03			−.02		−.44		+.27
37	4	35	59	0	+.06	−.40			−.09				+.42
38	4	36	24	40	+.04	−.19			+.11				+.01
39	2	37	14	31					+.03				−.03
40	2	37	42	20					−.02				+.02
41	4	38	12	38	+.11	−.53			+.23				+.16
42	2	38	43	36					−.05				+.08
43	5	39	4	38	+.16	−.17			+.24		−.49		+.25
44	2	39	39	57					−.18				+.18
45	2	40	2	32					−.20				+.23

TABLE VI.—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean N. P. D. 1888.0	$\Delta\pi$								
			90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		″	″	″	″	″	″	″	″	″	″
1	5	3441.27	+.09	−.19			−.45	+.73			−.17
2	4	4232.27	−.15	+.02			−.06				+.18
3	1	1356.20	.00								
4	3	3420.51	+.52	−.30			−.21				
5	5	768.55	−.42	−.08			+.13	−.02			+.40
6	2	5852.74		+.27			−.26				
7	2	4701.11		+.01			−.01				
8	5	3156.67	+.13	−.13			−.32	+.41			−.10
9	3	4675.09		−.12			−.14				+.25
10	2	4333.50		+.18			−.18				
11	2	6738.66		+.59			−.59				
12	1	5780.56					.00				
13	1	3818.15					.00				
14	2	5678.12		+.05			−.06				
15	2	5395.28		+.29			−.28				
16	3	4095.08		−.11			+.17				−.07
17	2	5788.53		+.31			−.31				
18	3	1988.54	+.10				−.27				+.17
19	1	6964.01					.00				
20	1	4964.64					.00				
21	3	5374.89		+.27			−.07				−.20
22	1	7275.93					.00				
23	1	4654.92									.00
24	1	6758.87					.00				
25	4	2976.90	+.17	−.22			+.08				−.03
26	2	2140.48	−.05								+.05
27	1	6254.01					.00				
28	4	3522.70	+.32	+.12			−.04				−.42
29	1	3490.55					.00				
30	1	6666.59					.00				
31	4	3279.90	+.46	−.18			−.18				−.09
32	2	4008.12					−.02				+.03
33	2	4112.62					+.05				−.05
34	2	4875.94					+.19				−.20
35	2	6473.08					+.14				−.15
36	5	3431.23	+.40	−.07			−.02		−.13		−.19
37	4	3148.15	+.41	−.04			−.22				−.16
38	4	2238.00	+.20	−.40			−.07				+.26
39	2	5936.36					−.10				+.10
40	2	4668.72					−.01				.00
41	4	2197.85	+.03	−.16			+.01				+.12
42	2	5337.19					+.17				−.17
43	5	1633.19	+.05	+.02			+.05		−.09		−.02
44	2	7144.65					+.15				−.15
45	2	5956.76					+.27				−.28

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean R. A. 1888.0			$\Delta\alpha$ sin π								
					90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		°	′	″	″	″	″	″	″	″	″	″	″
46	2	40	22	15					−.27				+.27
47	5	41	26	35	+.08	+.01			+.02		+.14		−.25
48	2	41	27	46					−.40				+.40
49	5	41	38	1	+.16	+.06			+.16		−.40		−.02
50	2	42	6	30					−.06				+.03
51	2	42	57	48					+.03				.00
52	2	43	17	0					+.16				−.13
53	2	43	38	18					−.20				+.17
54	4	44	28	28	+.15	−.01			−.10				−.03
55	2	44	39	4					−.47				+.43
56	2	45	14	23					−.10				+.10
57	2	45	23	37					−.14				+.14
58	2	46	6	59					−.04				+.04
59	2	46	58	28					.00				.00
60	1	52	33	7									.00
61	2	52	57	12					−.04				+.04
62	2	55	44	59	+.21								−.21
63	2	56	6	11					−.05				+.05
64	2	56	9	27					+.06				−.06
65	1	56	52	24									.00
66	5	57	50	51	+.07	+.06			−.06		−.29		+.25
67	4	58	38	53	−.01				+.01		−.27		+.26
68	1	59	23	14									.00
69	2	59	48	8					−.26				+.24
70	1	60	27	52									.00
71	3	60	45	19					+.02		−.22		+.22
72	1	61	58	5									.00
73	2	62	1	27					−.23				+.23
74	4	63	16	54	−.18				+.14		−.11		+.16
75	4	63	48	50	+.06				−.08		−.06		+.06
76	1	64	1	51									.00
77	3	64	10	48					.00		−.28		+.28
78	1	64	44	15									.00
79	1	65	4	41									.00
80	1	65	15	46									.00
81	1	66	8	15									.00
82	4	67	35	12	+.08				.00		−.21		+.17
83	4	67	39	56	+.19				−.10		−.20		+.14
84	1	68	35	42									.00
85	4	68	52	8	+.05				−.05		−.02		+.03
86	1	69	19	44									.00
87	1	69	26	15									.00
88	1	70	23	12									.00
89	5	70	48	0	+.14		−.37		+.29		−.28		+.23
90	1	72	50	57									.00

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean N. P. D 1888.0	Δπ								
			90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		″	″	″	″	″	″	″	″	″	″
46	2	3826.14					+.09				−.09
47	5	1297.20	+.22	−.07			+.02		+.04		−.19
48	2	6864.29					+.17				−.17
49	5	3039.63	+.01	+.09			−.07		−.04		+.03
50	2	6183.26					+.10				−.11
51	2	6220.84					+.34				−.34
52	2	6506.55					+.19				−.19
53	2	5859.74					+.13				−.12
54	4	1427.98	−.18	−.11			+.13				+.17
55	2	7343.74					+.41				−.41
56	2	4187.76					+.18				−.17
57	2	5738.26					+.21				−.22
58	2	7226.14					+.51				−.50
59	2	6159.32					+.46				−.47
60	1	3226.03									.00
61	2	4079.58					+.24				−.23
62	2	1909.41	+.05								−.05
63	2	3548.90					+.29				−.30
64	2	5924.64					+.15				−.14
65	1	5663.30									.00
66	5	2939.56	+.14	+.07			+.22		−.30		−.11
67	4	2652.74	+.08				+.20		−.18		−.09
68	1	7216.55									.00
69	2	5046.00					+.13				−.14
70	1	6812.77									.00
71	3	4049.68					+.35		−.33		−.01
72	1	5063.06									.00
73	2	4717.62					+.10				−.10
74	4	3614.02	+.22				+.16		−.28		−.10
75	4	3210.67	+.35				+.35		−.82		+.11
76	1	2726.66									.00
77	3	4684.80					+.21		−.06		−.16
78	1	5255.91									.00
79	1	3701.29									.00
80	1	2784.79									.00
81	1	5745.37									.00
82	4	3994.26	+.20				+.27		−.27		−.21
83	4	3559.18	+.14				+.33		−.38		−.10
84	1	6756.63									.00
85	4	3276.67	+.25				+.32		−.48		−.10
86	1	6755.63									.00
87	1	5132.52									.00
88	1	3904.08									.00
89	5	1238.76	+.36		−.26		+.45		−.65		+.09
90	1	6983.98									.00

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean R. A. 1888.0			$\Delta\alpha \sin \pi$								
					90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		°	′	″	″	″	″	″	″	″	″	″	″
91	1	73	6	31									.00
92	1	74	34	54									.00
93	5	75	0	17	+.32		−.54		−.03		−.14		+.40
94	2	75	5	56							−.10		+.10
95	2	78	15	16							−.23		+.23
96	1	78	17	5									.00
97	2	80	20	24							−.05		+.07
98	1	80	29	29									.00
99	2	82	29	16							−.17		+.20
100	2	83	5	16	.00								.00
101	2	83	37	16							−.13		+.11
102	1	84	28	5									.00
103	1	84	53	1									.00
104	2	84	57	26							+.03		−.03
105	1	88	10	41									.00
106	2	88	43	32	+.11								−.12
107	2	89	21	36							+.07		−.10
108	2	90	7	14							−.14		+.16
109	2	90	10	29							+.27		−.27
110	2	90	13	30							−.17		+.17
111	2	90	49	58							+.09		−.12
112	3	91	1	52	+.06						−.04		−.02
113	2	91	24	54							−.18		+.18
114	2	91	58	6							+.26		−.23
115	2	92	16	4							−.02		+.04
116	5	92	21	0	+.24		−.44		−.30		+.22		+.27
117	2	93	6	42							+.10		−.07
118	2	93	8	13							+.12		−.12
119	1	94	58	10									.00
120	4	97	5	0	−.26		−.44				+.22		+.24
121	5	98	0	13	+.06		−.24		−.51		+.42		+.30
122	1	99	38	29							.00		
123	5	100	1	26	+.43		−.03		−.30		−.12		+.02
124	5	100	44	47	+.27		−.24		−.46		+.44		+.02
125	2	101	19	21	.00								.00
126	1	104	55	58							.00		
127	4	107	49	10	+.29		−.65				+.15		+.21
128	1	110	25	59							.00		
129	4	113	12	10	−.03		−.42				+.51		−.02
130	2	115	13	10			−.21				+.18		
131	4	116	8	40	−.14		+.09				+.31		−.29
132	4	118	21	53	−.23		−.14				+.45		−.09
133	1	119	11	38							.00		
134	2	119	14	40			−.20				+.18		
135	5	120	18	56	+.10		+.10	−.60			+.35		+.06

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean N. P. D. 1888.0	Δπ 90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		″	″	″	″	″	″	″	″	″	″
91	1	5426.54	......	......	......	......	......	......	......	......	.00
92	1	6565.12	......	......	......	......	......	......	......	......	.00
93	5	2360.20	−.01	......	+.28	......	+.30	......	−.53	......	−.02
94	2	5223.14	......	......	......	......	......	......	−.29	......	+.29
95	2	4716.57	......	......	......	......	......	......	−.25	......	+.25
96	1	6891.66	......	......	......	......	......	......	......	......	.00
97	2	4761.43	......	......	......	......	......	......	−.43	......	+.43
98	1	6891.71	......	......	......	......	......	......	......	......	.00
99	2	5924.52	......	......	......	......	......	......	−.26	......	+.27
100	2	2179.08	−.06	......	......	......	......	......	......	......	+.05
101	2	4585.44	......	......	......	......	......	......	−.01	......	.00
102	1	2849.39	......	......	......	......	......	......	......	......	.00
103	1	3116.81	......	......	......	......	......	......	......	......	.00
104	2	5685.52	......	......	......	......	......	......	−.18	......	+.19
105	1	3508.76	......	......	......	......	......	......	......	......	.00
106	2	1771.80	−.04	......	......	......	......	......	......	......	+.05
107	2	5049.14	......	......	......	......	......	......	−.17	......	+.16
108	2	3798.78	......	......	......	......	......	......	+.08	......	−.09
109	2	5490.37	......	......	......	......	......	......	−.37	......	+.37
110	2	3600.64	......	......	......	......	......	......	+.04	......	−.05
111	2	4720.56	......	......	......	......	......	......	−.24	......	+.23
112	3	4014.98	+1.03	......	......	......	......	......	−.76	......	−.28
113	2	3234.52	......	......	......	......	......	......	−.33	......	+.34
114	2	5994.47	......	......	......	......	......	......	+.03	......	−.03
115	2	4024.14	......	......	......	......	......	......	+.05	......	−.04
116	5	1887.12	+.17	......	−.12	......	+.38	......	−.43	......	−.01
117	2	7282.10	......	......	......	......	......	......	−.41	......	+.41
118	2	4942.84	......	......	......	......	......	......	−.36	......	+.35
119	1	3473.50	......	......	......	......	......	......	......	......	.00
120	4	4154.93	−.36	......	+.51	......	......	......	−.27	......	+.12
121	5	3262.89	+.24	......	−.25	......	+.34	......	−.30	......	−.04
122	1	6825.05	......	......	......	......	......	......	.00	......	......
123	5	809.18	+.17	......	−.47	......	+.65	......	−.43	......	+.09
124	5	3444.25	+.13	......	−.18	......	+.32	......	−.24	......	−.02
125	2	1556.79	+.09	......	......	......	......	......	......	......	−.09
126	1	6934.39	......	......	......	......	......	......	.00	......	......
127	4	4382.62	+.21	......	+.06	......	......	......	−.28	......	.00
128	1	6680.53	......	......	......	......	......	......	.00	......	......
129	4	3530.38	+.33	......	−.11	......	......	......	−.41	......	+.20
130	2	4713.61	......	......	+.07	......	......	......	−.07	......	......
131	4	3728.24	−.12	......	+.13	......	......	......	+.06	......	−.09
132	4	3809.88	+.23	......	+.26	......	......	......	−.53	......	+.06
133	1	4444.69	......	......	......	......	......	......	.00	......	......
134	2	4620.28	......	......	+.07	......	......	......	−.06	......	......
135	5	1531.04	+.14	......	+.13	+.07	......	......	−.43	......	+.10

TABLE [illegible] (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean R.A. [illegible]			Δα [illegible]								
					10 [illegible]	10 no. [illegible]	10 no. 4	10 no. 5	10 no. [illegible]	18 no. [illegible]	18 no. 3	21 no. 13	21 no. 15
136	1	[illegible]	[illegible]	[illegible]							00		
137	4	[illegible]	[illegible]	[illegible]	− [illegible]		00				− 34		−.25
138	1	[illegible]	[illegible]	[illegible]							00		
139	2	[illegible]	[illegible]	[illegible]			− [illegible]				− 23		
140	2	[illegible]	[illegible]	[illegible]			− 45				− 45		
141	2	[illegible]	[illegible]	[illegible]			− [illegible]				− 32		
142	2	[illegible]	[illegible]	[illegible]			− [illegible]				− 41		
143	1	[illegible]	[illegible]	[illegible]			00						
144	4	[illegible]	[illegible]	[illegible]	− [illegible]		− [illegible]	− [illegible]			− 24		−.32
145	2	[illegible]	[illegible]	[illegible]			− 30				− 30		
146	1	[illegible]	[illegible]	[illegible]	00								
147	2	[illegible]	[illegible]	[illegible]			− [illegible]				− 17		
148	1	[illegible]	[illegible]	[illegible]			00						
149	2	[illegible]	[illegible]	[illegible]			− 30				− 30		
150	1	[illegible]	[illegible]	[illegible]			00						
151	2	[illegible]	[illegible]	[illegible]			− [illegible]				− 43		
152	1	[illegible]	[illegible]	[illegible]			00						
153	1	[illegible]	[illegible]	[illegible]			00						
154	1	[illegible]	[illegible]	[illegible]			00						
155	2	[illegible]	[illegible]	[illegible]			− [illegible]				− 06		
156	1	[illegible]	[illegible]	[illegible]			00						
157	3	[illegible]	[illegible]	[illegible]			− [illegible]	− [illegible]			− 50		
158	2	[illegible]	[illegible]	[illegible]			− 35				− 35		
159	2	[illegible]	[illegible]	[illegible]	− [illegible]		− [illegible]						
160	2	[illegible]	[illegible]	[illegible]			− [illegible]				− 14		
161	2	153	30	[illegible]			− [illegible]				− 05		
162	4	154	4	51	− [illegible]		− [illegible]	− [illegible]			− 43		
163	1	156	43	1			00						
164	2	158	10	31			− [illegible]	− [illegible]					
165	2	162	9	43			− 20	− 20					
166	3	162	10	20			− [illegible]	− 55			− 50		
167	1	162	24	12			00						
168	5	163	25	6	− 25		00	− 45			− 30	− 08	
169	1	165	22	24			00						
170	2	167	4	8			− 31	− 34					
171	2	167	51	4			− [illegible]	− [illegible]					
172	1	167	52	8			00						
173	2	169	10	39			− [illegible]	− [illegible]					
174	2	169	57	16			− 23	− 23					
175	5	171	19	0	−.04		− 45	−.44			− 80	−.94	
176	2	171	42	37			+ 48	−.48					
177	5	172	39	15	−.08		− 14	−.35			− 50	−.22	
178	2	174	4	22			−.22	− 30					
179	2	175	9	19			+.08	−.08					
180	2	177	13	30			+.22	−.19					

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean N. P. D. 1888.0	$\Delta\pi$								
			90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		″	″	″	″	″	″	″	″	″	″
136	1	6634.10	......	......	......	......	......	......	.00	......	......
137	4	3802.28	+.17	......	+.16	......	......	......	−.28	......	−.07
138	1	6314.87	......	......	......	......	......	......	.00	......	......
139	2	4722.49	......	......	+.36	......	......	......	−.36	......	......
140	2	6663.00	......	......	+.28	......	......	......	−.28	......	......
141	2	5963.29	......	......	+.39	......	......	......	−.39	......	......
142	2	7530.94	......	......	+.50	......	......	......	−.50	......	......
143	1	5422.97	......	......	.00	......	......	......	......	......	......
144	5	1297.90	+.36	......	−.11	−.14	......	......	−.32	......	+.20
145	2	4824.62	......	......	+.14	......	......	......	−.15	......	......
146	1	2450.81	.00	......	......	......	......	......	......	......	......
147	2	4343.32	......	......	+.25	......	......	......	−.25	......	......
148	1	6088.97	......	......	00	......	......	......	......	......	......
149	2	6235.06	......	......	+.15	......	......	......	−.16	......	......
150	1	6549.54	......	......	.00	......	......	......	......	......	......
151	2	5523.69	......	......	+.30	......	......	......	−.30	......	......
152	1	3071.25	......	......	.00	......	......	......	......	......	......
153	1	3037.83	......	......	.00	......	......	......	......	......	......
154	1	4848.91	......	......	.00	......	......	......	......	......	......
155	2	4245.51	......	......	+.37	......	......	......	−.37	......	......
156	1	5806.96	......	......	.00	......	......	......	......	......	......
157	3	3919.54	......	......	+.18	+.13	......	......	−.30	......	......
158	2	5111.14	......	......	+.44	......	......	......	−.44	......	......
159	2	1766.22	+.27	......	−.28	......	......	......	......	......	......
160	2	4956.66	......	......	+.17	......	......	......	−.18	......	......
161	2	5626.78	......	......	+.08	......	......	......	−.08	......	......
162	4	2013.98	+.26	......	+.06	−.26	......	......	−.08	......	......
163	1	3417.47	......	......	.00	......	......	......	......	......	......
164	2	4425.78	......	......	−.13	+.13	......	......	......	......	......
165	2	5392.96	......	......	−.14	+.14	......	......	......	......	......
166	3	4907.01	......	......	+.16	+.29	......	......	−.46	......	......
167	1	3324.30	......	......	.00	......	......	......	......	......	......
168	5	2301.01	+.08	......	−.07	+.07	......	......	+.21	−.29	......
169	1	6306.05	......	......	.00	......	......	......	......	......	......
170	2	5820.30	......	......	−.34	+.33	......	......	......	......	......
171	2	5165.76	......	......	+.28	−.27	......	......	......	......	......
172	1	6886.11	......	......	.00	......	......	......	......	......	......
173	2	5414.66	......	......	+.14	−.15	......	......	......	......	......
174	2	6036.96	......	......	+.19	−.19	......	......	......	......	......
175	5	2259.70	+.13	......	−.11	−.20	......	......	+.28	−.11	......
176	2	6094.27	......	......	−.03	+.03	......	......	......	......	......
177	5	1617.28	+.27	......	+.06	−.30	......	......	+.12	−.16	......
178	2	5745.32	......	......	−.17	+ 16	......	......	......	......	......
179	2	4364.62	......	......	+.08	−.07	......	......	......	......	......
180	2	5540.09	......	......	−.12	+.12	......	......	......	......	......

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean R. A. 1888.0 °	'	"	Δα sin z 90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		°	'	"	"	"	"	"	"	"	"	"	"
181	2	177	29	14	......	......	+.11	−.11	......	......	......	......	......
182	4	180	26	28	+.05	......	+.31	−.34	......	......	......	−.03	......
183	2	180	44	29	......	......	+.39	−.39	......	......	......	......	......
184	2	183	35	25	......	......	+.09	−.09	......	......	......	......	......
185	2	183	48	54	......	......	+.11	−.14	......	......	......	......	......
186	2	185	13	44	......	......	+.22	−.19	......	......	......	......	......
187	1	185	43	22	......	......	.00	......	......	......	......	......	......
188	2	188	7	47	......	......	−.04	+.04	......	......	......	......	......
189	4	191	12	58	+.37	......	+.46	−.27	......	......	......	−.58	......
190	4	191	16	36	+.16	......	+.35	−.01	......	......	......	−.48	......
191	5	191	21	58	+.11	......	+.11	−.24	......	......	+.32	−.30	......
192	2	191	53	11	......	......	+.20	−.20	......	......	......	......	......
193	5	193	33	28	+.05	......	−.05	−.29	......	......	+.34	−.04	......
194	2	194	48	34	......	......	+.14	−.14	......	......	......	......	......
195	2	194	52	32	......	......	+.15	−.15	......	......	......	......	......
196	4	195	34	52	+.31	......	+.48	−.09	......	......	......	−.72	......
197	3	195	39	2	......	......	+.34	−.11	......	......	......	−.21	......
198	2	196	25	52	......	......	+.17	−.17	......	......	......	......	......
199	1	196	34	39	......	......	......	.00	......	......	......	......	......
200	5	196	51	59	.00	......	+.24	−.30	......	......	+.79	−.74	......
201	3	197	5	20	......	......	+.04	+.16	......	......	......	−.20	......
202	3	197	9	8	+.36	......	+.07	......	......	......	......	−.43	......
203	2	198	23	10	......	......	+.10	−.12	......	......	......	......	......
204	2	199	9	8	......	......	......	+.06	......	......	......	−.04	......
205	3	199	30	38	......	......	+.31	+.29	......	......	......	−.62	......
206	3	200	9	24	......	......	+.25	.00	......	......	......	−.23	......
207	3	200	18	2	......	......	+.61	.00	......	......	......	−.63	......
208	1	200	43	12	......	......	......	.00	......	......	......	......	......
209	4	200	59	20	+.33	......	+.24	.00	......	......	......	−.60	......
210	1	201	43	29	.00	......	......	......	......	......	......	......	......
211	1	202	17	14	......	......	......	.00	......	......	......	......	......
212	4	202	47	34	+.14	......	+.28	−.12	......	......	......	−.34	......
213	1	202	55	17	......	......	......	.00	......	......	......	......	......
214	3	203	48	49	......	......	+.20	+.16	......	......	......	−.36	......
215	3	203	50	13	......	......	+.36	−.13	......	......	......	−.25	......
216	1	205	31	24	......	......	......	.00	......	......	......	......	......
217	3	206	4	26	......	......	+.70	−.06	......	......	......	−.64	......
218	2	206	30	35	......	......	......	+.44	......	......	......	−.44	......
219	1	208	52	46	......	......	......	.00	......	......	......	......	......
220	3	210	18	32	+.07	......	......	+.14	......	......	......	−.22	......
221	1	210	49	28	......	......	......	.00	......	......	......	......	......
222	2	211	4	40	......	......	......	+.16	......	......	......	−.14	......
223	5	211	36	53	−.32	......	−.02	+.19	......	+.41	......	−.27	......
224	2	212	29	2	......	......	......	+.36	......	......	......	−.36	......
225	2	213	31	18	......	......	......	+.30	......	......	......	−.32	......

TABLE VI (cont'd).—Preliminary Catalogue.

Star	Number Plates	Mean N. P. D. 1888.0	$\Delta\pi$ 90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		″	″	″	″	″	″	″	″	″	″
181	2	4558.98			+.16	−.17					
182	4	2687.05	+.22		+.13	−.42				+.06	
183	2	3061.13			+.02	−.02					
184	2	6045.46			−.08	+.09					
185	2	5756.56			+.11	−.11					
186	2	5470.48			+.19	−.18					
187	1	3246.20			.00						
188	2	4176.67			−.03	+.03					
189	4	2066.79	+.30		.00	−.30				.00	
190	4	2540.48	+.12		+.30	−.38				−.03	
191	5	2547.54	+.26		+.42	−.14			−.72	+.19	
192	2	5091.96			+.07	−.08					
193	5	3718.97	+.05		+.08	−.16			−.05	+.09	
194	2	5482.14			+.14	−.14					
195	2	4456.26			+.10	−.11					
196	4	2273.83	+.17		+.41	−.24				−.33	
197	3	3201.28			−.12	+.10				+.02	
198	2	4928.95			−.17	+.17					
199	1	6297.57				.00					
200	5	861.83	+.05		+.04	−.29			+.44	−.26	
201	3	4173.84			−.07	−.21				+.28	
202	3	2258.38	+.21		+.35					−.55	
203	2	5009.06			+.03	−.02					
204	2	4410.22				−.12				+.11	
205	3	4514.47			−.01	−.11				+.13	
206	3	3636.80			+.21	−.02				−.20	
207	3	4236.12			+.19	−.64				+.45	
208	1	4517.17				.00					
209	4	3106.94	+.10		+.39	−.60				+.10	
210	1	811.18	.00								
211	1	6750.56				.00					
212	4	4756.18	−.30		+.18	−.04				+.18	
213	1	4877.39				.00					
214	3	3634.03			+.24	−.14				−.10	
215	3	4011.33			−.11	−.29				+.41	
216	1	5082.95				.00					
217	3	3400.32			+.04	−.12				+.07	
218	2	4253.08				−.19				+.19	
219	1	6066.07				.00					
220	3	3710.63	−.01			−.10				+.12	
221	1	6397.42				.00					
222	2	5570.86				−.01				+.01	
223	5	1657.65	+.35		+.29	−.39		−.01		−.24	
224	2	6118.80				.00				.00	
225	2	5552.30				−.04				+.05	

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Number Plates	Mean R. A. 1888.0			$\Delta\alpha$ sin π								
				90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
	°	′	″	″	″	″	″	″	″	″	″	″
2	213	46	19				+.36				−.36	
4	215	48	31	+.01		+.50	−.14				−.37	
4	215	53	28	+.08		+.29	+.21				−.57	
2	215	58	32				+.40				−.38	
2	216	26	34				+.22				−.22	
2	216	34	38				+.23				−.23	
4	217	21	23	+.30		+.08	+.12				−.50	
2	218	8	57				+.21				−.21	
2	223	49	56				+.32				−.32	
2	224	30	28				+.16				−.14	
2	224	54	50				−.16				+.14	
2	225	18	59				+.09				−.09	
1	226	31	21								.00	
2	227	49	28				+.16				−.19	
2	228	6	40				+.27				−.24	
2	228	10	58	+.11							−.12	
5	228	28	51	−.08		+.06	+.08		+.15		−.25	
4	229	25	29	−.24			+.32		+.20		−.28	
1	229	42	13								.00	
2	230	19	14				+.48				−.45	
1	231	3	22								.00	
2	231	31	0				+.14				−.11	
1	231	58	21								.00	
1	236	26	9								.00	
5	237	22	4	−.23		+.01	+.23		+.31		−.32	
1	237	32	28								.00	
4	238	14	44	−.11			+.30		−.02		−.17	
1	238	22	44								.00	
1	238	27	6								.00	
1	240	3	16								.00	
1	240	30	53								.00	
1	241	25	23								.00	
4	242	43	22	−.18			+.41		−.14		−.09	
4	242	44	42	+.08			+.13		+.08		−.32	
3	242	58	50				+.07		−.07		−.02	
2	243	28	14				−.14				+.17	
1	243	50	26								.00	
1	243	53	6								.00	
2	244	10	30				−.06				+.08	
4	244	34	42	−.11			+.24		+.01		−.15	
5	246	47	5	+.01		+.01	+.35		−.27		−.13	
2	247	34	42				−.11				+.11	
3	247	35	41	−.07					+.17		−.09	
1	247	56	12								.00	
1	248	20	15								.00	

TABLE VI (cont'd).— PRELIMINARY CATALOGUE.

Star	Number Plates	Mean N. P. D. 1888.0	$\Delta\pi$								
			90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		″	″	″	″	″	″	″	″	″	″
226	2	5776.84				+.04				−.05	
227	4	2185.51	+.13		+.30	−.43				.00	
228	4	2654.86	+.27		+.30	−.45				−.12	
229	2	5543.74				−.06				+.05	
230	2	6360.33				+.20				−.20	
231	2	4662.68				−.31				+.32	
232	4	2425.80	+.07		+.22	−.05				−.24	
233	2	4842.60				−.08				+.08	
234	2	4150.28				+.09				−.10	
235	2	4209.48				−.15				+.15	
236	2	5596.56				+.15				−.16	
237	2	6485.89				−.45				+.45	
238	1	2390.02								.00	
239	2	5611.26				+.05				−.05	
240	2	7096.93				+.06				−.06	
241	2	2102.64	+.20							−.20	
242	5	3834.59	−.17		+.10	−.23		+.17		+.11	
243	4	1374.02	+.21		−.44			+.52		−.31	
244	1	5929.39								.00	
245	2	6516.76			−.14					+.14	
246	1	7269.19								.00	
247	2	5672.97			−.03					+.03	
248	1	4405.96								.00	
249	1	1771.32								.00	
250	5	1153.67	+.27		+.11	−.28		+.23		−.35	
251	1	4361.29								.00	
252	4	1912.58	+.29			−.37		+.27		−.18	
253	1	3393.05								.00	
254	1	3746.22								.00	
255	1	6984.37								.00	
256	1	6305.97								.00	
257	1	3733.07								.00	
258	4	3793.36	−.28			−.23		+.49		+.03	
259	4	3868.84	−.09			−.16		+.24		+.01	
260	3	3804.21				−.12		−.06		+.18	
261	2	5818.24				+.01				−.02	
262	1	5826.46								.00	
263	1	3099.19								.00	
264	2	5817.20				+.17				−.18	
265	4	2811.28	−.06			−.02		+.32		−.25	
266	5	2686.10	−.02		−.15	−.10		+.44		−.17	
267	2	5506.64				−.03				+.02	
268	3	1932.48	+.02					+.30		−.31	
269	1	1795.55								.00	
270	1	4136.04								.00	

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean R. A. 1888.0 °	′	″	$\Delta\alpha \sin x$ 90° ″	16 no. 2 ″	16 no. 4 ″	16 no. 5 ″	16 no. 7 ″	18 no. 1 ″	18 no. 3 ″	21 no. 13 ″	21 no. 15 ″
271	1	248	42	1								.00	
272	3	249	3	45				−.10		−.18		−.08	
273	1	249	14	54								.00	
274	1	250	23	33								.00	
275	1	251	28	43								.00	
276	1	253	1	33								.00	
277	1	254	30	41								.00	
278	1	254	36	25								.00	
279	5	254	56	46	−.37	−.42		−.36		−.09		+.52	
280	1	255	4	30								.00	
281	1	256	5	41								.00	
282	1	256	17	24								.00	
283	1	256	59	28								.00	
284	2	257	4	38						+.30		−.32	
285	2	258	26	12						+.10		−.06	
286	1	260	21	29								.00	
287	1	260	53	12								.00	
288	5	262	59	9	−.22	+.11		+.40		−.16		−.12	
289	1	263	15	20								.00	
290	1	263	36	24								.00	
291	4	263	51	32	−.18			+.72		−.30		−.22	
292	1	264	18	4								.00	
293	2	265	21	48						−.16		+.16	
294	2	265	28	58						−.09		+.12	
295	1	265	29	32	.00								
296	2	265	40	24						+.02		.00	
297	3	265	43	30				+.29		.00		−.29	
298	5	266	14	6	−.18	+.19		−.08		+.14		−.06	
299	1	267	6	7								.00	
300	2	268	4	8						.00		+.02	
301	5	268	28	55	−.07	+.08		+.38		−.03		−.34	
302	2	269	49	39						−.11		+.11	
303	1	270	24	30						.00			
304	2	270	30	0						−.06		+.06	
305	2	270	46	0						+.04		−.07	
306	2	271	55	32						+.07		−.07	
307	2	271	56	55						+.12		−.12	
308	2	271	58	15						−.41		+.41	
309	1	273	10	37								.00	
310	5	273	57	50	−.08	−.23		+.40		−.02		−.10	
311	1	274	26	1								.00	
312	2	274	45	52						−.19		+.22	
313	5	275	5	22	−.02	−.25		+.23		+.02		−.02	
314	2	275	53	45						−.28		+.28	
315	1	276	25	19	.00								

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean N. P. D. 1888.0	$\Delta\pi$								
			90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		″	″	″	″	″	″	″	″	″	″
271	1	5939.73								.00	
272	3	4119.75				−.29		+.34		−.04	
273	1	6455.18								.00	
274	1	6884.80								.00	
275	1	4129.25								.00	
276	1	6145.37								.00	
277	1	6542.06								.00	
278	1	6052.71								.00	
279	5	731.80	+.32	+.33		−.24		+.13		−.53	
280	1	4286.98								.00	
281	1	4986.46								.00	
282	1	5652.74								.00	
283	1	6720.11								.00	
284	2	5473.84						+.09		−.10	
285	2	6521.14						−.18		+.18	
286	1	5291.05								.00	
287	1	6031.51								.00	
288	5	1230.50	+.10	+.34		−.10		+.13		−.49	
289	1	4662.49								.00	
290	1	2852.46								.00	
291	4	3212.78	−.17			+.12		+.18		−.13	
292	1	4229.31								.00	
293	2	6548.05						+.28		−.28	
294	2	6382.06						+.17		−.17	
295	1	329.24	.00								
296	2	4726.12						−.17		+.16	
297	3	4256.54				−.18		+.25		−.08	
298	5	340.70	−.06	+.59		−.05		+.02		−.51	
299	1	5118.16								.00	
300	2	5098.98						−.04		+.03	
301	5	2978.23	−.26	+.15		−.13		+.03		+.22	
302	2	7334.60						+.34		−.34	
303	1	6918.38						.00			
304	2	6296.22						−.14		+.15	
305	2	4549.76						−.06		+.05	
306	2	3411.91						+.20		−.20	
307	2	4727.60						+.15		−.16	
308	2	7041.16						+.40		−.40	
309	1	1971.86								.00	
310	5	3995.34	+.04	−.04		−.50		+.30		+.19	
311	1	2913.21								.00	
312	2	5486.96						+.39		−.39	
313	5	3730.28	+.06	+.06		−.16		+.09		−.07	
314	2	7230.50						+.14		−.14	
315	1	1228.87	.00								

[illegible]

Star	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21 [illegible] 15	21 no. 15
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	− [illegible]	− [illegible]		− [illegible]		− [illegible]		− .00	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						− [illegible]		− .18	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]								.00	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						− [illegible]		− .11	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						− [illegible]		− .03	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						− [illegible]		− .07	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	− [illegible]	− [illegible]				− [illegible]		− [illegible]	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	− [illegible]	− [illegible]		− [illegible]		− [illegible]		− [illegible]	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						− [illegible]		− .05	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						− [illegible]		− .20	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]			......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]		.00	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]			......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	− [illegible]	− [illegible]		− [illegible]		− [illegible]		− .18	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]		..	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						− [illegible]		− .16	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		− [illegible]				− [illegible]		− .14	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]		...	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]							..	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	− [illegible]	− [illegible]		− [illegible]		− [illegible]		− .20	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]		...	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]		..	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]					− [illegible]		− .57	......
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						[illegible]			......
340	5	[illegible]	[illegible]	[illegible]	− [illegible]	− [illegible]		− [illegible]		− [illegible]		− .04	......
341	3	[illegible]	[illegible]	1		− [illegible]				− [illegible]		− .13	......
342	1	[illegible]	[illegible]	[illegible]						[illegible]		....	......
343	1	[illegible]	[illegible]	4						[illegible]		....	......
344	1	[illegible]	[illegible]	[illegible]						[illegible]		...	......
345	1	295	[illegible]	[illegible]						.00	...	.....	......
346	3	295	33	33	.	− .22				− .22	. .	− .12	......
347	1	296	51	5						.00		...	......
348	5	297	14	54	− .0[illegible]	− .22		− .25		− [illegible]	.	− .14	......
349	1	297	57	36		. .				.00	.	......	......
350	1	298	4	2	...	.	. .			.00	....	....	......
351	4	298	14	31	− .14	− .05	...	.		− .05	. .	− .23	......
352	1	299	0	22	. . .		. . .	.		.00	. . .	......	......
353	4	300	12	35	− .34	− .22	...	. .		− .30	.....	− .12	......
354	2	300	40	16	.....	− .12	......	.....		− .15	......	......	......
355	1	301	27	27	.....	...	......	.....	. . . .	.00	......	......	......
356	4	302	20	41	− .19	.00	......	......	...	− .02	....	− .19	......
	1	303	46	53	.....	...	......	......	......	.00	....	....	......
	1	304	50	48	.....	...	......	......	......	.00	....	....	......
	2	304	51	16	.....	+ .03	......	......	......	− .05	....	....	......
	2	304	57	8	.....	+ .02	......	......	......	.00	....	....	......

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean N. P. D. 1888.0	$\Delta\pi$								
			90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		"	"	"	"	"	"	"	"	"	"
316	5	1759.18	−.16	+.40	......	−.07	......	+.02	......	−.18	......
317	2	5090.84	......	......	......	......	......	+.10	......	−.09	......
318	1	2286.27	......	......	......	......	......	......	......	.00	......
319	2	3335.18	......	......	......	......	......	+.26	......	−.27	......
320	2	5821.74	......	......	......	......	......	+.24	......	−.24	......
321	2	4923.50	......	......	......	......	......	+.14	......	−.13	......
322	4	1212.12	−.15	+.40	......	......	......	+.11	......	−.37	......
323	5	2622.05	−.07	+.05	......	+.18	......	+.07	......	−.21	......
324	2	5182.51	......	......	......	......	......	+.26	......	−.26	......
325	2	6543.34	......	......	......	......	......	−.23	......	+.23	......
326	1	4964.68	......	......	......	......	......	.00	......	......	......
327	2	4961.06	......	......	......	......	......	−.02	......	+.01	......
328	1	6411.01	......	......	......	......	......	.00	......	......	......
329	5	2430.73	−.05	+.21	......	.00	......	−.02	......	−.14	......
330	1	6788.39	......	......	......	......	......	.00	......	......	......
331	2	3709.08	......	......	......	......	......	+.14	......	−.15	......
332	3	4749.89	......	+.05	......	......	......	+.14	......	−.19	......
333	1	6706.78	......	......	......	......	......	.00	......	......	......
334	1	502.74	.00	......	......	......	......	......	......	......	......
335	5	1593.84	−.28	+.11	......	−.07	......	+.29	......	−.07	......
336	1	4832.64	......	......	......	......	......	.00	......	......	......
337	1	3078.28	......	......	......	......	......	.00	......	......	......
338	3	1055.52	+.14	......	......	......	......	+.36	......	−.50	......
339	1	5223.72	......	......	......	......	......	.00	......	......	......
340	5	3735.63	−.06	−.22	......	−.03	......	+.05	......	+.24	......
341	3	4444.31	......	−.05	......	......	......	+.24	......	−.18	......
342	1	5552.44	......	......	......	......	......	.00	......	......	......
343	1	5487.95	......	......	......	......	......	.00	......	......	......
344	1	5287.07	......	......	......	......	......	.00	......	......	......
345	1	5880.00	......	......	......	......	......	.00	......	......	......
346	3	5217.73	......	−.07	......	......	......	+.23	......	−.16	......
347	1	5854.96	......	......	......	......	......	.00	......	......	......
348	5	1291.66	−.35	+ 08	......	−.01	......	+.20	......	+.10	......
349	1	6877.70	......	......	......	......	......	.00	......	......	......
350	1	4661.40	......	......	......	......	......	.00	......	......	......
351	4	4843.21	−.46	+.09	......	......	......	+.13	......	+.23	......
352	1	7217.67	......	......	......	......	......	.00	......	......	......
353	4	4069.50	−.09	+.01	......	......	......	+.19	......	−.11	......
354	2	5257.42	......	−.12	......	......	......	+.12	......	......	......
355	1	3275.37	......	......	......	......	......	.00	......	......	......
356	4	4351.60	−.04	−.02	......	......	......	+.01	......	+.07	......
357	1	4856.29	......	......	......	......	......	.00	......	......	......
358	1	5745.98	......	......	......	......	......	.00	......	......	......
359	2	5341.66	......	−.31	......	......	......	+.31	......	......	......
360	2	5173.86	......	−.23	......	......	......	+.24	......	......	......

TABLE VI (cont'd).—PRELIMINARY CATALOGUE.

Star	Number Plates	Mean R. A. 1888.0			Δα sin π								
					90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
		°	′	″	″	″	″	″	″	″	″	″	″
361	1	305	6	24						.00			
362	4	307	21	12	+.21	−.34				−.20		+.34	
363	2	309	41	4		−.16				+.12			
364	2	310	3	15		−.03				+.03			
365	2	310	9	14		−.15				+.15			
366	1	310	43	41						.00			
367	5	311	7	22	+.26	−.34			+.12	−.39		+.36	
368	2	312	2	26		+.04				−.02			
369	2	312	44	9		+.06				−.06			
370	2	314	31	24		−.08				+.08			
371	2	314	47	2		+.07				−.03			
372	5	314	53	31	−.04	−.32			+.23	−.38		+.52	
373	5	315	29	20	−.31	−.22			+.24	−.70		+.98	
374	2	317	8	28		+.12				−.12			
375	2	317	17	52		+.16				−.18			
376	5	317	46	25	−.21	−.51			+.38	−.18		+.51	
377	2	318	31	58		−.12				+.09			
378	2	318	52	16		−.16				+.16			
379	2	319	41	53		−.02				+.02			
380	2	320	7	40		+.05				−.02			
381	5	322	7	1	+.07	−.40			+.10	−.50		+.74	
382	2	322	26	56		−.12				+.09			
383	1	324	28	59	.00								
384	4	326	1	57	+.36	−.12			+.10	−.32			
385	1	328	6	35		.00							
386	2	330	15	6		+.12				−.14			
387	4	334	27	24	−.13	−.13			+.02	+.23			
388	1	337	35	34		.00							
389	4	338	39	32	+.37	−.14			+.05	−.32			
390	4	344	19	18	+.04	.00			−.09	+.02			
391	1	345	4	43		.00							
392	5	348	36	4	+.18	−.32			−.08	−.43			+.67
393	2	349	47	40		−.14			+.17				
394	1	351	34	1		.00							
395	2	353	4	3		+.19			−.19				
396	1	353	8	1		.00							
397	2	353	37	37		+.05			−.05				
398	1	353	41	47					.00				
399	2	353	47	56		+.02			−.05				
400	1	354	56	21		.00							
401	2	356	1	50		+.16			−.13				
402	2	356	2	51		.00			.00				
403	2	356	37	9		+.09			−.09				
404	2	356	38	6		+.31			−.28				
405	5	357	12	20	−.02	−.23			−.15	−.04			+.40
406	4	358	7	15	+.36	−.23			−.19				+.08
407	2	359	1	18		+.04			−.07				
408	2	359	33	22		−.05			+.03				

TABLE VI (concl'd).—PRELIMINARY CATALOGUE.

Number Plates	Mean N. P. D. 1888.0	$\Delta\pi$								
		90°	16 no. 2	16 no. 4	16 no. 5	16 no. 7	18 no. 1	18 no. 3	21 no. 13	21 no. 15
	″	″	″	″	″	″	″	″	″	″
1	5390.72						.00			
4	1542.46	−.46	+.48				+.14		−.18	
2	6478.64		−.05				+.05			
2	5507.34		−.17				+.18			
2	3977.84		+.10				−.11			
1	5188.78						.00			
5	851.89	−.29	+.27			−.15	+.34		−.19	
2	4438.12		+.10				−.11			
2	5725.42		−.23				+.23			
2	5393.14		−.22				+.23			
2	6962.54		−.21				+.20			
5	1581.62	−.11	+.09			−.08	+.09		+.01	
5	1353.84	−.31	+.17			−.10	+.38		−.15	
2	6212.38		−.07				+.08			
2	5435.92		−.26				+.27			
5	3252.78	+.06	+.19			−.52	+.09		+.20	
2	5903.71		−.19				+.19			
2	4739.66		−.28				+.27			
2	5197.71		−.17				+.17			
2	5237.84		+.02				−.03			
5	1185.99	−.05	+.06			+.15	−.01		−.17	
2	6396.76		−.20				+.19			
1	460.55	.00								
4	4132.19	+.38	−.17			−.35	+.15			
1	4839.22		.00							
2	6034.77		+.01				−.01			
4	3953.95	−.19	+.22			−.12	+.10			
1	5646.66		.00							
4	4786.44	+.40	−.12			−.42	+.14			
4	4459.16	+.26	−.23			−.37	+.32			
1	7048.20		.00							
5	2904.74	−.05	+.10			−.26	+.18			+.05
2	7095.32		+.50			−.49				
1	4156.97		.00							
2	4383.43		+.05			−.05				
1	6502.08		.00							
2	4938.10		+.32			−.32				
1	6665.37					.00				
2	5143.24		+.20			−.19				
1	5114.04		.00							
2	5453.80		−.11			+.10				
2	6396.74		+.05			−.06				
2	5923.66		+.03			−.04				
2	6335.08		+.28			−.27				
5	3889.77	+.19	−.09			−.07	+.08			−.09
4	4241.30	+.28	−.11			−.13				−.02
2	7372.48		+.64			−.64				
2	5421.18		+.26			−.27				

III.

INTER-ADJUSTMENT OF THE PLATES.

The process next entered upon is the inter-adjustment of the results obtained from the different plates. Since the problem in question is similar to that discussed by Jacoby in the paper already referred to (Col. Con. no. [illegible]), where the south polar plates are treated, the method devised by him will be followed here. A full statement of the problem is given on p. [illegible] of the above paper, and the method of solution on pp. 6[illegible]–75. Its application in the present case is as follows. The right ascensions and polar distances of the unknown stars upon any plate depend fundamentally upon the standard stars which appear upon that plate. For the [illegible] plates the number of standard stars on each plate varies from six to nine, and on the average four of these appear on two adjacent plates. Thus it may be expected, that, since the common data are so few, the agreement between the α's and π's of any particular star as found on two plates will not necessarily be very close. The largest actual residual from the mean in Table VI is $1''.03$. Only ten residuals are greater than $0''.75$, and but twenty-five lie between $0''.60$ and $0''.75$.

The adjustment of these differences is carried out by means of equations (2), Publication 1. These may be written in a more convenient form without impairing the correctness of their derivation by assuming that the X axis points to the vernal equinox, thus replacing the angle B by a. They then read:

$$\begin{aligned} \omega \sin a d\xi - \omega \cos a d\eta + p dA' + (a - a') \sin \pi &= 0\,, \\ -\omega \cos a d\xi - \omega \sin a d\eta + p d\omega + (\pi - \pi') &= 0\,. \end{aligned} \qquad (2)$$

These equations are general in their application. They express the relation of small changes in the constants $d\xi$, $d\eta$, dA' and $d\omega$ to changes in the corresponding α's and π's without regard to the cause of the change. If the changes in the α's and π's of a sufficient number of stars arising from the same cause are known, the quantities $d\xi$, $d\eta$, dA' and $d\omega$ may be obtained by the method of least squares. If on the other hand these corrections are known, the corrections $\Delta\alpha$ and $\Delta\pi$ to the positions of the stars may be computed. In the present case the quantities $d\xi$, etc., represent corrections to the plate constants of one plate which are required to adjust the α's and π's of the stars on that plate to a closer agreement with the α's and π's of

the same stars when found on an adjacent plate. They are obtained by a least squares solution of equations (2) in which the quantities $(\alpha - \alpha') \sin \pi$ and $(\pi - \pi')$ are the differences in right ascension and polar distance of all the stars, standard and unknown, common to the two plates in question. Their numerical values are taken from the original α's and π's of which Table VI contains the means. Hence the X axis points to the equinox of 1888.0 and the α's and π's are the mean places for the same epoch. Each pair of adjacent plates will furnish such a set of corrections to their plate constants, and there will be eight combinations of the 89° plates one with another. There will also be eight combinations of the 90° plate with each of the 89° plates, making sixteen combinations in all. The results of these least squares determinations are found in Table VII which follows. The numbers in the first column designate the plates which are included in each comparison. The number of equations involved in the least squares solution stands in the second column. It is equal to twice the number of stars common to the two plates. The remaining columns give the corrections $d\xi$, $d\eta$, dA' and $d\omega$ to the plate constants. For example, in the first comparison the values inserted in equations (2) are in the sense plate 16–2 minus 16–7. Hence if the corrections in the first line of Table VII are substituted in equations (2) they will give the corrections to be added to the results of plate 16–2 in order to secure closer agreement with 16–7.

TABLE VII.—INTER-COMPARISON OF PLATES.

Plates.	No. eq.	$d\xi$	$d\eta$	da'	$d\omega$
		mm.	mm.	"	"
16 no. 7 and 16 no. 7	90	− .0025	− .0065	− .0036	− .0055
16 no. 7 and 21 no. 15	122	− .0033	+ .0036	+ .0066	− .0034
21 no. 15 and 18 no. 3	82	− .0024	+ .0100	+ .0018	+ .0027
18 no. 3 and 16 no. 4	74	− .0033	− .0010	− .0046	+ .0070
16 no. 4 and 16 no. 5	96	− .0093	− .0075	− .0005	+ .0058
16 no. 5 and 21 no. 13	128	+ .0033	− .0019	− .0029	+ .0010
21 no. 13 and 18 no. 1	120	+ .0028	+ .0118	+ .0013	− .0058
18 no. 1 and 16 no. 2	96	+ .0010	+ .0029	+ .0023	− .0039
90° and 16 no. 2	86	− .0005	+ .0059	− .0001	− .0033
90° and 16 no. 7	76	− .0011	− .0044	− .0055	− .0055
90° and 21 no. 15	78	− .0021	− .0011	+ .0017	− .0050
90° and 18 no. 3	58	− .0004	+ .0096	+ .0006	+ .0003
90° and 16 no. 4	64	− .0029	+ .0045	− .0043	+ .0023
90° and 16 no. 5	78	− .0110	− .0032	− .0052	+ .0042
90° and 21 no. 13	98	− .0067	− .0065	− .0073	+ .0067
90° and 18 no. 1	84	− .0015	+ .0056	− .0018	− .0008

Table VIII contains the probable errors of Table VII.

TABLE VIII.—PROBABLE ERRORS OF TABLE VII.

	$d\xi$ or $d\eta$.	dA' or $d\omega$.	r_0.
	mm.	″	″
16 no 2 and 16 no. 7	± .0009	± .0007	± 0.173
16 no. 7 and 21 no. 15	.0007	.0006	.176
21 no. 15 and 18 no. 3	.0001	.0010	.198
18 no. 3 and 16 no. 4...........	.0010	.0008	.194
16 no. 4 and 16 no. 5	.0008	.0008	.167
16 no. 5 and 21 no. 13	.0007	.0007	.204
21 no. 13 and 18 no. 1	.0008	.0007	.205
18 no. 1 and 16 no. 2	.0007	.0006	.152
90° and 16 no. 2..............	.0006	.0007	.171
90° and 16 no. 7..............	.0007	.0008	.177
90° and 21 no. 15.............	.0006	.0007	.154
90° and 18 no. 3..............	.0009	.0011	.199
90° and 16 no. 4..............	.0006	.0009	.162
90° and 16 no. 5..............	.0007	.0009	.177
90° and 21 no. 13.............	.0007	.0009	.205
90° and 18 no. 1..............	.0007	.0008	.183

Since the values of the differential reduction constants given in Table VII enable us merely to form the best combination of results derived from pairs of adjacent plates, it becomes necessary to find a method of reducing all of these combinations to the most uniform standard. This may be performed as follows (*cf.* Col. Con. no. 21, p. 70). If a star is found on plates 16–2, 16–7, and 90°, we may consider that by substituting the proper values from Table VII in equations (2) we may pass from plate 16–2 to 16–7, by a second negative substitution from 16–7 to 90°, and by a third substitution from 90° to 16–2. The final α and π should agree with the initial value of plate 16–2. Since this substitution is valid for every star common to the three plates, we should have theoretically

$$d\xi_1 - d\xi_{10} + d\xi_9 = 0\,,$$

$$d\eta_1 - d\eta_{10} + d\eta_9 = 0\,,$$

etc.

The subscripts denote the respective combinations in Table VII in which they are numbered in order from 1 to 16.

Let u represent a general expression for any differential constant. Since there are eight possible combinations of overlapping plates by means of which we may start with plate 16–2 and return to it, they

will give rise to the following eight equations of condition of which there are four repetitions, u standing in turn for $d\xi_1$, $d\eta$, dA', and $d\omega$.

$$
\begin{aligned}
&u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 &&= 0\,, \\
&u_1 - u_{10} + u_9 &&= 0\,, \\
&u_1 + u_2 - u_{11} + u_9 &&= 0\,, \\
&u_1 + u_2 + u_3 - u_{12} + u_9 &&= 0\,, \\
&u_1 + u_2 + u_3 + u_4 - u_{13} + u_9 &&= 0\,, \\
&u_1 + u_2 + u_3 + u_4 + u_5 - u_{14} + u_9 &&= 0\,, \\
&u_1 + u_2 + u_3 + u_4 + u_5 + u_6 - u_{15} + u_9 &&= 0\,, \\
&u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 - u_{16} + u_9 &&= 0\,.
\end{aligned}
\tag{3}
$$

The second terms of these equations will not actually be equal to zero, but there will exist small discrepancies $w_1 \ldots w_8$, and these must be distributed among the various quantities $u_1 \ldots u_{16}$ in such a way as to cause the final differences to become as small as possible. This problem is solved by the method of correlatives (Merriman, Least Squares, pp. 59–64). Let $k_1 \ldots k_8$ be the multipliers or correlatives of the equations of condition. We shall then have the following eight normal equations which are to be solved by the ordinary processes of elimination.

$$
\begin{aligned}
8k_1 + k_2 + 2k_3 + 3k_4 + 4k_5 + 5k_6 + 6k_7 + 7k_8 + w_1 &= 0\,, \\
3k_2 + 2k_3 + 2k_4 + 2k_5 + 2k_6 + 2k_7 + 2k_8 + w_2 &= 0\,, \\
4k_3 + 3k_4 + 3k_5 + 3k_6 + 3k_7 + 3k_8 + w_3 &= 0\,, \\
5k_4 + 4k_5 + 4k_6 + 4k_7 + 4k_8 + w_4 &= 0\,, \\
6k_5 + 5k_6 + 5k_7 + 5k_8 + w_5 &= 0\,, \\
7k_6 + 6k_7 + 6k_8 + w_6 &= 0\,, \\
8k_7 + 7k_8 + w_7 &= 0\,, \\
9k_8 + w_8 &= 0\,.
\end{aligned}
\tag{4}
$$

The values $k_1 \ldots k_8$, when substituted in the following formulas, will give the corrections to the various quantities $u_1 \ldots u_{16}$.

$$
\begin{aligned}
du_1 &= k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7 + k_8\,, \\
du_2 &= k_1 + k_3 + k_4 + k_5 + k_6 + k_7 + k_8\,, \\
du_3 &= k_1 + k_4 + k_5 + k_6 + k_7 + k_8\,, \\
du_4 &= k_1 + k_5 + k_6 + k_7 + k_8\,, \\
du_5 &= k_1 + k_6 + k_7 + k_8\,, \\
du_6 &= k_1 + k_7 + k_8\,,
\end{aligned}
$$

$$du_7 = k_1 + k_8\,,$$
$$du_8 = k_1\,,$$
$$du_9 = k_2 + k_3 + k_4 + k_5 + k_6 + k_7 + k_8\,,$$
$$du_{10} = -k_2\,,$$
$$du_{11} = -k_3\,,$$
$$du_{12} = -k_4\,,$$
$$du_{13} = -k_5\,,$$
$$du_{14} = -k_6\,,$$
$$du_{15} = -k_7\,,$$
$$du_{16} = -k_8\,.$$

Table IX gives the numerical values of the quantities $w_1 \ldots w_8$ obtained from equations (3).

TABLE IX.—VALUES OF $w_1 \ldots w_8$.

	$d\xi$	$d\eta$	dA'	$d\omega$
	mm.	mm.	"	"
w_1	− .0137	+ .0114	+ .0004	− .0021
w_2	− .0019	+ .0038	+ .0018	− .0033
w_3	− .0042	+ .0041	+ .0012	− .0072
w_4	− .0083	+ .0034	+ .0041	− .0098
w_5	− .0091	+ .0075	+ .0044	− .0048
w_6	− .0103	+ .0077	+ .0048	− .0009
w_7	− .0113	+ .0091	+ .0040	− .0024
w_8	− .0137	+ .0088	− .0002	− .0007

Table X gives the values of the correlatives $k_1 \ldots k_8$ as determined from equations (4).

TABLE X.—VALUES OF THE CORRELATIVES $k_1 \ldots k_8$.

	$d\xi$	$d\eta$	dA'	$d\omega$
	mm.	mm.	"	"
k_1	+ .0011	− .0019	+ .0001	+ .0011
k_2	− .0005	− .0012	− .0002	− .0001
k_3	− .0004	− .0003	+ .0008	+ .0014
k_4	+ .0010	+ .0012	− .0007	+ .0030
k_5	+ .0003	− .0009	− .0005	.0000
k_6	.0000	.0000	− .0009	− .0018
k_7	− .0003	− .0002	− .0009	.0000
k_8	+ .0005	+ .0010	+ .0016	− .0014

Table XI gives the values of $du_1 \ldots du_{16}$ in equations (5). These numbers are to be applied to the corresponding quantities in Table VII.

TABLE XI.—CORRECTIONS REQUIRED BY TABLE VII.

	$d\xi$	$d\eta$	dA'	$d\omega$
	mm.	mm.	"	"
u_1	+ .0017	− .0023	− .0007	+ .0022
u_2	+ .0022	− .0011	− .0005	+ .0023
u_3	+ .0026	− .0008	− .0013	+ .0009
u_4	+ .0016	− .0020	− .0006	− .0021
u_5	+ .0013	− .0011	− .0001	− .0021
u_6	+ .0013	− .0011	+ .0008	− .0003
u_7	+ .0016	− .0009	+ .0017	− .0003
u_8	+ .0011	− .0019	+ .0001	+ .0011
u_9	+ .0006	− .0004	− .0008	+ .0011
u_{10}	+ .0005	+ .0012	+ .0002	+ .0001
u_{11}	+ .0004	+ .0003	− .0008	− .0014
u_{12}	− .0010	− .0012	+ .0007	− .0030
u_{13}	− .0003	+ .0009	+ .0005	.0000
u_{14}	.0000	.0000	+ .0009	+ .0018
u_{15}	+ .0003	+ .0002	+ .0009	.0000
u_{16}	− .0005	− .0010	− .0016	+ .0014

The quantities in Table XII are the sums of the corresponding values in Tables VII and XI. They form the best possible set of relative reduction constants, which adjust the separate inter-comparisons to a uniform basis.

TABLE XII.—FINAL INTER-COMPARISON OF PLATES.

	$d\xi$	$d\eta$	dA'	$d\omega$
	mm.	mm.	"	"
u_1	− .0008	− .0088	− .0043	− .0033
u_2	− .0011	+ .0025	+ .0061	− .0011
u_3	+ .0002	+ .0092	+ .0005	+ .0036
u_4	− .0017	− .0030	− .0052	+ .0049
u_5	− .0080	.0086	− .0006	+ .0037
u_6	+ .0046	− .0030	− .0021	+ .0007
u_7	+ .0044	+ .0109	+ .0030	− .0061
u_8	+ .0021	+ .0010	+ .0024	− .0028
u_9	+ .0001	+ .0055	− .0009	− .0022
u_{10}	− .0006	− .0032	− .0053	− .0054
u_{11}	− .0017	− .0008	+ .0009	− .0064
u_{12}	− .0014	+ .0084	+ .0013	− .0027
u_{13}	− .0032	+ .0054	− .0038	+ .0023
u_{14}	− .0110	− .0032	− .0043	+ .0060
u_{15}	− .0064	− .0063	− .0084	+ .0067
u_{16}	− .0020	+ .0046	− .0034	+ .0006

IV.

Final Standardization. Catalogue.

It now becomes possible to make use of the preceding inter-adjustment to obtain a more complete agreement with the sky by reducing the right ascensions and polar distances of the standard stars as obtained from the several plates to the system of some one particular plate and comparing the results thus deduced with the heliometer places as in the first determination of the plate constants. (*Cf.* Col. Con. no. 21, p. 74.) The 90° plate was selected as the medium of comparison and the eight 89° plates were reduced to its standard. This was performed by substituting the last eight values from Table XII in equations (2) for such of the reference stars as appeared on the respective plates. Since the signs of the quantities as they stand in this table give the results which must be added to the 90° plate to produce agreement with any 89° plate, therefore the corrections obtained above must be subtracted from the α's and π's of the 89° plates to produce agreement with the 90° plate. This was accordingly done and the resulting right ascensions and polar distances were compared with the heliometer positions. The differences, Photographic minus Heliometer, were inserted in equations (2), and a least squares solution gave the corrections to the plate constants of the 90° plates upon which the improvement of the entire system is based. Equations (2) are then of the form

$$\omega \sin a\, d\xi - \omega \cos a\, d\eta + p\, dA' + \Delta a \sin \pi = 0\,,$$
$$\omega \cos a\, d\xi + \omega \sin a\, d\eta + p\, d\omega + \Delta\pi = 0\,,$$

in which the photographic place has been reduced to the 90° standard as just described. The quantities forming the last terms of these equations were taken from the values of which Table VI is the mean. They are in the sense Photographic minus Heliometer. The distribution of the standard stars was such that 124 equations were available for the least squares solution, and the following differential constants are the resulting values:

$$d\xi = +\overset{\text{mm.}}{0.0006} \pm 0.0005\,,$$
$$d\eta = +\overset{\text{mm.}}{0.0033} \pm 0.0005\,,$$
$$dA' = -\overset{\prime\prime}{0.0019} \pm 0.0004\,,$$
$$d\omega = -\overset{\prime\prime}{0.0013} \pm 0.0004\,.$$

These values when substituted in equations (2) for the unknown stars will give the corrections to be added to the 90° places to produce the most improved values. To obtain the definitive corrections to the 89° plates it is necessary to subtract from these quantities the last eight sets of numbers in Table XII, $u_9 \dots u_{16}$, in turn. The results when substituted in equations (2) will give the corrections to be added to the places from the 89° plates. Table XIII contains these definitive correction constants.

TABLE XIII.—DEFINITIVE CORRECTION CONSTANTS.

	$d\xi$	$d\eta$	dA'	$d\omega$
90°	+ .0006	+ .0033	− .0019	− .0013
16–2	+ .0005	− .0022	− .0010	+ .0009
16–7	+ .0012	+ .0065	+ .0034	+ .0041
21–15	+ .0023	+ .0041	− .0028	+ .0051
18–3	+ .0020	− .0051	− .0032	+ .0014
16–4	+ .0038	− .0021	+ .0019	− .0036
16–5	+ .0116	+ .0065	+ .0024	− .0073
21–13	+ .0070	+ .0096	+ .0065	− .0080
18–1	+ .0026	− .0013	+ .0015	− .0019

The corrections thus obtained were added directly to the right ascensions and polar distances of which Table VI contains the means, $\Delta\alpha \sin \pi$, having been previously multiplied by cosec π. The means of these corrected places form the final catalogue which is found in Table XVI. The residuals from the mean are given in Table XV, in the sense plate minus mean. A comparison of these residuals with those of Table VI shows that the process of inter-adjustment has considerably reduced them. The largest one, 1″.03, has become 0″.68, and only four residuals of the second solution are greater than 0″.60 as against thirty-five of the preliminary solution. The improvement may also be shown by comparing the sums of the squares of the residuals for all the stars, using the values which occur in Tables VI and XV, respectively:

	Preliminary solution.	*Final solution.*
$\Delta\alpha \sin \pi$	53″.10,	22″.82,
$\Delta\pi$	46.25,	23.46.

The following table shows the differences, Photographic minus Heliometer, of the standard stars from both preliminary and final solutions. The preliminary solution being based solely on the standard stars gives results which agree more closely with the heliometer

places than does the final solution, since the latter involves the unknown stars also, which far outnumber the standards. However, the deviation is not so great as to throw doubt upon the validity of the method of inter-adjustment and its results.

TABLE XIV.—RESIDUALS, PHOTOGRAPHIC MINUS HELIOMETER.

Star.	No. Plates.	Preliminary solution.		Final solution.	
		$\Delta\alpha \sin\pi$	$\Delta\pi$	$\Delta\alpha \sin\pi$	$\Delta\pi$
		"	"	"	"
α	4	− .17	− .01	− .38	+ .06
β	2	+ .11	− .16	+ .11	.00
δ	5	+ .02	− .05	− .01	− .11
ε	2	− .05	+ .28	− .24	+ .38
ζ	2	+ .13	− .34	− .03	− .21
η	2	− .04	+ .34	− .15	+ .60
θ	2	− .08	+ .04	− .25	+ .38
ι	4	+ .13	− .13	+ .14	− .06
κ	2	− .27	− .03	− .46	+ .12
λ	1	− .16	− .13	− .03	− .26
μ	2	+ .17	+ .05	+ .49	− .03
ν	4	− .09	+ .09	+ .05	+ .18
ξ	1	− .03	+ .03	+ .40	+ .04
ο	2	+ .08	− .13	+ .40	− .13
π	2	+ .06	+ .03	+ .22	− .03
ρ	5	+ .14	+ .18	+ .03	+ .28
σ	2	.00	− .01	+ .12	− .28
τ	2	+ .29	− .18	+ .29	− .32
υ	2	.00	− .21	.00	− .45
φ	5	− .18	− .18	− .41	− .33
ψ	4	+ .02	− .11	− .09	− .28
χ	2	− .03	+ .13	+ .03	− .06
ω	4	+ .04	+ .16	− .06	+ .17

MAGNITUDES. These were determined both photographically and visually by estimation. In the examination of the plates, the faintest stars measured were assumed to be of the twelfth magnitude. For the brighter stars the photographic magnitudes contained in the Harvard Annals, Vol. XVIII, p. 149, were taken as standards of comparison. Fortunately these were so distributed that some of them appeared on every plate. The mean results of the estimates from the different plates were taken as the adopted magnitudes.

The visual magnitudes were determined with the aid of the 12-inch equatorial, using the Argelander method and basing the comparisons upon the magnitudes found in Carrington's Redhill Catalogue of Circumpolar Stars. These had been determined by Carrington with great care, using the method of extinction (*cf.* Sec. 10, p. xxv), and

while in the process of making the comparisons variations from his values were suspected, it did not seem advisable to delay the conclusion of this work long enough to make a thorough investigation, especially as the main object in view was the formation of a catalogue of positions. Further, in most cases only a single comparison was made, and the adopted values may, therefore, be subject to some error. Hence neither the photographic nor the visual magnitudes are to be considered as final.

The chart used for comparing the photographic positions with the sky was made by plotting the rectangular coördinates of the stars upon millimeter cross-section paper. By means of this chart a few errors in the reduction were detected by the disagreement of plate with sky.

Table XVI, which follows, contains the final catalogue. The headings of the several columns explain themselves. The rectangular coördinates and the precession coefficients were determined according to the method of Fabritius given in Publication 1, p. 58. The identification with the Carrington catalogue and with the Durchmusterung was made by using the above formulas and carrying the stars back to 1855, the epoch of both catalogues.

TABLE XV.—RESIDUALS FROM MEAN OF FINAL CATALOGUE.

λ	Δα sin π									Δπ								
	90°	16-2	16-4	16-5	16-7	18-1	18-3	21-13	21-15	90°	16-2	16-4	16-5	16-7	18-1	18-3	21-13	21-15
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
1	−.02	+.17	...	...	.00	−.03	...	...	−.08	−.02	−.15	...	...	−.29	+.48	...	...	−.02
2	−.21	+.38	...	...	−.08	...	...	...	−.06	−.36	−.01	...	...	+.06	...	...	...	+.32
3	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...	...	...	...	...
4	+.08	+.10	...	...	−.17	...	...	...	...	+.38	−.28	...	...	−.09	...	...	...	...
5	+.02	+.21	...	...	+.08	−.17	...	...	−.14	−.41	−.01	...	...	+.15	−.11	...	...	+.38
6	...	−.06	...	...	+.08	...	...	...	...	...	+.16	...	...	−.15	...	...	...	...
7	...	−.02	...	...	.00	...	...	...	...	...	−.06	...	...	+.07	...	...	...	...
8	.00	+.16	...	...	+.02	−.24	...	...	+.09	+.03	−.07	...	...	−.19	+.20	...	...	+.04
9	...	−.09	...	...	+.14	...	...	...	−.05	...	−.22	...	...	−.11	...	...	...	+.32
10	...	−.21	...	...	+.21	...	...	...	...	...	+.13	...	...	−.13	...	...	...	...
11	...	−.13	...	...	−.16	...	...	...	...	...	+.48	...	...	−.48	...	...	...	...
12	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
13	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
14	...	−.06	...	...	+.06	...	...	...	...	...	−.02	...	...	+.01	...	...	...	...
15	...	+.03	...	...	−.03	...	...	...	...	...	+.22	...	...	−.22	...	...	...	...
16	...	+.12	...	...	+.04	...	...	...	−.16	...	−.16	...	...	+.18	...	...	...	−.02
17	...	.00	...	...	−.03	...	...	...	...	...	+.24	...	...	−.24	...	...	...	...
18	.00	...	...	...	−.33	...	...	...	+.32	+.03	...	...	...	−.24	...	...	...	+.20
19	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
20	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
21	...	.00	...	...	−.05	...	...	...	+.05	...	+.19	...	...	−.06	...	...	...	−.12
22	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
23	...	...	...	...	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00
24	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
25	+.14	−.24	...	...	+.04	...	...	...	+.03	+.01	−.13	...	...	+.08	...	...	...	+.02
26	+.27	...	...	...	...	...	...	...	−.27	−.11	...	...	...	...	...	...	...	+.11
27	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
28	+.42	+.08	...	...	−.12	...	...	...	−.37	+.14	+.21	...	...	−.02	...	...	...	−.34
29	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
30	...	...	...	...	.00	...	...	...	...	...	...	...	...	.00	...	...	...	...
31	−.06	−.27	...	...	+.14	...	...	...	+.16	+.28	−.08	...	...	−.18	...	...	...	−.03
32	...	...	...	...	−.11	...	...	...	+.10	...	...	...	...	−.06	...	...	...	+.07
33	...	...	...	...	+.04	...	...	...	−.02	...	...	...	...	+.01	...	...	...	−.01
34	...	...	...	...	+.10	...	...	...	−.10	...	...	...	...	+.15	...	...	...	−.14
35	...	...	...	...	+.06	...	...	...	−.09	...	...	...	...	+.08	...	...	...	−.08
36	.00	+.19	...	...	−.02	...	−.22	...	+.07	+.18	+.03	...	...	−.07	...	+.03	...	−.16

37	-.04	-.20			-.04				+.30	+.24	+.10			-.25				-.10
38	-.03	+.03			+.10				-.08	+.08	-.23			-.13				+.27
39					+.26				-.26					-.17				+.17
40					+.16				-.14					-.07				+.06
41	+.03	-.31			+.22				+.08	-.09	+.01			-.06				+.13
42					+.16				-.13					+.11				-.11
43	+.02	-.03			+.13		-.22		+.11	-.09	+.15			-.09		+.10		-.09
44					+.10				-.10					+.07				-.07
45					.00				-.03					+.21				-.20
46					-.13				+.15					+.04				-.03
47	-.05	+.13			-.11		+.40		-.36	+.09	+.08			-.14		+.24		-.29
48					-.13				+.13					+.09				-.09
49	+.02	+.21			+.16		-.18		-.18	-.22	+.20			-.17		+.15		+.02
50					+.21				-.21					+.03				-.03
51					+.27				-.27					+.27				-.26
52					+.42				-.38					+.11				-.11
53					+.06				-.06					+.05				-.05
54	+.08	+.18			-.17				-.09	-.26	+.10			+.02				+.14
55					-.14				+.14					+.32				-.32
56					+.04				-.04					+.12				-.11
57					+.08				-.08					+.13				-.14
58					+.28				-.28					+.42				-.41
59					+.27				-.24					+.39				-.38
60									.00									.00
61					+.10				-.12					+.16				-.16
62	+.19								-.20	-.01								.00
63					+.07				-.07					+.23				-.22
64					+.29				-.29					+.05				-.06
65									.00									.00
66	-.07	+.14			-.01		-.15		+.13	-.10	+.22			+.06		-.03		-.13
67	-.13				+.06		-.12		+.17	-.10				+.07		+.14		-.09
68									.00									.00
69					-.07				+.05					+.04				-.05
70									.00									.00
71					+.14		-.18		+.02					+.16		-.13		-.04
72									.00									.00
73					-.05				+.05					+.01				-.02
74	-.32				+.27		-.02		+.04	-.03				+.05		+.04		-.05
75	-.06				+.03		+.03		-.03	+.12				+.22		-.49		+.13
76									.00									.00
77					+.18		-.25		+.09					+.02		+.14		-.17
78									.00									.00
79									.00									.00
80									.00									.00
81									.00									.00
82	-.06				+.17		-.15		+.06	-.08				+.16		+.06		-.15

TABLE XV (cont'd).—Residuals from Mean of Final Catalogue.

No.	$\Delta\alpha \sin\pi$									$\Delta\pi$								
	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
83	+.07				+.05		−.14		+.03	−.09				+.21		−.04		−.06
84									.00									.00
85	−.06				+.10		+.06		−.06	+.01				+.19		−.13		−.07
86									.00									.00
87									.00									.00
88									.00									.00
89	−.01		−.14		+.22		−.21		+.14	+.22		−.20		+.23		−.28		+.01
90									.00									.00
91									.00									.00
92									.00									.00
93	−.15		−.30		−.01		−.15		+.29	−.20		+.26		+.12		−.15		−.04
94							−.05		+.08							−.18		+.18
95							−.21		+.18							−.12		+.12
96									.00									.00
97							−.05		+.02							−.31		+.30
98									.00									.00
99							−.17		+.20							−.18		+.17
100	−.04								+.03	−.15								+.14
101							−.11		+.11							+.13		−.13
102									.00									.00
103									.00									.00
104							+.03		.00							−.09		+.08
105									.00									.00
106	+.07								−.08	−.12								+.11
107							+.07		−.05							−.05		+.04
108							−.18		+.16							+.24		−.24
109							+.24		−.27							−.27		+.26
110							−.19		+.19							+.21		−.21
111							+.07		−.07							−.10		+.11
112	+.04						−.06		.00	+.68						−.43		−.25
113							−.21		+.19							−.16		+.16
114							+.20		−.20							+.12		−.12
115							−.06		+.06							+.19		−.20
116	+.13		−.26		−.21		+.13		+.23	−.02		−.07		+.17		−.03		−.04
117							+.04		−.04							−.36		+.36
118							+.07		−.07							−.23		+.23

119									.00									.00
120	− .14		− .12				+ .06		+ .20	− .68		+ .37				+ .10		+ .21
121	− .10		− .03		− .30		+ .21		+ .21	− .02		− .29		+ .19		+ .09		+ .02
122							.00									.00		
123	+ .34		+ .08		− .24		− .22		+ .04	.00		− .32		+ .38		− .05		− .02
124	+ .12		− .03		− .24		+ .20		− .08	− .14		− .23		+ .18		+ .15		+ .05
125	− .04								+ .05	+ .02								− .02
126							.00									.00		
127	+ .23		− .36				− .08		+ .19	− .12		− .07				+ .09		+ .12
128							.00									.00		
129	− .07		− .19				+ .27		+ .02	+ .01		− .20				− .08		+ .25
130			+ .09				− .09					− .19				+ .18		
131	− .16		+ .34				+ .07		− .25	− .43		+ .05				+ .40		− .01
132	− .25		+ .09				+ .20		− .05	− .10		+ .16				− .20		+ .14
133							.00									.00		
134			+ .09				− .09					− .18				+ .18		
135	− .06		+ .04	+ .08			− .04		− .01	− .07		+ .24	− .13			− .09		+ .07
136							.00									.00		
137	− .09		+ .23				+ .07		− .20	− .15		+ .06				+ .04		+ .03
138							.00									.00		
139			+ .07				− .07					+ .12				− .12		
140			− .03				+ .06					− .04				+ .03		
141			+ .03				− .03					+ .11				− .11		
142			+ .04				.00					+ .15				− .16		
143			.00									.00						
144	− .24		+ .01	− .17			+ .26		− .32	+ .13		− .02	− .18			− .07		+ .15
145			+ .02				.00					− .08				+ .09		
146	.00									.00								
147			+ .10				− .13					+ .04				− .04		
148			.00									.00						
149			.00				.00					− .12				+ .12		
150			.00									.00						
151			− .11				+ .08					+ .06				− .06		
152			.00									.00						
153			.00									.00						
154			.00									.00						
155			+ .25				− .23					+ .18				− .19		
156			.00									.00						
157			+ .08	− .08			− .02					+ .11	− .08			− .03		
158			− .02				+ .02					+ .22				− .22		
159	+ .09		− .08							+ .15		− .15						
160			+ .19				− .17					− .03				+ .03		
161			+ .38				− .38					− .15	......			+ .15		
162	+ .16		+ .11	− .21			− .06			+ .03		+ .07	− .18			+ .08		
163			.00									.00						
164			− .17	+ .17								− .12	+ .12					

TABLE XV (cont'd).—Residuals from Mean of Final Catalogue.

No.	Δα sin π									Δπ								
	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
165			−.05	+.05								−.12	+.12					
166			+.05	+.10			−.14					+.05	+.17			−.21		
167			.00									.00						
168	+.12		−.24	−.10			−.34	+.56		−.09		−.05	+.24			+.36	−.46	
169			.00									.00						
170			.00	.00								+.18	−.17					
171			+.40	−.42								+.26	−.26					
172			.00									.00						
173			+.16	−.13								+.12	−.13					
174			−.06	+.09								+.19	−.19					
175	−.06		+.20	−.09			+.22	−.29		−.04		−.13	+.02			+.37	−.22	
176			+.15	−.15								−.03	+.03					
177	−.14		−.12	−.02			−.09	+.38		+.07		+.03	−.03			+.15	−.21	
178			−.08	+.11								−.19	+.20					
179			−.21	+.21								.00	.00					
180			−.11	+.08								−.17	+.17					
181			−.18	+.20								+.08	−.09					
182	−.21		−.09	−.21				+.48		+.08		+.08	−.17				.00	
183			+.12	−.12								−.13	+.12					
184			−.17	+.17								−.14	+.15					
185			−.14	+.14								+.04	−.04					
186			−.08	+.08								+.10	−.10					
187			.00									.00						
188			−.28	+.26								−.17	+.17					
189	+.23		+.10	−.20				−.13		+.12		−.14	.00				+.04	
190	−.01		−.01	+.05				−.01		−.04		+.17	−.10				−.02	
191	+.08		−.11	−.04			−.25	+.31		+.12		+.30	+.15			−.76	+.21	
192			−.02	+.02								−.05	+.05					
193	−.02		−.25	−.07			−.32	+.68		−.04		−.04	+.06			−.01	+.04	
194			−.08	+.08								+.02	−.02					
195			−.07	+.07								−.05	+.05					
196	+.19		+.13	−.06				−.26		.00		+.25	+.05				−.28	
197			−.11	−.14				+.24				−.30	+.31				−.02	
198			−.05	+.02								−.31	+.31					
199				.00									.00					
200	+.08		+.02	−.16			+.37	−.31		−.15		−.10	+.11			+.22	−.08	

201			−.40	+.10				+.28				−.21	−.01				+.22	
202	+.25		−.28					+.02		+.13		+.27					−.40	
203			−.07	+.07								−.12	+.13					
204				−.27				+.27					+.01				−.01	
205			−.11	+.22				−.09				−.15	+.08				+.07	
206			−.18	−.07				+.23				+.03	+.19				−.22	
207			+.19	−.06				−.13				−.30	+.23				+.08	
208				.00									.00					
209	+.18		−.09	+.03				−.10		−.01		+.22	−.34				+.13	
210	.00									.00								
211				.00									.00					
212	−.12		−.05	−.12				+.25		−.31		+.04	+.15				+.13	
213				.00									.00					
214			−.20	+.09				+.11				+.04	+.07				−.11	
215			−.02	−.21				+.23				−.29	−.09				+.39	
216				.00									.00					
217			+.35	−.13				−.21				−.17	+.10				+.08	
218				+.15				−.15					−.09				+.08	
219				.00									.00					
220	−.18			.00				+.18		−.15			+.06				+.09	
221				.00									.00					
222				−.19				+.22					+.09				−.09	
223	−.29		−.26	+.17		+.25		+.11		+.21		+.08	−.04		−.21		−.05	
224				.00				.00					+.10				−.11	
225				−.03				+.05					+.06				−.05	
226				.00				−.03					+.15				−.14	
227	−.01		+.22	−.24				+.01		−.02		+.03	−.15				+.13	
228	+.03		+.03	+.12				−.17		+.16		+.05	−.19				.00	
229				+.05				−.03					+.03				−.04	
230				−.16				+.16					+.29				−.30	
231				−.09				+.09					−.23				+.24	
232	+.26		−.18	+.01				−.11		−.06		−.04	+.22				−.11	
233				−.12				+.12					−.01				.00	
234				+.02				−.04					+.15				−.16	
235				−.16				+.14					−.09				+.09	
236				−.49				+.51					+.22				−.21	
237				−.28				+.28					−.38				+.38	
238								.00									.00	
239				−.19				+.16					+.11				−.10	
240				−.14				+.14					+.12				−.13	
241	−.01							+.01		+.05							−.05	
242	−.11		−.10	−.06		+.06		+.23		−.14		−.13	.00		+.03		+.25	
243	−.15			+.11		+.08		−.04		+.05			−.16		+.22		−.09	
244								.00									.00	
245				+.06				−.06					−.09				+.09	
246								.00									.00	

TABLE XV (cont'd).—RESIDUALS FROM MEAN OF FINAL CATALOGUE.

No.	$\Delta\alpha \sin \pi$									$\Delta\pi$								
	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
247				−.17				+.14					+.02				−.01	
248								.00									.00	
249								.00									.00	
250	−.07		−.11	−.01½		+.28		−.11		+.11		−.16	+.06		−.01		−.02	
251								.00									.00	
252	−.04			+.06		−.07		+.04		+.18			−.17		−.04		+.01	
253								.00									.00	
254								.00									.00	
255								.00									.00	
256								.00									.00	
257								.00									.00	
258	−.20			+.16		−.18		+.23		−.27			−.13		+.27		+.12	
259	+.04			−.11		+.04		+.04		−.07			−.07		+.03		+.10	
260				−.18		−.11		+.31					−.02		−.27		+.28	
261				−.53				+.50					+.03				−.03	
262								.00									.00	
263								.00									.00	
264				−.45				+.42					+.19				−.18	
265	−.07			−.03		.00		+.11		−.09			+.13		+.07		−.09	
266	+.09		−.04	+.10		−.26		+.13		+.02		−.44	+.11		+.25		+.07	
267				−.46				+.46					−.02				+.03	
268	−.06					+.08		−.03		.00					+.06		−.05	
269								.00									.00	
270								.00									.00	
271								.00									.00	
272				−.40		+.16		+.24					−.22		+.16		+.06	
273								.00									.00	
274								.00									.00	
275								.00									.00	
276								.00									.00	
277								.00									.00	
278								.00									.00	
279	−.14	−.26		−.03		−.04		+.47		+.30	−.01		+.03		−.14		−.17	
280								.00									.00	
281								.00									.00	
282								.00									.00	

283								.00									.00	
284						+.16		−.14							+.03		−.02	
285						−.13		+.13							−.19		+.20	
286								.00									.00	
287								.00									.00	
288	−.06	+.25		+.02		−.08		−.12		+.13	+.09		+.07		−.11		−.18	
289								.00									.00	
290								.00									.00	
291	−.11			+.37		−.19		−.05		−.10			+.13		−.04		.00	
292								.00									.00	
293						−.32		+.32							+.27		−.27	
294						−.28		+.25							+.16		−.15	
295	.00									.00								
296						−.07		+.09							−.27		+.26	
297				+.06		+.10		−.06					−.18		+.13		+.04	
298	+.07	+.36		−.48		+.21		−.15		−.06	+.28		+.15		−.25		−.14	
299								.00									.00	
300						−.10		+.10							−.11		+.10	
301	.00	+.18		+.01		+.07		−.24		−.15	+.04		−.10		−.16		+.39	
302						−.29		+.29							+.39		−.39	
303						.00									.00			
304						−.19		+.19							−.15		+.15	
305						.00		.00							−.14		+.15	
306						+.07		−.07							+.05		−.05	
307						+.16		−.16							+.24		−.23	
308						−.54		+.58							+.43		−.44	
309								.00									.00	
310	−.06	−.13		+.04		+.11		+.08		+.22	−.04		−.58		+.15		+.27	
311								.00									.00	
312						−.27		+.27							+.36		−.35	
313	+.02	−.13		−.14		+.16		+.13		+.23	+.04		−.22		−.06		+.03	
314						−.42		+.42							+.20		−.19	
315	.00									.00								
316	+.12	+.10		−.07		+.01		−.14		−.07	+.23		−.04		−.19		+.05	
317						−.20		+.22							+.06		−.05	
318								.00									.00	
319						−.08		+.06							+.13		−.14	
320						−.08		+.08							+.24		−.24	
321						−.07		+.07							+.09		−.09	
322	+.01	−.17				+.05		+.12		−.04	+.23				−.09		−.10	
323	.00	−.04		+.32		−.03		−.27		+.08	−.03		+.12		−.10		−.06	
324						−.08		+.08							+.23		−.23	
325						−.35		+.32							−.19		+.18	
326						.00									.00			
327						.00		.00							−.05		+.04	
328						.00									.00			

TABLE XV (concl'd).—Residuals from Mean of Final Catalogue.

No.	$\Delta\alpha$ sin =									$\Delta\delta$								
	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15	90°	16–2	16–4	16–5	16–7	18–1	18–3	21–13	21–15
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
329	−.18	+.26		−.23		−.01		−.13		−.11	−.14		−.08		−.18		+.01	
330						.00									.00			
331						−.11		−.09							+.06		−.06	
332		+.05				−.14		−.12			+.17				+.05		−.22	
333						.00									.00			
334	.00									.00								
335	+.02	+.10		−.24		+.03		+.10		−.12	+.03		−.15		+.14		+.11	
336						.00									.00			
337						.00									.00			
338	+.10					−.42		+.32		+.22					+.13		−.34	
339						.00									.00			
340	+.11	+.23		−.54		+.16		+.05		+.18	−.15		−.25		−.05		+.25	
341		+.04				−.11		+.07			+.07				+.15		−.23	
342						.00									.00			
343						.00									.00			
344						.00									.00			
345						.00									.00			
346		+.10				−.18		+.10			+.12				+.17		−.28	
347						.00									.00			
348	+.21	+.07		−.18		−.05		−.04		−.19	.00		−.11		+.05		+.27	
349						.00									.00			
350						.00									.00			
351	−.02	+.12				+.18		−.23		−.24	+.20				.00		+.06	
352						.00									.00			
353	+.22	−.12				−.10		+.04		+.11	+.07				+.05		−.23	
354		+.08				−.10					+.01				−.01			
355						.00									.00			
356	+.02	+.08				+.15		−.27		+.16	+.07				−.13		−.10	
357						.00									.00			
358						.00									.00			
359		−.03				.00					−.17				+.17			
360		−.02				+.05					−.10				+.10			
361						.00									.00			
362	+.18	−.11				−.05		−.01		−.30	+.44				−.01		−.12	
363		−.22				+.22					+.12				−.12			
364		−.08				+.08					−.03				+.02			

365		−.17				+.17					+.22				−.22			
366						.00									.00			
367	+.28	−.10			+.06	−.24		−.02		−.21	+.13			+.12	+.12		−.17	
368		.00				−.02					+.23				−.23			
369		.00				.00					−.07				+.07			
370		−.13				+.13					−.07				+.07			
371		−.03				+.03					−.02				+.01			
372	−.06	−.07			+.18	−.21		+.16		−.03	.00			+.25	−.12		−.09	
373	−.32	+.05			+.19	−.51		+.60		−.23	+.08			+.22	+.17		−.23	
374		+.03				−.06					+.10				−.10			
375		+.13				−.13					−.11				+.11			
376	−.34	+.35			+.37	.00		+.30		+.14	+.17			−.05	−.14		−.12	
377		−.17				+.17					−.02				+.02			
378		−.21				+.18					−.14				+.13			
379		−.08				+.08					−.01				+.02			
380		.00				+.02					+.18				−.18			
381	+.05	−.13			+.02	−.30		+.35		+.04	−.01			+.44	−.20		−.28	
382		−.19				+.19					−.01				+.02			
383	.00									.00								
384	+.12	−.02			+.06	−.16				+.34	−.24			+.06	−.18			
385		.00									.00							
386		+.06				−.06					+.20				−.19			
387	−.36	.00			−.04	+.42				−.23	+.19			+.24	−.21			
388		.00									.00							
389	+.12	−.02			+.02	−.09				+.33	−.12			−.02	−.19			
390	−.22	+.13			−.13	+.26				+.20	−.21			−.01	+.03			
391		.00									.00							
392	+.03	−.11			−.17	−.17			+.41	−.12	+.08			−.06	−.09			+.20
393		−.14			+.17						+.27			−.28				
394		.00									.00							
395		+.29			−.27						−.08			+.07				
396		.00									.00							
397		+.12			−.14						+.18			−.18				
398					.00									.00				
399		+.12			−.10						+.05			−.05				
400		.00									.00							
401		+.21			−.21						−.25			+.25				
402		+.03			−.03						−.11			+.11				
403		+.14			−.12						−.12			+.12				
404		+.34			−.31						+.11			−.11				
405	−.19	.00			−.17	+.28			+.11	+.07	−.08			+.13	−.21			+.11
406	+.23	+.04			−.13				−.17	+.06	−.18			.00				+.11
407		+.04			−.04						+.46			−.46				
408		+.03			−.03						+.14			−.14				

TABLE XVI.—Final Catalogue.

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0 h	m	s	N. P. D.	Decl. 1888.0 °	′	″	Y	X	Number in B. D. M.		Number in Car.
			h	m	s	″	°	′	″	″	″			
1	10.4	10.3	1	45	46	3441.26	89	2	38.74	+ 105.85	+3439.48	88	1	9
2	8.6	8.3	3	38	4	4232.34	88	49	27.66	+ 268.27	+4223.53	88	2	23
3	12.0	12.0	3	49	35	1356.12	89	37	23.88	+ 90.50	+1353.09	....		....
4	10.7	11.5	5	12	36	3420.52	89	2	59.48	+ 310.59	+3406.23	....		....
5	10.3	10.1	5	41	21	768.47	89	47	11.53	+ 76.18	+ 764.68	89	1	6
6	11.5	10.5	5	46	30	5852.92	88	22	27.08	+ 588.86	+5822.44	....		....
7	11.0	11.3	6	22	46	4701.24	88	41	38.76	+ 522.32	+4671.72	....		....
8	11.1	11.0	6	36	23	3156.64	89	7	23.36	+ 363.15	+3135.55	....		....
9	11.0	10.1	8	33	17	4675.25	88	42	4.75	+ 695.40	+4622.83	88	3	73
10	11.9	11.5	9	10	2	4333.61	88	47	46.39	+ 690.36	+4277.94	....		....
11	11.2	10.5	10	18	2	6738.87	88	7	41.13	+1204.78	+6629.07	....		....
12	12.0	10.8	10	32	53	5780.82	88	23	39.18	+1058.10	+5682.39	....		....
13	12.0	12.0	10	48	56	3818.27	88	56	21.73	+ 716.45	+3750.23	....		....
14	6.6	6.7	13	9	31	5678.28	88	25	21.72	+1292.48	+5528.50	88	4	117
15	10.2	10.2	13	37	31	5395.43	88	30	4.57	+1270.86	+5242.99	....		....
16	11.5	11.5	13	45	7	4095.19	88	51	44.81	+ 973.44	+3977.53	....		....
17	9.3	9.9	14	6	30	5788.69	88	23	31.31	+1410.84	+5613.35	88	5	127
18	12.0	11.0	14	9	22	1988.48	89	26	51.52	+ 486.31	+1928.07	....		....
19	9.8	9.1	14	24	0	6964.32	88	3	55.68	+1731.62	+6744.23	87	8	137
20	12.0	10.6	18	15	20	4964.79	88	37	15.21	+1555.10	+4714.45	....		....
21	9.5	9.7	18	20	53	5375.06	88	30	24.94	+1691.82	+5101.22	88	6	172
22	9.5	8.0	18	43	35	7276.24	87	58	43.76	+2335.55	+6889.62	87	12	183
23	9.5	9.5	19	18	30	4655.10	88	42	24.90	+1539.09	+4392.89	88	7	178
24	11.5	10.4	21	18	3	6759.12	88	7	20.88	+2454.91	+6296.25	87	14	197
25	10.5	9.7	24	5	22	2976.88	89	10	23.12	+1215.01	+2717.52	89	2	201
26	12.0	11.5	24	6	7	2140.38	89	24	19.62	+ 874.03	+1953.75	....		....
27	11.5	11.5	24	26	46	6254.21	88	15	45.79	+2587.83	+5692.66	....		....
28	11.1	11.0	25	7	42	3522.69	89	1	17.31	+1495.83	+3189.15	....		....
29	12.0	12.0	27	26	55	3490.54	89	1	49.46	+1608.90	+3097.44	....		....
30	8.7	9.3	27	51	58	6666.80	88	8	53.20	+3115.56	+5892.70	87	16	263
31	11.5	11.0	28	15	52	3279.88	89	5	20.12	+1553.10	+2888.71	....		....
32	12.0	11.5	29	38	40	4008.18	88	53	11.82	+1982.39	+3483.34	....		....
33	11.3	11.5	30	21	22	4112.69	88	51	27.31	+2078.30	+3548.61	....		....
34	8.2	8.4	31	50	35	4876.04	88	38	43.96	+2572.33	+4141.80	88	9	284
35	8.4	8.9	32	59	46	6473.31	88	12	6.69	+3524.68	+5428.34	88	11	305
36	9.8	9.6	35	16	0	3431.23	89	2	48.77	+1981.05	+2801.38	88	10	303
37	10.8	11.0	35	58	51	3148.11	89	7	31.89	+1849.49	+2547.39	....		....
38	10.7	10.3	36	24	28	2237.92	89	22	42.08	+1328.24	+1801.07	89	5	288
39	10.4	9.9	37	14	25	5936.54	88	21	3.46	+3592.06	+4725.44	88	12	331
40	11.9	11.6	37	42	12	4668.80	88	42	11.20	+2855.07	+3693.58	....		....
41	10.8	11.2	38	12	26	2197.77	89	23	22.23	+1359.31	+1726.93	....		....
42	8.9	9.1	38	43	30	5337.32	88	31	2.68	+3338.57	+4163.48	88	13	347
43	10.4	10.4	39	4	32	1633.14	89	32	46.86	+1029.43	+1267.82	89	4	6
44	10.8	10.6	39	39	53	7144.92	88	0	55.08	+4559.64	+5499.00	....		....
45	10.2	9.7	40	2	28	5956.92	88	20	43.08	+3831.78	+4559.88	88	14	362

PRECESSION COEFFICIENTS.

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	″	″	″	″	″	″
1	+0.7685	−20.0740	−0.4480	−0.0118	−0.0009	+0.0119
2	+0.9437	−20.1088	−0.4487	−0.0165	−0.0010	+0.0119
3	+0.3023	−20.0729	−0.4483	+0.0006	−0.0006	+0.0119
4	+0.7611	−20.1197	−0.4491	−0.0117	−0.0009	+0.0119
5	+0.1709	−20.0700	−0.4484	+0.0041	−0.0005	+0.0119
6	+1.3009	−20.1766	−0.4500	−0.0261	−0.0012	+0.0120
7	+1.0438	−20.1646	−0.4499	−0.0192	−0.0010	+0.0120
8	+0.7006	−20.1318	−0.4494	−0.0102	−0.0008	+0.0119
9	+1.0329	−20.2034	−0.4508	−0.0189	−0.0010	+0.0120
10	+0.9558	−20.2029	−0.4508	−0.0169	−0.0010	+0.0120
11	+1.4812	−20.3116	−0.4529	−0.0310	−0.0013	+0.0121
12	+1.2697	−20.2816	−0.4524	−0.0252	−0.0012	+0.0120
13	+0.8380	−20.2098	−0.4511	−0.0138	−0.0009	+0.0120
14	+1.2353	−20.3343	−0.4535	−0.0244	−0.0011	+0.0121
15	+1.1715	−20.3302	−0.4535	−0.0227	−0.0011	+0.0121
16	+0.8887	−20.2666	−0.4524	−0.0151	−0.0009	+0.0120
17	+1.2542	−20.3604	−0.4541	−0.0249	−0.0011	+0.0121
18	+0.4308	−20.1608	−0.4501	−0.0030	−0.0007	+0.0120
19	+1.5069	−20.4286	−0.4555	−0.0316	−0.0013	+0.0121
20	+1.0534	−20.3948	−0.4550	−0.0196	−0.0010	+0.0121
21	+1.1398	−20.4243	−0.4556	−0.0219	−0.0011	+0.0121
22	+1.5394	−20.5625	−0.4585	−0.0326	−0.0013	+0.0122
23	+0.9815	−20.3919	−0.4550	−0.0177	−0.0010	+0.0121
24	+1.4068	−20.5908	−0.4591	−0.0291	−0.0012	+0.0122
25	+0.6072	−20.3225	−0.4537	−0.0078	−0.0008	+0.0121
26	+0.4365	−20.2473	−0.4521	−0.0031	−0.0007	+0.0120
27	+1.2720	−20.6221	−0.4600	−0.0256	−0.0012	+0.0122
28	+0.7126	−20.3844	−0.4551	−0.0105	−0.0008	+0.0121
29	+0.6921	−20.4097	−0.4556	−0.0100	−0.0008	+0.0121
30	+1.3167	−20.7387	−0.4626	−0.0268	−0.0012	+0.0123
31	+0.6454	−20.3975	−0.4553	−0.0088	−0.0008	+0.0121
32	+0.7783	−20.4922	−0.4574	−0.0124	−0.0009	+0.0122
33	+0.7929	−20.5135	−0.4579	−0.0128	−0.0009	+0.0122
34	+0.9254	−20.6223	−0.4601	−0.0164	−0.0009	+0.0122
35	+1.2129	−20.8307	−0.4647	−0.0242	−0.0011	+0.0124
36	+0.6259	−20.4929	−0.4575	−0.0083	−0.0008	+0.0122
37	+0.5692	−20.4639	−0.4568	−0.0068	−0.0007	+0.0121
38	+0.4024	−20.3487	−0.4544	−0.0023	−0.0006	+0.0121
39	+1.0558	−20.8474	−0.4651	−0.0200	−0.0010	+0.0124
40	+0.8253	−20.6859	−0.4617	−0.0137	−0.0009	+0.0123
41	+0.3859	−20.3556	−0.4546	−0.0019	−0.0006	+0.0121
42	+0.9303	−20.7923	−0.4640	−0.0166	−0.0010	+0.0123
43	+0.2833	−20.2824	−0.4530	+0.0010	−0.0006	+0.0120
44	+1.2287	−21.0599	−0.4698	−0.0247	−0.0011	+0.0125
45	+1.0188	−20.9009	−0.4664	−0.0190	−0.0010	+0.0124

TABLE XVI.—FINAL CATALOGUE (cont'd).

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0 h	m	s	N. P. D. ″	Decl. 1888.0 °	′	″	Y ″	X ″	Number in B. D. M.	Number in Car.
46	11.8	11.6	40	22	6	3326.14	88	56	13.36	−2478.04	+2914.95	....	....
47	8.7	9.3	41	26	30	1297.14	89	38	22.36	+ 858.52	+ 972.37	89 3	b
48	9.0	8.5	41	27	42	6864.53	88	5	35.47	−4544.30	+5143.32	87 23	375
49	10.2	10.3	41	37	56	3039.63	89	9	20.37	−2019.29	+2271.81	89 6	345
50	10.7	11.2	42	6	25	6133.44	88	16	56.56	−4145.48	+4586.77	....	....
51	11.5	11.2	42	57	44	6221.02	88	16	18.98	−4239.07	+4551.86	....	....
52	10.9	10.7	43	16	56	6506.76	88	11	33.24	−4460.25	+4736.04	....	....
53	11.5	11.2	43	38	13	5859.90	88	22	20.10	+4043.29	+4240.39	....	....
54	11.1	11.2	44	28	15	1427.86	89	36	12.14	−1000.27	+1018.92	....	....
55	9.6	9.1	44	39	0	7344.01	87	57	35.99	+5160.10	+5223.52	87. 27	412
56	10.8	10.3	45	14	17	4157.78	88	50	12.22	+2973.28	+2948.67	88 15	397
57	9.6	9.9	45	23	33	5738.40	88	24	21.60	+4084.84	+4029.25	88 16	409
58	11.2	10.4	46	6	56	7226.40	87	59	33.60	+5207.28	+5008.37	....	....
59	11.2	9.7	46	58	24	6159.48	88	17	20.52	+4502.13	+4202.22	88 17	428
60	11.5	12.0	52	32	55	3226.03	89	6	13.97	+2560.94	+1961.63	....	....
61	11.2	11.2	52	57	8	4079.58	88	52	0.42	+3255.85	+2457.72	....	....
62	11.8	11.4	55	44	46	1909.24	89	28	10.76	+1578.06	+1074.62	....	....
63	11.4	11.5	56	6	7	3548.84	89	0	51.16	+2945.51	+1979.15	....	....
64	10.8	10.3	56	9	25	5924.78	88	21	15.22	+4920.23	+3299.16	88 18	500
65	12.0	12.0	56	52	13	5663.50	88	25	36.50	+4742.21	+3094.91	....	....
66	9.8	10.1	57	50	49	2939.55	89	11	00.45	+2488.63	+1564.32	89 7	475
67	11.2	11.0	58	38	50	2652.68	89	15	47.32	+2265.27	+1380.17	....	....
68	8.0	8.7	59	23	4	7216.88	87	59	43.12	+6209.60	+3674.63	87 33	545
69	10.8	9.9	59	48	6	5046.06	88	35	53.94	+4360.81	+2537.88	88 19	529
70	10.5	10.7	60	27	42	6813.07	88	6	26.93	+5926.47	+3358.28	88 21	....
71	11.1	10.8	60	45	19	4049.78	88	52	30.22	+3533.38	+1978.36	....	....
72	12.0	11.8	61	57	56	5063.21	88	35	36.79	+4468.67	+2379.48	....	....
73	11.2	11.2	62	1	26	4717.66	88	41	22.34	+4166.00	+2212.88	....	....
74	10.1	9.7	63	16	52	3614.00	88	59	46.00	+3227.94	+1624.82	88 20	538
75	11.4	11.5	63	48	48	3210.64	89	6	29.36	+2880.99	+1416.79	....	....
76	12.0	12.0	64	1	43	2726.61	89	14	33.39	+2451.18	+1194.01	....	....
77	11.2	11.3	64	10	47	4684.93	88	41	55.07	+4216.85	+2040.35	....	....
78	10.8	11.3	64	44	6	5256.08	88	32	23.92	+4752.78	+2243.08	....	....
79	11.8	11.8	65	4	33	3701.33	88	58	18.67	+3356.43	+1559.72	....	....
80	12.0	11.8	65	15	38	2784.75	89	13	35.25	+2529.09	+1165.36	....	....
81	11.5	11.2	66	8	6	5745.58	88	24	14.42	+5253.66	+2324.27	....	....
82	10.2	10.3	67	35	10	3994.26	88	53	25.74	+3692.28	+1522.90	88 22	585
83	11.0	10.6	67	39	54	3559.16	89	0	40.84	+3291.97	+1352.49	....	....
84	10.5	10.3	68	35	33	6756.93	88	7	23.07	+6289.61	+2465.82	88 23	....
85	11.1	10.3	68	52	6	3276.64	89	5	23.36	+3056.17	+1181.22	89 8	579
86	10.2	10.3	69	19	36	6755.93	88	7	24.07	+6319.77	+2384.68	88 24	627
87	12.0	11.0	69	26	7	5132.68	88	34	27.32	+4805.11	+1802.74	....	....
88	11.8	11.5	70	23	5	3904.14	88	54	55.86	+3677.35	+1310.55	....	....
89	10.6	10.8	70	48	13	1238.67	89	39	21.33	+1169.78	+ 407.28	....	....
90	10.8	10.3	72	50	49	6984.30	88	3	35.70	+6672.37	+2059.45	88 25	660

PRECESSION COEFFICIENTS (cont'd).

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	″	″	″	″	″	″
46	+0.6513	−20.6023	−0.4600	−0.0090	−0.0008	+0.0122
47	+0.2173	−20.2445	−0.4523	+0.0027	−0.0005	+0.0120
48	+1.1492	−21.0574	−0.4698	−0.0225	−0.0011	+0.0125
49	+0.5076	−20.5021	−0.4578	−0.0052	−0.0007	+0.0122
50	+1.0249	−20.9704	−0.4680	−0.0193	−0.0010	+0.0124
51	+1.0171	−20.9912	−0.4685	−0.0190	−0.0010	+0.0125
52	+1.0582	−21.0397	−0.4694	−0.0202	−0.0010	+0.0125
53	+0.9475	−20.9484	−0.4675	−0.0172	−0.0010	+0.0124
54	+0.2277	−20.2761	−0.4530	+0.0024	−0.0005	+0.0120
55	+1.1671	−21.1934	−0.4728	−0.0231	−0.0011	+0.0126
56	+0.6588	−20.7132	−0.4624	−0.0093	−0.0008	+0.0123
57	+0.9003	−20.9580	−0.4677	−0.0159	−0.0009	+0.0124
58	+1.1191	−21.2043	−0.4731	−0.0219	−0.0011	+0.0126
59	+0.9389	−21.0500	−0.4698	−0.0169	−0.0010	+0.0125
60	+0.4383	−20.6228	−0.4605	−0.0034	−0.0007	+0.0122
61	+0.5492	−20.7766	−0.4640	−0.0064	−0.0007	+0.0123
62	+0.2401	−20.4048	−0.4558	+0.0020	−0.0005	+0.0121
63	+0.4422	−20.7082	−0.4624	−0.0036	−0.0007	+0.0123
64	+0.7372	−21.1442	−0.4720	−0.0117	−0.0008	+0.0126
65	+0.6915	−21.1051	−0.4712	−0.0105	−0.0008	+0.0125
66	+0.3495	−20.6071	−0.4602	−0.0010	−0.0006	+0.0122
67	+0.3084	−20.5576	−0.4591	+0.0001	−0.0006	+0.0122
68	+0.8210	−21.4283	−0.4783	−0.0141	−0.0010	+0.0127
69	+0.5671	−21.0215	−0.4693	−0.0071	−0.0007	+0.0125
70	+0.7504	−21.3663	−0.4769	−0.0122	−0.0009	+0.0127
71	+0.4420	−20.8387	−0.4653	−0.0037	−0.0007	+0.0124
72	+0.5317	−21.0455	−0.4699	−0.0061	−0.0007	+0.0125
73	+0.4944	−20.9786	−0.4684	−0.0051	−0.0007	+0.0125
74	+0.3630	−20.7712	−0.4639	−0.0014	−0.0006	+0.0123
75	+0.3166	−20.6943	−0.4622	−0.0002	−0.0006	+0.0123
76	+0.2668	−20.5990	−0.4600	+0.0012	−0.0006	+0.0122
77	+0.4559	−20.9901	−0.4687	−0.0041	−0.0007	+0.0125
78	+0.5012	−21.1086	−0.4713	−0.0054	−0.0007	+0.0125
79	+0.3485	−20.7999	−0.4646	−0.0012	−0.0006	+0.0123
80	+0.2604	−20.6164	−0.4604	+0.0014	−0.0006	+0.0122
81	+0.5193	−21.2192	−0.4738	−0.0059	−0.0007	+0.0126
82	+0.3403	−20.8743	−0.4662	−0.0009	−0.0006	+0.0124
83	+0.3022	−20.7857	−0.4643	+0.0001	−0.0006	+0.0123
84	+0.5510	−21.4476	−0.4789	−0.0070	−0.0008	+0.0127
85	+0.2639	−20.7334	−0.4631	+0.0012	−0.0006	+0.0123
86	+0.5328	−21.4544	−0.4790	−0.0064	−0.0008	+0.0127
87	+0.4028	−21.1205	−0.4717	−0.0028	−0.0006	+0.0125
88	+0.2928	−20.8712	−0.4662	+0.0003	−0.0006	+0.0124
89	+0.0910	−20.3141	−0.4538	+0.0060	−0.0005	+0.0121
90	+0.4602	−21.5325	−0.4808	−0.0045	−0.0008	+0.0128

TABLE XVI.—Final Catalogue (cont'd).

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0 °	′	″	N. P. D. ″	Decl. 1888.0 °	′	″	Y ″	X ″	Number in B. D. M.	Number in Car.
91	11.2	11.8	73	6	24	5426.73	89	29	33.27	+5191.96	+1576.78	....	......
92	10.2	10.3	74	34	46	6565.41	88	10	34.59	+6328.00	+1745.46	88 27	679
93	10.8	10.3	75	0	24	2360.14	89	20	39.86	+2279.74	+ 610.57	....	......
94	10.4	10.4	75	5	50	5223.42	88	32	56.58	+5047.19	+1343.22	88 26	679
95	11.5	11.5	78	15	12	4716.83	88	41	23.17	+4617.64	+ 960.19	....	......
96	11.5	10.6	78	16	58	6891.98	88	5	8.02	+6747.10	+1399.37	....	......
97	11.0	10.7	80	20	20	4761.70	88	40	38.30	+4693.75	+ 799.04	....	......
98	11.0	10.5	80	29	22	6892.03	88	5	7.97	+6796.04	+1138.55	88 28	734
99	10.1	10.3	82	29	10	5924.86	88	21	15.14	+5873.18	+ 774.67	88 30	744
100	11.2	10.8	83	5	14	2178.92	89	23	41.08	+2163.04	+ 262.25	....	......
101	9.6	9.3	83	37	11	4585.70	88	43	34.30	+4556.92	+ 509.55	88 29	739
102	12.0	11.5	84	28	4	2849.37	89	12	30.63	+2836.01	+ 274.69	....	......
103	12.0	11.5	84	52	59	3116.82	89	8	3.18	+3104.28	+ 277.98	....	......
104	10.4	9.3	84	57	20	5685.86	88	25	14.14	+5663.12	+ 499.88	88 31	763
105	12.0	11.1	88	10	39	3508.81	89	1	31.19	+3506.87	+ 111.59	....	......
106	11.8	11.2	88	43	34	1771.64	89	30	28.36	+1771.18	+ 39.39	....	......
107	9.6	10.2	89	21	30	5049.44	88	35	50.56	+5048.61	+ 56.54	88 33	791
108	11.1	9.7	90	7	12	3799.01	88	56	40.99	+3798.78	− 7.96	88 32	778
109	11.4	10.8	90	10	24	5490.70	88	28	29.30	+5490.04	− 16.61	....	......
110	11.4	11.4	90	13	27	3600.86	88	59	59.14	+3600.65	− 14.09	....	......
111	11.4	10.8	90	49	53	4720.84	88	41	19.16	+4719.93	− 68.49	....	......
112	11.0	11.0	91	1	48	4015.05	88	53	4.95	+4014.15	− 72.17	....	......
113	11.6	10.8	91	24	52	3234.73	89	6	5.27	+3233.61	− 79.84	....	......
114	9.1	9.4	91	58	1	5994.83	88	20	5.17	+5990.46	− 205.73	88 35	823
115	11.4	10.4	92	16	1	4024.40	88	52	55.60	+4020.99	− 159.18	....	......
116	10.2	9.3	92	21	11	1887.07	89	28	32.93	+1885.45	− 77.48	89 9	*d*
117	11.0	10.2	93	6	36	7282.53	87	58	37.47	+7270.30	− 395.02	87 44	......
118	10.6	10.4	93	8	8	4943.14	88	37	36.86	+4935.26	− 270.36	88 34	822
119	12.0	11.2	94	58	10	3473.56	89	2	6.44	+3460.33	− 300.88	....	......
120	10.7	10.1	97	5	1	4154.97	88	50	45.03	+4122.99	− 512.35	88 36	847
121	10.2	10.3	98	0	20	3262.89	89	5	37.11	+3230.96	− 454.40	89 10	833
122	11.5	10.3	99	38	20	6825.53	88	6	14.47	+6727.91	−1142.64	88 37	912
123	10.7	10.5	100	1	57	809.14	89	46	30.86	+ 796.77	− 140.96	....	......
124	10.4	10.3	100	44	54	3444.26	89	2	35.74	+3383.68	− 642.31	89 11	862
125	12.0	12.0	101	19	30	1556.64	89	34	3.36	+1526.31	− 305.68	....	......
126	11.4	10.5	104	55	48	6934.88	88	4	25.12	+6699.50	−1786.36	....	......
127	10.6	10.4	107	49	11	4382.68	88	46	57.32	+4172.10	−1341.10	88 38	......
128	11.0	10.2	110	25	48	6681.01	88	8	38.99	+6259.67	−2331.68	88 40	1022
129	11.0	10.6	113	12	12	3530.46	89	1	9.54	+3244.72	−1390.92	....	......
130	9.5	9.7	115	13	10	4713.80	88	41	26.20	+4264.12	−2008.31	88 41	1046
131	6.8	7.0	116	8	41	3728.31	88	57	51.69	+3346.65	−1642.75	89 13	1035
132	10.3	10.3	118	21	54	3809.97	88	56	30.03	+3352.35	−1809.96	89 14	1060
133	11.4	10.5	119	11	25	4445.12	88	45	54.88	+3880.32	−2167.77	88 42	1086
134	9.6	9.7	119	14	39	4620.47	88	42	59.53	+4031.24	−2257.06	88 43	1089
135	9.6	10.0	120	19	27	1531.06	88	34	28.94	+1321.57	− 773.01	89 12	*e*

PRECESSION COEFFICIENTS (cont'd).

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	"	"	"	"	"	"
91	+0.3523	−21.2062	−0.4736	−0.0015	−0.0006	+0.0126
92	+0.3900	−21.4568	−0.4792	−0.0027	−0.0007	+0.0127
93	+0.1364	−20.5612	−0.4593	+0.0047	−0.0005	+0.0122
94	+0.3001	−21.1744	−0.4729	−0.0001	−0.0006	+0.0126
95	+0.2145	−21.0796	−0.4709	+0.0023	−0.0005	+0.0125
96	+0.3127	−21.5495	−0.4813	−0.0006	−0.0007	+0.0128
97	+0.1785	−21.0965	−0.4712	+0.0033	−0.0005	+0.0125
98	+0.2544	−21.5604	−0.4815	+0.0009	−0.0007	+0.0128
99	+0.1731	−21.3571	−0.4771	+0.0032	−0.0006	+0.0127
100	+0.0586	−20.5353	−0.4588	+0.0067	−0.0004	+0.0122
101	+0.1129	−21.0663	−0.4706	+0.0050	−0.0005	+0.0125
102	+0.0614	−20.6849	−0.4622	+0.0066	−0.0004	+0.0123
103	+0.0621	−20.7444	−0.4635	+0.0065	−0.0004	+0.0123
104	+0.1117	−21.3109	−0.4760	+0.0048	−0.0006	+0.0127
105	+0.0249	−20.8338	−0.4655	+0.0074	−0.0004	+0.0124
106	+0.0088	−20.4480	−0.4568	+0.0082	−0.0004	+0.0121
107	+0.0126	−21.1752	−0.4731	+0.0076	−0.0004	+0.0126
108	−0.0018	−20.8985	−0.4670	+0.0081	−0.0004	+0.0124
109	−0.0037	−21.2727	−0.4753	+0.0079	−0.0005	+0.0126
110	−0.0031	−20.8545	−0.4660	+0.0082	−0.0004	+0.0124
111	−0.0153	−21.1024	−0.4715	+0.0083	−0.0004	+0.0125
112	−0.0161	−20.9462	−0.4680	+0.0084	−0.0004	+0.0124
113	−0.0178	−20.7731	−0.4641	+0.0087	−0.0004	+0.0123
114	−0.0460	−20.3831	−0.4778	+0.0090	−0.0005	+0.0127
115	−0.0356	−20.9477	−0.4681	+0.0089	−0.0004	+0.0124
116	−0.0173	−20.4735	−0.4574	+0.0088	−0.0004	+0.0122
117	−0.0883	−21.6651	−0.4842	+0.0099	−0.0004	+0.0129
118	−0.0604	−21.1500	−0.4726	+0.0095	−0.0004	+0.0126
119	−0.0672	−20.8235	−0.4653	+0.0099	−0.0004	+0.0124
120	−0.1145	−20.9702	−0.4687	+0.0110	−0.0003	+0.0124
121	−0.1015	−20.7725	−0.4642	+0.0109	−0.0003	+0.0123
122	−0.2553	−21.5454	−0.4816	+0.0145	−0.0003	+0.0128
123	−0.0315	−20.2310	−0.4521	+0.0093	−0.0004	+0.0120
124	−0.1435	−20.8063	−0.4650	+0.0119	−0.0003	+0.0123
125	−0.0683	−20.3935	−0.4557	+0.0102	−0.0004	+0.0121
126	−0.3991	−21.5387	−0.4814	+0.0183	−0.0003	+0.0128
127	−0.2997	−20.9807	−0.4690	+0.0160	−0.0002	+0.0125
128	−0.5210	−21.4411	−0.4793	+0.0215	−0.0002	+0.0127
129	−0.3108	−20.7751	−0.4644	+0.0165	−0.0002	+0.0123
130	−0.4487	−21.0007	−0.4695	+0.0199	−0.0001	+0.0125
131	−0.3671	−20.7976	−0.4649	+0.0178	−0.0002	+0.0123
132	−0.4044	−20.7987	−0.4650	+0.0189	−0.0002	+0.0123
133	−0.4844	−20.9154	−0.4677	+0.0210	−0.0001	+0.0124
134	−0.5043	−20.9487	−0.4683	+0.0214	−0.0001	+0.0124
135	−0.1727	−20.3478	−0.4548	+0.0130	−0.0003	+0.0121

TABLE XVI.—FINAL CATALOGUE (cont'd).

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0			N. P. D.	Decl. 1888.0			Y	X	Number in B. D. M.		Number in Car.
			°	′	″	″	°	′	″	″	″			
136	11.4	10.6	121	20	58	6634.58	88	9	25.42	+5665.02	−3451.08	88	45	1135
137	9.9	9.7	121	39	2	3802.37	88	56	37.63	+3236.63	−1995.13	89	15	1095
138	11.3	10.2	121	59	18	6315.34	88	14	44.66	+5355.56	−3345.01	88	46	1140
139	9.6	10.3	124	46	17	4722.68	88	41	17.32	+3879.02	−2693.12	88	47	1149
140	10.2	10.3	129	1	50	6663.16	88	8	56.84	+5175.10	−4195.29	88	49	1214
141	10.8	10.6	131	57	23	5963.46	88	20	36.54	+4434.13	−3986.39	88	51	....
142	10.4	10.3	132	37	15	7531.08	87	54	28.92	+5540.51	−5098.49	88	53	1255
143	12.0	12.0	133	1	15	5422.89	88	29	37.11	+3964.25	−3699.42	....		....
144	10.9	10.5	133	34	30	1297.98	89	38	22.02	+ 940.34	− 894.70	....		....
145	10.2	10.7	134	13	34	4824.80	88	39	35.20	+3457.10	−3364.95	88	52	....
146	12.0	12.0	136	29	50	2450.65	89	19	9.35	+1686.96	−1777.51	....		....
147	9.6	10.3	138	7	20	4343.52	88	47	36.48	+2899.28	−3233.82	88	54	1287
148	11.0	11.0	138	15	41	6088.86	88	18	31.14	+4052.97	−4542.78	....		....
149	9.6	9.7	140	13	45	6235.21	88	16	4.79	+3988.17	−4791.70	88	55	1336
150	11.0	10.3	140	36	44	6549.40	88	10	50.60	+4155.34	−5060.99	88	56	1344
151	10.2	10.3	142	20	42	5523.86	88	27	56.14	+3374.15	−4372.73	88	57	1359
152	11.2	12.0	142	52	57	3071.32	89	8	48.68	+1853.32	−2448.98	....		....
153	11.0	12.0	143	58	37	3037.90	89	9	22.10	+1786.55	−2456.91	....		....
154	11.0	12.0	144	28	55	4848.88	88	39	11.12	+2816.74	−3946.30	....		....
155	11.0	11.5	145	43	40	4245.70	88	49	14.30	+2390.69	−3508.27	....		....
156	11.0	10.6	146	34	5	5806.87	88	23	13.13	+3198.84	−4845.42	....		....
157	11.3	11.5	146	39	41	3919.63	88	54	40.37	+2154.04	−3274.40	....		....
158	10.0	10.2	146	44	17	5111.31	88	34	48.69	+2803.09	−4273.49	88	58	1413
159	11.0	10.3	150	30	52	1766.24	89	30	33.76	+ 869.34	−1537.46	89	16	f
160	10.4	10.3	151	5	58	4956.82	88	37	23.18	+2395.36	−4339.08	88	59	1456
161	8.8	8.7	153	39	23	5621.93	88	26	18.07	+2494.43	−5037.46	88	60	1490
162	11.0	11.0	154	5	6	2014.11	89	26	25.89	+ 880.23	−1811.55	....		....
163	12.0	12.0	156	43	6	3417.52	89	3	2.48	+1350.72	−3139.09	....		....
164	11.0	10.4	158	16	52	4425.76	88	46	14.24	+1637.64	−4111.26	88	62	1549
165	10.5	10.9	162	10	1	5392.87	88	30	7.13	+1651.35	−5133.18	....		....
166	9.6	10.2	162	10	25	4907.08	88	38	12.92	+1502.08	−4671.04	88	63	1592
167	12.0	12.0	162	24	16	3324.35	89	4	35.65	+1004.89	−3168.67	....		....
168	9.2	9.7	163	25	30	2301.11	89	21	38.89	+ 656.42	−2205.44	89	17	g
169	6.5	7.5	165	22	28	6305.92	88	14	54.08	+1592.00	−6100.61	88	64	1639
170	11.4	11.3	167	4	23	5819.68	88	23	0.32	+1301.74	−5671.44	....		....
171	11.5	11.8	167	51	22	5165.72	88	33	54.28	+1086.58	−5049.59	....		....
172	11.2	11.5	167	52	12	6885.95	88	5	14.05	+1446.68	−6730.96	....		....
173	9.2	9.9	169	10	54	5414.60	88	29	45.40	+1016.18	−5317.75	88	65	1676
174	12.0	11.5	169	57	30	6036.84	88	19	23.16	+1052.46	−5943.52	....		....
175	10.6	11.0	171	19	21	2259.83	89	22	20.17	+ 340.94	−2233.92	89	19	....
176	9.5	9.9	171	42	51	6094.15	88	18	25.85	+ 878.11	−6029.66	88	67	1705
177	9.9	9.9	172	39	42	1617.45	89	33	2.55	+ 206.59	−1604.18	89	18	h
178	9.8	10.3	174	4	36	5745.24	88	24	14.76	+ 592.82	−5713.82	88	68	1737
179	10.0	9.9	175	9	34	4364.68	88	47	15.32	+ 368.28	−4348.79	88	69	1748
180	10.8	10.3	177	13	44	5540.04	88	27	39.96	+ 267.81	−5532.89	88	70	1767

PRECESSION COEFFICIENTS (cont'd).

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	″	″	″	″	″	″
136	−0.7711	−21.3085	−0.4766	+0.0283	0.0000	+0.0127
137	−0.4458	−20.7729	−0.4645	+0.0201	−0.0001	+0.0123
138	−0.7474	−21.2403	−0.4750	+0.0278	0.0000	+0.0126
139	−0.6017	−20.9145	−0.4677	+0.0241	0.0000	+0.0124
140	−0.9374	−21.1989	−0.4742	+0.0328	+0.0002	+0.0126
141	−0.8907	−21.0354	−0.4706	+0.0317	+0.0001	+0.0125
142	−1.1392	−21.2777	−0.4761	+0.0381	+0.0002	+0.0126
143	−0.8266	−20.9319	−0.4682	+0.0301	+0.0001	+0.0124
144	−0.1999	−20.2628	−0.4529	+0.0138	−0.0003	+0.0120
145	−0.7519	−20.8200	−0.4657	+0.0281	0.0000	+0.0124
146	−0.3972	−20.4285	−0.4566	+0.0190	−0.0002	+0.0121
147	−0.7226	−20.6964	−0.4629	+0.0274	0.0000	+0.0123
148	−1.0150	−20.9500	−0.4687	+0.0350	+0.0002	+0.0124
149	−1.0706	−20.9351	−0.4685	+0.0365	+0.0002	+0.0124
150	−1.1308	−20.9714	−0.4693	+0.0381	+0.0003	+0.0125
151	−0.9770	−20.7998	−0.4653	+0.0341	+0.0002	+0.0123
152	−0.5472	−20.4650	−0.4575	+0.0229	−0.0001	+0.0121
153	−0.5490	−20.4501	−0.4572	+0.0230	−0.0001	+0.0121
154	−0.8818	−20.6769	−0.4625	+0.0317	+0.0001	+0.0123
155	−0.7839	−20.5830	−0.4604	+0.0291	+0.0001	+0.0122
156	−1.0827	−20.7599	−0.4646	+0.0370	+0.0002	+0.0123
157	−0.7316	−20.5308	−0.4592	+0.0277	0.0000	+0.0122
158	−0.9549	−20.6733	−0.4625	+0.0336	+0.0002	+0.0123
159	−0.3435	−20.2465	−0.4525	+0.0176	−0.0002	+0.0120
160	−0.9695	−20.5825	−0.4605	+0.0341	+0.0002	+0.0122
161	−1.1256	−20.6030	−0.4610	+0.0382	+0.0003	+0.0122
162	−0.4048	−20.2488	−0.4527	+0.0193	−0.0002	+0.0120
163	−0.7014	−20.3521	−0.4551	+0.0271	0.0000	+0.0121
164	−0.9186	−20.4144	−0.4568	+0.0328	+0.0001	+0.0121
165	−1.1469	−20.4152	−0.4568	+0.0389	+0.0003	+0.0121
166	−1.0437	−20.3830	−0.4560	+0.0361	+0.0002	+0.0121
167	−0.7080	−20.2750	−0.4534	+0.0273	0.0000	+0.0120
168	−0.4928	−20.1985	−0.4516	+0.0216	−0.0001	+0.0120
169	−1.3631	−20.3994	−0.4565	+0.0446	+0.0004	+0.0121
170	−1.2672	−20.3360	−0.4552	+0.0421	+0.0004	+0.0121
171	−1.1283	−20.2896	−0.4540	+0.0384	+0.0003	+0.0121
172	−1.5040	−20.3651	−0.4559	+0.0484	+0.0005	+0.0121
173	−1.1882	−20.2733	−0.4537	+0.0401	+0.0003	+0.0120
174	−1.3280	−20.2797	−0.4539	+0.0438	+0.0004	+0.0120
175	−0.4991	−20.1281	−0.4500	+0.0218	−0.0001	+0.0119
176	−1.3473	−20.2406	−0.4531	+0.0443	+0.0004	+0.0120
177	−0.3584	−20.0987	−0.4493	+0.0181	−0.0002	+0.0119
178	−1.2767	−20.1778	−0.4517	+0.0424	+0.0004	+0.0120
179	−0.9717	−20.1309	−0.4504	+0.0343	+0.0002	+0.0119
180	−1.2363	−20.1056	−0.4500	+0.0414	+0.0003	+0.0119

TABLE XVI.—FINAL CATALOGUE (cont'd).

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0 °	′	″	N. P. D. ″	Decl. 1888.0 °	′	″	Y ″	X ″	Number in B. D. M.	Number in Car.
181	10.5	11.3	177	29	28	4559.02	88	44	0.98	+ 199.55	−4554.28	....	....
182	10.8	10.9	180	26	56	2687.17	89	15	12.83	− 21.05	−2687.01	89 20	....
183	12.0	11.5	180	44	45	3061.32	89	8	58.68	− 39.85	−3060.95	....	....
184	6.4	6.2	183	35	36	6045.38	88	19	14.62	− 378.84	−6032.63	88 71	1834
185	9.0	9.9	183	49	4	5756.50	88	24	3.50	− 383.24	−5742.98	88 72	1837
186	9.3	10.1	185	13	56	5470.46	88	28	49.54	− 498.81	−5447.04	88 73	1855
187	12.0	11.8	185	43	19	3246.22	89	5	53.78	− 323.64	−3229.91	....	....
188	11.1	11.5	188	7	58	4176.76	88	50	23.24	− 590.84	−4134.48	....	....
189	10.7	11.0	191	13	24	2067.00	89	25	33.00	− 402.30	−2027.44	....	....
190	10.7	11.5	191	16	59	2540.66	89	17	39.34	− 497.08	−2491.49	....	....
191	8.8	9.7	191	22	9	2547.70	89	17	32.30	− 502.22	−2497.64	89 21	i
192	8.8	9.6	191	53	20	5091.97	88	35	8.03	−1048.91	−4982.24	88 75	1922
193	9.4	9.1	193	33	36	3719.06	88	58	0.94	− 871.93	−3615.19	89 22	1951
194	11.0	11.0	194	48	42	5482.12	88	28	37.88	−1401.30	−5299.34	....	....
195	10.8	11.3	194	52	40	4456.33	88	45	43.67	−1144.11	−4306.59	....	....
196	10.5	10.3	195	35	14	2274.04	89	22	5.96	− 611.03	−2190.36	89 23	k
197	12.0	11.0	195	39	25	3201.46	89	6	38.54	− 863.96	−3082.54	....	....
198	10.8	10.8	196	26	0	4928.98	88	37	51.02	−1394.27	−4727.18	....	....
199	8.0	7.5	196	34	53	6297.58	88	15	2.42	−1796.90	−6034.76	88 76	1972
200	10.6	10.3	196	52	15	862.10	89	45	37.90	− 250.19	− 824.99	89 26	m
201	11.3	11.2	197	5	39	4173.91	88	50	26.09	−1226.81	−3989.25	....	....
202	12.0	11.2	197	9	28	2258.50	89	22	21.50	− 666.25	−2157.94	....	....
203	11.5	11.2	198	23	16	5009.08	88	36	30.92	−1579.94	−4752.86	....	....
204	11.6	11.5	199	9	39	4410.34	88	46	29.66	−1447.45	−4165.70	....	....
205	11.1	11.2	199	30	55	4514.51	88	44	45.49	−1507.99	−4254.82	....	....
206	11.7	11.0	200	9	43	3636.93	88	59	23.07	−1253.49	−3413.88	....	....
207	10.9	11.2	200	18	19	4236.52	88	49	23.48	−1470.06	−3972.96	....	....
208	12.0	12.0	200	43	26	4517.41	88	44	42.59	−1598.43	−4224.78	....	....
209	10 9	10.7	200	59	35	3107.09	89	8	12.91	−1113.09	−2900.75	....	....
210	11.5	12.0	201	44	5	811.27	89	46	28.73	− 300.42	− 753.59	....	....
211	7.8	8.5	202	17	25	6750.52	88	7	29.48	−2560.01	−6244.97	88 77	2048
212	9.6	9.7	202	47	46	4756.20	88	40	43.80	−1842.64	−4384.30	88 78	2068
213	11.0	10.6	202	55	29	4877.59	88	38	42.41	−1899.74	−4491.92	88 79	2069
214	10.8	11.5	203	49	6	3634.17	88	59	25.83	−1467.54	−3324.48	....	....
215	11.7	11.8	203	50	29	4011.43	88	53	8.57	−1621.34	−3668.90	....	....
216	12.0	11.5	205	31	34	5083.12	88	35	16.88	−2190.21	−4586.50	....	....
217	11.9	10.8	206	4	42	3400.48	89	3	19.52	−1494.78	−3054.15	....	....
218	12.0	10.9	206	30	59	4253.26	88	49	6.74	−1898.75	−3805.57	....	....
219	10.0	10.8	208	52	54	6066.12	88	18	53.88	−2929.52	−5310.84	....	....
220	10.8	10.7	210	18	48	3710.82	88	58	9.18	−1872.85	−3203.29	....	....
221	10.5	10.6	210	49	35	6397.43	88	13	22.57	−3277.77	−5492.75	88 81	....
222	10.2	10.3	211	5	0	5570.87	88	27	9.13	−2875.80	−4770.40	88 82	2148
223	9.2	10.1	211	37	1	1657.89	89	32	22.11	− 869.12	−1411.80	89 25	l
224	10.0	10.3	212	29	21	6118.76	88	18	1.24	−3286.14	−5160.38	88 23	2160
225	10.3	10.3	213	31	36	5552.32	88	27	27.68	−3066.31	−4628.02	88 84	2173

PRECESSION COEFFICIENTS (cont'd).

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	"	"	"	"	"	"
181	−1.0176	−20.0928	−0.4495	+0.0356	+0.0002	+0.0119
182	−0.6004	−20.0467	−0.4483	+0.0246	0.0000	+0.0119
183	−0.6839	−20.0420	−0.4482	+0.0268	0.0000	+0.0119
184	−1.3479	−19.9599	−0.4468	+0.0445	+0.0004	+0.0118
185	−1.2832	−19.9597	−0.4468	+0.0428	+0.0004	+0.0118
186	−1.2171	−19.9345	−0.4462	+0.0411	+0.0003	+0.0118
187	−0.7217	−19.9783	−0.4468	+0.0278	0.0000	+0.0119
188	−0.9238	−19.9170	−0.4457	+0.0332	+0.0001	+0.0118
189	−0.4530	−19.9621	−0.4463	+0.0208	−0.0001	+0.0118
190	−0.5567	−19.9404	−0.4458	+0.0235	−0.0001	+0.0118
191	−0.5581	−19.9393	−0.4458	+0.0235	−0.0001	+0.0118
192	−1.1132	−19.8126	−0.4434	+0.0383	+0.0003	+0.0118
193	−0.8078	−19.8550	−0.4442	+0.0302	+0.0001	+0.0118
194	−1.1841	−19.7329	−0.4416	+0.0403	+0.0003	+0.0117
195	−0.9622	−19.7928	−0.4429	+0.0344	+0.0002	+0.0117
196	−0.4894	−19.9154	−0.4453	+0.0217	−0.0001	+0.0118
197	−0.6888	−19.8577	−0.4441	+0.0270	0.0000	+0.0118
198	−1.0562	−19.7359	−0.4417	+0.0369	+0.0002	+0.0117
199	−1.3484	−19.6422	−0.4397	+0.0446	+0.0004	+0.0117
200	−0.1843	−19.9970	−0.4470	+0.0135	−0.0003	+0.0119
201	−0.8914	−19.7749	−0.4425	+0.0325	+0.0001	+0.0117
202	−0.4822	−19.9030	−0.4450	+0.0215	−0.0001	+0.0118
203	−1.0620	−19.6942	−0.4407	+0.0370	+0.0002	+0.0117
204	−0.9308	−19.7251	−0.4414	+0.0336	+0.0002	+0.0117
205	−0.9507	−19.7114	−0.4410	+0.0340	+0.0002	+0.0117
206	−0.7628	−19.7699	−0.4423	+0.0291	+0.0001	+0.0117
207	−0.8877	−19.7203	−0.4412	+0.0324	+0.0001	+0.0117
208	−0.9440	−19.6911	−0.4405	+0.0339	+0.0002	+0.0117
209	−0.6481	−19.8021	−0.4429	+0.0260	0.0000	+0.0118
210	−0.1684	−19.9859	−0.4467	+0.0131	−0.0003	+0.0119
211	−1.3954	−19.4703	−0.4359	+0.0461	+0.0004	+0.0116
212	−0.9796	−19.6360	−0.4393	+0.0349	+0.0002	+0.0117
213	−1.0037	−19.6230	−0.4390	+0.0356	+0.0002	+0.0116
214	−0.7428	−19.7221	−0.4412	+0.0285	0.0000	+0.0117
215	−0 8198	−19.6870	−0.4404	+0.0306	+0.0001	+0.0117
216	−1 0248	−19.5576	−0.4376	+0.0362	+0.0002	+0.0116
217	−0.6824	−19.7164	−0.4410	+0.0269	0.0000	+0.0117
218	−0.8503	−19.6246	−0.4390	+0.0315	+0.0001	+0.0116
219	−1.1866	−19.3898	−0.4340	+0.0405	+0.0003	+0.0115
220	−0.7157	−19.6314	−0.4391	+0.0279	0.0000	+0.0117
221	−1.2273	−19.3110	−0.4322	+0.0417	+0.0003	+0.0115
222	−1.0659	−19.4032	−0.4342	+0.0373	+0.0002	+0.0115
223	−0.3154	−19.8582	−0.4440	+0.0171	−0.0002	+0.0118
224	−1.1530	−19.3101	−0.4322	+0.0398	+0.0003	+0.0115
225	−1.0341	−19.3607	−0.4332	+0.0365	+0.0002	+0.0115

TABLE XVI.—FINAL CATALOGUE (cont'd).

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0 °	′	″	N. P. D. ″	Decl. 1888.0 °	′	″	Y ″	X ″	Number in B. D. M.	Number in Car.
226	9.8	9.7	213	46	38	5776.82	88	23	43.18	−3211.29	−4801.10	88 85	2174
227	11.8	11.5	215	48	40	2185.76	89	23	34.24	−1278.89	−1772.51	....	....
228	11.8	11.5	215	53	36	2655.06	89	15	44.94	−1556.56	−2150.84	89 24	....
229	10.2	10.6	215	58	50	5543.76	88	27	36.24	−3256.64	−4485.57	88 87	2205
230	11.4	11.8	216	26	51	6360.26	88	13	59.74	−3777.94	−5115.38	....	....
231	11.0	11.0	216	34	56	4662.82	88	42	17.18	−2778.69	−3743.94	....	....
232	11.1	11.5	217	21	31	2426.02	89	19	33.98	−1472.08	−1928.29	....	....
233	10.2	10.3	218	9	14	4842.72	88	39	17.28	−2991.45	−3807.75	88 88	2246
234	12.0	12.0	223	50	10	4150.48	88	50	49.52	−2874.42	−2993.64	....	....
235	10.8	11.2	224	30	42	4209.68	88	49	50.32	−2951.01	−3001.75	....	....
236	11.8	11.2	224	55	2	5596.56	88	26	43.44	−3951.15	−3962.60	....	....
237	11.5	10.3	225	19	12	6485.80	88	11	54.20	−4610.93	−4559.72	88 89	2297
238	12.0	12.0	226	31	51	2390.40	89	20	9.60	−1734.78	−1644.47	....	....
239	9.2	9.0	227	49	40	5611.26	88	26	28.74	−4158.15	−3766.71	88 90	2333
240	10.8	11.2	228	6	53	7096.76	88	1	43.24	−5282.38	−4737.14	....	....
241	12.0	11.0	228	11	14	2102.92	89	24	57.08	−1567.34	−1401.99	....	....
242	9.8	10.3	228	28	52	3834.65	88	56	5.35	−2870.99	−2541.72	89 27	2369
243	11.7	10.5	229	25	26	1374.32	89	37	5.68	−1043.85	− 893.93	....	....
244	12.0	11.8	229	42	37	5929.31	88	21	10.69	−4522.16	−3833.68	....	....
245	9.8	10.1	230	19	26	6516.65	88	11	23.35	−5014.81	−4159.85	88 91	2351
246	11.5	11.2	231	3	45	7268.94	87	58	51.06	−5652.83	−4567.38	....	....
247	11.0	11.0	231	31	8	5672.96	88	25	27.04	−4440.32	−3529.59	....	....
248	12.0	12.0	231	58	45	4406.08	88	46	33.92	−3470.78	−2713.71	....	....
249	12.0	12.0	236	26	28	1771.79	89	30	28.21	−1476.45	− 979.42	....	....
250	9.8	10.3	237	21	44	1153.89	89	40	46.11	− 971.68	− 622.32	89 29	0
251	12.0	12.0	237	32	48	4361.42	88	47	18.58	−3680.02	−2340.22	....	....
252	11.2	10.5	238	14	37	1912.84	89	28	7.16	−1626.46	−1006.73	....	....
253	12.0	12.0	238	23	3	3393.30	89	3	26.70	−2889.54	−1778.76	....	....
254	12.0	12.0	238	27	25	3746.42	88	57	33.58	−3192.69	−1959.79	....	....
255	9.8	10.3	240	3	36	6984.14	88	3	35.86	−6050.94	−3485.07	88 93	2449
256	11.5	10.6	240	31	13	6305.83	88	14	54.17	−5488.55	−3102.71	....	....
257	12.0	11.0	241	25	40	3733.29	88	57	46.71	−3278.45	−1785.41	....	....
258	11.4	10.8	242	43	20	3793.46	88	56	46.54	−3371.42	−1738.46	....	....
259	10.3	10.5	242	44	40	3868.93	88	55	31.07	−3439.17	−1771.71	....	....
260	11.2	10.7	242	58	48	3804.30	88	56	35.70	−3388.86	−1728.20	....	....
261	10.2	8.8	243	28	20	5818.18	88	23	1.82	−5204.93	−2598.23	88 94	2494
262	12.0	12.0	243	50	44	5826.38	88	22	53.62	−5229.11	−2567.88	....	....
263	12.0	11.0	243	53	20	3099.47	89	8	20.53	−2783.04	−1364.07	....	....
264	10.6	10.6	244	10	36	5817.12	88	23	2.88	−5235.54	−2533.59	88 95	....
265	11.3	11.0	244	34	36	2811.44	89	13	8.56	−2539.10	−1206.92	....	....
266	9.9	9.0	246	46	56	2686.20	89	15	13.80	−2468.58	−1058.94	89 28	[illegible]
267	9.8	9.7	247	34	45	5506.58	88	28	13.42	−5089.74	−2100.00	88 96	2533
268	11.5	11.5	247	35	38	1932.65	89	27	47.35	−1786.72	− 736.66	....	....
269	12.0	12.0	247	56	15	1796.00	89	30	4.00	−1664.46	− 674.60	....	....
270	11.5	12.0	248	20	28	4136.18	88	51	3.82	−3843.90	−1526.48	....	....

PRECESSION COEFFICIENTS (cont'd).

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	"	"	"	"	"	"
226	−1.0727	−19.3277	−0.4326	+0.0375	+0.0002	+0.0115
227	−0.3960	−19.7662	−0.4418	+0.0193	−0.0002	+0.0117
228	−0.4806	−19.7036	−0.4405	+0.0216	−0.0001	+0.0117
229	−1.0022	−19.3181	−0.4322	+0.0357	+0.0002	+0.0115
230	−1.1430	−19.1995	−0.4297	+0.0395	+0.0003	+0.0114
231	−0.8365	−19.4271	−0.4345	+0.0312	+0.0001	+0.0115
232	−0.4308	−19.7228	−0.4410	+0.0203	−0.0001	+0.0117
233	−0.8508	−19.3791	−0.4335	+0.0316	+0.0001	+0.0115
234	−0.6689	−19.4067	−0.4341	+0.0268	0.0000	+0.0115
235	−0.6707	−19.3895	−0.4337	+0.0268	0.0000	+0.0115
236	−0.8854	−19.1629	−0.4287	+0.0326	+0.0001	+0.0114
237	−1.0188	−19.0128	−0.4254	+0.0363	+0.0002	+0.0113
238	−0.3674	−19.6641	−0.4396	+0.0185	−0.0002	+0.0117
239	−0.8416	−19.1165	−0.4276	+0.0316	+0.0001	+0.0113
240	−1.0585	−18.8609	−0.4222	+0.0375	+0.0002	+0.0112
241	−0.3133	−19.7018	−0.4404	+0.0171	−0.0002	+0.0117
242	−0.5679	−19.4081	−0.4341	+0.0241	−0.0001	+0.0115
243	−0.1997	−19.8194	−0.4430	+0.0140	−0.0003	+0.0118
244	−0.8566	−19.0344	−0.4258	+0.0320	+0.0001	+0.0113
245	−0.9295	−18.9225	−0.4234	+0.0340	+0.0002	+0.0112
246	−1.0205	−18.7775	−0.4202	+0.0365	+0.0001	+0.0112
247	−0.7886	−19.0534	−0.4262	+0.0302	+0.0001	+0.0113
248	−0.6063	−19.2730	−0.4311	+0.0252	0.0000	+0.0114
249	−0.2188	−19.7224	−0.4407	+0.0146	−0.0003	+0.0117
250	−0.1390	−19.8357	−0.4434	+0.0124	−0.0003	+0.0118
251	−0.5229	−19.2263	−0.4299	+0.0230	−0.0001	+0.0114
252	−0.2249	−19.6888	−0.4400	+0.0148	−0.0003	+0.0117
253	−0.3974	−19.4048	−0.4338	+0.0196	−0.0002	+0.0115
254	−0.4379	−19.3364	−0.4324	+0.0206	−0.0001	+0.0115
255	−0.7787	−18.6896	−0.4181	+0.0301	0.0000	+0.0111
256	−0.6933	−18.8173	−0.4208	+0.0278	−0.0001	+0.0112
257	−0.3989	−19.3173	−0.4318	+0.0197	−0.0002	+0.0115
258	−0.3884	−19.2964	−0.4314	+0.0194	−0.0002	+0.0115
259	−0.3959	−19.2812	−0.4310	+0.0196	−0.0002	+0.0114
260	−0.3861	−19.2925	−0.4313	+0.0194	−0.0002	+0.0115
261	−0.5806	−18.8821	−0.4224	+0.0247	−0.0001	+0.0112
262	−0.5738	−18.8767	−0.4222	+0.0246	−0.0001	+0.0112
263	−0.3048	−19.4290	−0.4343	+0.0171	−0.0002	+0.0115
264	−0.5661	−18.8753	−0.4221	+0.0243	−0.0001	+0.0112
265	−0.2697	−19.4839	−0.4355	+0.0162	−0.0002	+0.0116
266	−0.2366	−19.4998	−0.4358	+0.0152	−0.0003	+0.0116
267	−0.4692	−18.9087	−0.4228	+0.0218	−0.0001	+0.0112
268	−0.1646	−19.6530	−0.4392	+0.0132	−0.0003	+0.0117
269	−0.1507	−19.6804	−0.4398	+0.0128	−0.0003	+0.0117
270	−0.3411	−19.1901	−0.4290	+0.0182	−0.0002	+0.0114

TABLE XVI.—FINAL CATALOGUE (cont'd).

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0 ° ′ ″	N. P. D. ″	Decl. 1888.0 ° ′ ″	Y ″	X ″	Number in B. D. M.	Number in Car.
271	11.0	10.6	248 42 17	5939.63	88 21 0.37	−5533.31	−2156.82	....	......
272	11.2	11.0	249 3 42	4119.78	88 51 20.22	−3847.48	−1472.16	....	......
273	10.2	10.3	249 15 10	6455.00	88 12 25.00	−6035.41	−2286.28	88 97	2538
274	10.0	10.5	250 23 54	6884.56	88 5 15.44	−6484.37	−2309.19	88 98	2551
275	12.0	12.0	251 28 55	4129.38	88 51 10.62	−3915.32	−1311.42	....	......
276	9.8	10.2	253 1 47	6145.21	88 17 34.79	−5876.76	−1793.38	88 99	2584
277	10.8	10.6	254 30 55	6541.86	88 10 58.14	−6303.34	−1746.26	....	......
278	11.0	11.5	254 36 39	6052.57	88 19 7.43	−5834.71	−1605.96	....	......
279	10.2	10.0	254 55 53	732.01	89 47 47.99	− 706.84	− 190.30	89 35	*r*
280	12.0	11.8	255 4 40	4287.07	88 48 32.93	−4142.20	−1103.88	....	......
281	12.0	12.0	256 5 52	4986.45	88 36 53.55	−4839.91	−1197.96	....	......
282	11.0	11.0	256 17 36	5652.64	88 25 47.36	−5490.97	−1339.23	....	......
283	11.0	10.6	256 59 42	6719.86	88 8 0.14	−6546.33	−1511.94	....	......
284	11.4	11.0	258 4 42	5473.68	88 28 46.32	−5354.99	−1130.59	....	......
285	9.1	8.7	258 26 19	6520.90	88 11 19.10	−6387.54	−1306.69	88 100	2639
286	11.8	11.0	260 21 39	5290.98	88 31 49.02	−5215.70	− 885.84	....	......
287	12.0	11.8	260 53 23	6031.33	88 19 28.67	−5954.40	− 954.84	....	......
288	11.0	10.3	262 58 34	1230.64	89 39 29.36	−1221.39	− 150.49	89 31	*p*
289	11.8	12.0	263 15 27	4662.48	88 42 17.52	−4629.84	− 547.36	....	......
290	12.0	12.0	263 36 21	2852.70	89 12 27.30	−2834.86	− 317.69	....	......
291	11.0	11.0	263 51 20	3212.84	89 6 27.16	−3194.25	− 343.87	....	......
292	12.0	12.0	264 18 9	4229.36	88 49 30.64	−4208.17	− 419.85	....	......
293	11.0	11.0	265 21 53	6547.79	88 10 52.21	−6525.27	− 529.05	....	......
294	10.6	10.5	265 29 4	6381.80	88 13 38.20	−6360.98	− 502.36	....	......
295	12.0	12.0	265 29 13	329.43	89 54 30.57	− 328.41	− 25.92	....	......
296	9.4	9.3	265 40 26	4726.00	88 41 14.00	−4712.13	− 356.47	88 101	2740
297	11.1	10.5	265 43 23	4256.46	88 49 3.54	−4244.31	− 317.41	....	......
298	10.2	10.1	266 11 39	340.89	89 54 19.11	− 340.14	− 22.63	89 37	*t*
299	12.0	12.0	267 6 14	5118.07	88 34 41.93	−5111.00	− 258.56	....	......
300	9.3	9.9	268 4 11	5098.82	88 35 1.18	−5095.41	− 171.73	88 102	2762
301	9.9	10.0	268 28 42	2978.25	89 10 21.75	−2977.10	− 79.08	89 30	2818
302	9.6	9.7	269 49 45	7334.24	87 57 45.76	−7332.65	− 21.86	87 168	2752
303	12.0	10.3	270 24 31	6918.08	88 4 41.92	−6916.60	+ 49.33	88 103	2766
304	8.0	8.0	270 30 4	6295.95	88 15 4.05	−6294.73	+ 55.06	88 104	2770
305	9.0	8.4	270 46 0	4549.62	88 44 10.38	−4548.84	+ 60.87	88 105	2793
306	11.6	11.5	271 55 28	3411.87	89 3 8.13	−3409.78	+ 114.57	....	......
307	11.6	10.7	271 56 56	4727.44	88 41 12.56	−4724.29	+ 160.76	....	......
308	11.4	10.5	271 58 20	7040.82	88 2 39.18	−7035.29	+ 242.26	....	......
309	12.0	12.0	273 10 13	1972.15	89 27 7.85	−1969.10	+ 109.07	....	......
310	9.9	10.2	273 57 40	3995.27	88 53 24.73	−3985.47	+ 275.97	88 106	2848
311	12.0	12.0	274 25 50	2913.36	89 11 26.64	−2904.55	+ 225.05	....	......
312	12.0	11.5	274 45 55	5486.72	88 28 33.28	−5467.11	+ 455.75	....	......
313	10.7	9.7	275 5 11	3730.22	88 57 49.78	−3715.32	+ 330.69	88 107	2866
314	9.8	8.5	275 53 50	7230.12	87 59 29.88	−7190.38	+ 742.70	87 173	2819
315	12.0	12.0	276 25 2	1229.04	89 39 30.96	−1221.33	+ 137.37	....	......

PRECESSION COEFFICIENTS (cont'd).

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	″	″	″	″	″	″
271	−0.4819	−18.8084	−0.4206	+0.0222	−0.0002	+0.0112
272	−0.3289	−19.1893	−0.4290	+0.0178	−0.0002	+0.0114
273	−0.5108	−18.6948	−0.4180	+0.0230	−0.0002	+0.0111
274	−0.5160	−18.5930	−0.4158	+0.0232	−0.0002	+0.0110
275	−0.2930	−19.1742	−0.4286	+0.0169	−0.0002	+0.0114
276	−0.4007	−18.7311	−0.4188	+0.0200	−0.0003	+0.0111
277	−0.3902	−18.6346	−0.4166	+0.0199	−0.0003	+0.0111
278	−0.3588	−18.7408	−0.4190	+0.0190	−0.0003	+0.0111
279	−0.0425	−19.8951	−0.4446	+0.0098	−0.0004	+0.0118
280	−0.2466	−19.1232	−0.4275	+0.0157	−0.0003	+0.0114
281	−0.2677	−18.9658	−0.4239	+0.0164	−0.0002	+0.0113
282	−0.2992	−18.8186	−0.4207	+0.0174	−0.0003	+0.0112
283	−0.3378	−18.5797	−0.4153	+0.0185	−0.0003	+0.0110
284	−0.2526	−18.8495	−0.4214	+0.0160	−0.0003	+0.0112
285	−0.2920	−18.6158	−0.4161	+0.0173	−0.0003	+0.0111
286	−0.1979	−18.8811	−0.4220	+0.0146	−0.0003	+0.0112
287	−0.2134	−18.7141	−0.4183	+0.0150	−0.0004	+0.0111
288	−0.0336	−19.7798	−0.4420	+0.0097	−0.0004	+0.0117
289	−0.1223	−19.0135	−0.4249	+0.0124	−0.0003	+0.0113
290	−0.0710	−19.4178	−0.4339	+0.0109	−0.0004	+0.0115
291	−0.0768	−19.3369	−0.4321	+0.0110	−0.0004	+0.0115
292	−0.0938	−19.1086	−0.4271	+0.0117	−0.0003	+0.0113
293	−0.1182	−18.5850	−0.4153	+0.0126	−0.0004	+0.0110
294	−0.1122	−18.6222	−0.4162	+0.0125	−0.0004	+0.0111
295	−0.0058	−19.9796	−0.4465	+0.0086	−0.0004	+0.0119
296	−0.0797	−18.9949	−0.4244	+0.0114	−0.0004	+0.0113
297	−0.0709	−19.1005	−0.4268	+0.0111	−0.0004	+0.0113
298	−0.0051	−19.9770	−0.4464	+0.0087	−0.0004	+0.0119
299	−0.0578	−18.9049	−0.4224	+0.0108	−0.0004	+0.0112
300	−0.0384	−18.9085	−0.4225	+0.0103	−0.0004	+0.0112
301	−0.0177	−19.3858	−0.4331	+0.0095	−0.0004	+0.0115
302	−0.0049	−18.4020	−0.4112	+0.0097	−0.0005	+0.0109
303	+0.0110	−18.4964	−0.4133	+0.0093	−0.0005	+0.0110
304	+0.0123	−18.6372	−0.4165	+0.0092	−0.0005	+0.0111
305	+0.0136	−19.0318	−0.4252	+0.0088	−0.0004	+0.0113
306	+0.0256	−19.2885	−0.4310	+0.0084	−0.0004	+0.0114
307	+0.0359	−18.9922	−0.4243	+0.0083	−0.0004	+0.0113
308	+0.0541	−18.4694	−0.4127	+0.0082	−0.0005	+0.0110
309	+0.0244	−19.6121	−0.4382	+0.0082	−0.0004	+0.0116
310	+0.0617	−19.1588	−0.4281	+0.0075	−0.0004	+0.0114
311	+0.0503	−19.4021	−0.4335	+0.0077	−0.0004	+0.0115
312	+0.1018	−18.8244	−0.4205	+0.0067	−0.0006	+0.0112
313	+0.0739	−19.2197	−0.4294	+0.0071	−0.0004	+0.0114
314	+0.1659	−18.4342	−0.4118	+0.0052	−0.0006	+0.0109
315	+0.0307	−19.7798	−0.4420	+0.0080	−0.0004	+0.0117

TABLE XVI.—FINAL CATALOGUE (concl'd).

No.	Mag. Phot.	Mag. Vis.	R. A. 1888.0 °	′	″	N. P. D. ″	Decl. 1888.0 °	′	″	Y ″	X ″	Number in B. D. M.	Car.
361	11.5	10.8	305	6	26	5390.40	88	30	9.60	−4409.25	+3099.71	....	....
362	11.6	11.2	307	20	48	1542.40	89	34	17.60	−1226.17	+ 935.67	....	....
363	9.2	9.4	309	41	4	6478.45	88	12	1.55	−4984.81	+4136.18	88 118	3193
364	10.0	10.3	310	3	16	5507.16	88	28	12.84	−4214.86	+3543.52	88 119	3204
365	11.4	12.0	310	9	15	3977.66	88	53	42.34	−3039.99	+2564.82	....	....
366	11.2	11.0	310	43	43	5188.45	88	33	31.55	−3931.44	+3384.98	....	....
367	10.3	10.6	311	6	32	851.92	89	45	48.08	− 641.89	+ 560.13	....	....
368	11.4	11.5	312	2	28	4437.94	88	46	2.06	−3295.65	+2971.69	....	....
369	12.0	10.8	312	44	10	5725.23	88	24	34.77	−4204.56	+3884.77	88 120	....
370	11.0	9.9	314	31	25	5392.96	88	30	7.04	−3844.53	+3781.12	88 122	3241
371	11.2	10.1	314	47	4	6962.34	88	3	57.66	−4940.66	+4903.62	87 191	3230
372	10.5	10.4	314	53	7	1581.62	89	33	38.38	−1120.60	+1116.12	....	....
373	10.8	10.4	315	28	50	1353.84	89	37	26.16	− 949.24	+ 965.30	....	....
374	9.9	10.0	317	8	30	6212.19	88	16	27.81	−4224.81	+4553.07	88 123	3259
375	10.6	10.4	317	17	53	5435.74	88	29	24.26	−3686.01	+3994.21	....	....
376	10.9	10.8	317	46	16	3252.74	89	5	47.26	−2186.06	+2408.44	88 126	....
377	10.6	9.6	318	31	59	5903.52	88	21	36.48	−3908.71	+4423.13	88 124	3276
378	10.5	10.3	318	52	18	4739.48	88	41	0.52	−3117.11	+3569.64	88 125	3292
379	11.5	12.0	319	41	55	5197.52	88	33	22.48	−3361.45	+3963.48	....	....
380	10.9	10.3	320	7	42	5237.65	88	32	42.35	−3357.34	+4019.36	88 127	3307
381	10.9	10.6	322	6	25	1185.97	89	40	14.03	− 728.40	+ 935.91	....	....
382	10.0	10.3	322	26	57	6396.56	88	13	23.44	−3897.85	+5070.46	88 128	3317
383	12.0	12.0	324	27	30	460.63	89	52	19.37	− 267.76	+ 374.81	....	....
384	10.5	10.3	326	1	53	4132.22	88	51	7.78	−2308.67	+3426.79	88 129	3374
385	11.2	12.0	328	6	36	4839.20	88	39	20.80	−2556.27	+4108.42	....	....
386	8.6	9.5	330	15	8	6034.58	88	19	25.42	−2993.82	+5238.59	88 130	3411
387	8.7	9.5	334	27	19	3953.96	88	54	6.04	−1704.90	+3567.24	88 131	3465
388	10.4	10.3	337	35	35	5646.67	88	25	53.33	−2152.14	+5219.69	88 132	3473
389	9.4	9.7	338	39	28	4786.44	88	40	13.56	−1741.81	+4457.80	88 133	3487
390	9.9	9.7	344	19	14	4459.14	88	45	40.86	−1205.01	+4292.87	88 134	3543
391	11.0	10.1	345	4	43	7048.24	88	2	31.76	−1814.52	+6809.23	87 211	3544
392	9.4	9.7	348	35	54	2904.75	89	11	35.25	− 574.21	+2847.34	89 39	3601
393	10.4	10.3	349	47	40	7095.60	88	1	44.40	−1256.95	+6981.95	87 215	3600
394	12.0	11.8	351	34	4	4156.98	88	50	43.02	− 609.53	+4111.76	....	....
395	11.5	11.3	353	4	0	4383.58	88	46	56.42	− 529.12	+4351.19	....	....
396	11.0	10.8	353	8	2	6502.13	88	11	37.87	− 777.20	+6454.43	....	....
397	10.2	10.3	353	37	36	4938.27	88	37	41.73	− 548.13	+4907.28	88 136	3642
398	12.0	11.1	353	41	46	6665.80	88	8	54.20	− 731.79	+6624.33	....	....
399	10.0	10.1	353	47	54	5143.42	88	34	16.58	− 555.58	+5112.79	88 137	3645
400	11.2	10.1	354	56	23	5114.08	88	34	45.92	− 451.03	+5093.61	88 138	3654
401	11.0	10.8	356	1	50	5453.98	88	29	6.02	− 377.50	+5440.25	....	....
402	9.2	9.7	356	2	51	6396.96	88	13	23.04	− 440.87	+6380.73	88 139	3670
403	11.3	10.3	356	37	8	5923.86	88	21	16.14	− 349.33	+5912.74	88 140	....
404	10.6	10.7	356	38	6	6335.31	88	14	24.69	− 371.80	+6323.40	....	....
405	10.1	10.1	357	12	12	3889.78	88	55	10.22	− 189.78	+3884.91	88 141	3678
406	9.9	9.6	358	7	5	4241.40	88	49	18.60	− 139.28	+4238.81	88 142	3689
7	11.2	10.8	359	1	18	7372.74	87	57	7.26	− 125.86	+7370.10	87 219	....
	10.7	10.3	359	33	21	5421.35	88	29	38.65	− 42.02	+5420.56	88 143	....

PRECESSION COEFFICIENTS (concl'd).

No.	dy	dx	100 d^2y	100 d^2x	10,000 d^3y	10,000 d^3x
	"	"	"	"	"	"
361	+0.6926	−19.0610	−0.4255	−0.0092	−0.0008	+0.0113
362	+0.2091	−19.7785	−0.4419	+0.0032	−0.0005	+0.0117
363	+0.9242	−18.9294	−0.4223	−0.0153	−0.0009	+0.0112
364	+0.7918	−19.1041	−0.4264	−0.0118	−0.0009	+0.0113
365	+0.5731	−19.3701	−0.4324	−0.0062	−0.0007	+0.0115
366	+0.7563	−19.1684	−0.4278	−0.0110	−0.0008	+0.0114
367	+0.1252	−19.9095	−0.4448	+0.0054	−0.0005	+0.0118
368	+0.6640	−19.3120	−0.4311	−0.0085	−0.0008	+0.0115
369	+0.8680	−19.1059	−0.4264	−0.0138	−0.0009	+0.0113
370	+0.8448	−19.1872	−0.4282	−0.0134	−0.0009	+0.0114
371	+1.0957	−18.9378	−0.4224	−0.0198	−0.0011	+0.0112
372	+0.2494	−19.8021	−0.4423	+0.0021	−0.0005	+0.0118
373	+0.2157	−19.8406	−0.4433	+0.0029	−0.0005	+0 0118
374	+1.0173	−19.1000	−0.4262	−0.0178	−0.0010	+0.0113
375	+0.8925	−19.2225	−0.4289	−0.0146	−0.0009	+0.0114
376	+0.5381	−19.5622	−0.4368	−0.0054	−0.0007	+0.0116
377	+0.9883	−19.1715	−0.4278	−0.0172	−0.0010	+0.0114
378	+0.7976	−19.3513	−0.4318	−0.0122	−0.0009	+0.0115
379	+0.8856	−19.2956	−0.4306	−0.0144	−0.0009	+0.0115
380	+0.8981	−19.2964	−0.4305	−0.0148	−0.0009	+0.0115
381	+0.2091	−19.8900	−0.4444	+0.0031	−0.0005	+0.0118
382	+1.1329	−19.1725	−0.4276	−0.0210	−0.0011	+0.0114
383	+0.0838	−19.9932	−0.4467	+0.0064	−0.0004	+0.0119
384	+0.7657	−19.5332	−0.4360	−0.0115	−0.0009	+0.0116
385	+0.9180	−19.4763	−0.4345	−0.0154	−0.0009	+0.0116
386	+1.1705	−19.3756	−0.4323	−0.0221	−0.0011	+0.0115
387	+0.7971	−19.6685	−0.4390	−0.0124	−0.0009	+0.0117
388	+1.1663	−19.5646	−0.4365	−0.0221	−0.0011	+0.0116
389	+0.9960	−19.6585	−0.4386	−0.0177	−0.0010	+0.0117
390	+0.9592	−19.7792	−0.4414	−0.0167	−0.0010	+0.0117
391	+1.5214	−19.6360	−0.4379	−0.0316	−0.0013	+0.0117
392	+0.6362	−19.9228	−0.4448	−0.0082	−0.0008	+0.0118
393	+1.5600	−19.7603	−0.4405	−0.0327	−0.0013	+0.0117
394	+0.9187	−19.9128	−0.4444	−0.0157	−0.0009	+0.0118
395	+0.9722	−19.9303	−0.4448	−0.0171	−0.0010	+0.0118
396	+1.4422	−19.8694	−0.4430	−0.0296	−0.0013	+0.0118
397	+1.0965	−19.9248	−0.4445	−0.0205	−0.0011	+0.0118
398	+1.4801	−19.8791	−0.4433	−0.0306	−0.0013	+0.0118
399	+1.1424	−19.9228	−0.4444	−0.0217	−0.0011	+0.0118
400	+1.1381	−19.9462	−0.4450	−0.0215	−0.0011	+0.0118
401	+1.2156	−19.9616	−0.4452	−0.0236	−0.0011	+0.0118
402	+1.4257	−19.9449	−0.4447	−0.0292	−0.0012	+0.0118
403	+1.3211	−19.9668	−0.4454	−0.0265	−0.0012	+0.0119
404	+1.4129	−19.9605	−0.4451	−0.0288	−0.0012	+0.0119
405	+0.8680	−20.0071	−0.4466	−0.0144	−0.0009	+0.0119
406	+0.9471	−20.0177	−0.4467	−0.0166	−0.0010	+0.0119
407	+1.6468	−20.0122	−0.4462	−0.0352	−0.0014	+0.0119
408	+1.2112	−20.0367	−0.4470	−0.0236	−0.0011	+0.0119

An Investigation into the Elastic Constants of Rocks, More Especially with Reference to Cubic Compressibility

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AN INVESTIGATION INTO THE ELASTIC CONSTANTS OF ROCKS, MORE ESPECIALLY WITH REFERENCE TO CUBIC COMPRESSIBILITY.

INTRODUCTION.

The question as to the amount of cubic compression which rocks may undergo under the stresses to which they are subjected in the earth's crust is one which has a direct bearing on many very important problems in geophysics. It is, however, a subject which has been but little investigated as the experimental difficulties connected with it are very considerable. The importance of a series of determinations of the cubic compressibility of a few typical plutonic igneous rocks was some time since impressed upon the authors by Mr. G. K. Gilbert, with a request that if possible they should make such determinations in connection with the researches on rock deformation which are now being carried out at McGill University under the auspices of the Carnegie Institution of Washington. An examination of all the direct methods proposed or adopted for the measurement of the cubic compressibility of solids showed that none of these could be satisfactorily applied to such materials as rocks, but the indirect methods based on Hook's law and which have been applied to metals and other compact isotropic bodies having an approximately perfect elasticity promised to give satisfactory results if applied to certain rocks, more especially to the class of rocks referred to above, viz, the acid and basic plutonic rocks, which form the greater part at least of the outer portions of the earth's crust. The present paper sets forth the methods adopted and the results obtained.

The work which was carried out in the laboratories of McGill University was commenced by the authors whose names appear on the title page, and was carried well towards completion when Dr. Coker was called to take the professorship of mechanical engineering in the Finsbury Technical Institute of London, England. He was accordingly obliged to give up the work of the research and his place was taken by Mr. Charles McKergow, lecturer in mechanical engineering in McGill University, but who immediately on the completion of the work was appointed to the professorship in mechanical engineering in the University of Virginia. A large number of the very careful measurements of elastic constants which are given in the paper were made by the latter gentleman.

METHODS WHICH MAY BE USED IN THE DETERMINATION OF THE ELASTIC CONSTANTS OF MATERIALS.

The determination of the cubic compressibility of solid substances is, as above mentioned, beset with serious difficulties. On the one hand, every direct method which has been suggested presents experimental difficulties which tend to impair its accuracy, while on the other hand the indirect methods are based on assumptions as to the isotropy of the materials, which are not warranted in the case of certain rocks. The indirect methods depending on the theory of elasticity are capable of considerable variation, and it is of interest to examine them in some detail in order to see whether certain of them at least may not be depended upon to give reliable and satisfactory results.

The determination of the elastic constants of metals has engaged the attention of many physicists and at the present time a large amount of information exists as to the values of these constants for various metals.

It is well known that in homogeneous elastic substances a simple compression stress causes a lateral strain, which bears a fixed ratio to the compression strain for any particular substance within the limit of elasticity. If, then,* we call p_x the stress on a plane perpendicular to x in the direction x, and e_x the corresponding strain, then for a direct compression stress p_x there will be a strain in the direction of this stress of amount p_x/E, where E is Young's modulus, and lateral strain of magnitude p_x/mE, where m is the ratio of the longitudinal compression to the lateral extension per unit of length.

If we suppose further that a body is subjected to cubical stress of intensity p_x, we easily see that for small and therefore superposable strains the cubical strain e_c is

$$e_c = 3p_x \frac{m - 2}{mE}$$

and since the modulus of cubical compressibility D is the ratio of the stress per unit of area to the cubical strain produced, we have

$$D = \frac{p_x}{e_c} = \frac{1}{3}\frac{m}{m-2}E.$$

Hence if we know E and m we can calculate the value of D.

Further, it is shown in treatises on elasticity that if C is the modulus of shear, then

$$C = \frac{1}{2}\frac{m}{m+1}E$$

*See Ewing's Strength of Materials, Chapters I & II.

and since C and E are quantities which can be ascertained by experiment, we can from them calculate m and D.

In an important paper by Nagaoka* this latter method has been used to determine the elastic constants of a series of rocks. The value of E was determined by supporting a bar at the ends and measuring the angular change at the support due to a given load applied at the center; the value of E is then obtained by the formula $E = 3wl^3 / 4bd^3\theta$, where l is the length of the bar between the supports, b is the breadth of the bar, d the depth, and θ the angular change at the ends for a load, W. In order to determine the value of m, a specimen of rectangular section was twisted by a given torque, T, and the amount of the strain measured. It has been shown by St. Venant that for such a case the value of C is given by the formula

$$T = C\theta b^3 h\left[\frac{16}{3} - \frac{32^2 b^4}{\pi^5}\sum_0^\infty \frac{\tan h\,(2n+1)\dfrac{\pi h}{2b}}{(2n+1)^5}\right]$$

where θ is the angular change, and from this formula values of C were calculated from the observations.

This method appears to us to be open to some minor objections in that the formula for determining E is based upon a theory of flexure, which although sufficient for many purposes is nevertheless only approximate, and it is well known that values of E obtained by flexure experiments in this manner often differ from the values of E obtained by direct compression experiments by not inconsiderable amounts.

Further, in experiments upon the deflection of beams cut from rocks, it is difficult to obtain consistent readings, because of the time effect of the loading, and this difficulty is noticed in the paper cited.

As an example of the results obtained in this way, we may quote the results of certain experiments made by us with a pure white marble from Vermont.

Lath-shaped pieces of the marble were carefully prepared and were suspended on two wedge-shaped supports and then loaded in the middle. The weights were placed in a light brass pan, hanging from a thick wire which passed over the middle of the lath and lay flat upon it.

Each experiment occupied about half an hour, and the deflection was measured by attaching a scale to the marble and reading it with reference to a thin wire stretched in front of the specimen, a properly mounted telescope being employed for this purpose. The marble was in all cases placed so that its broader surface rested on the terminal supports.

*Elastic Constants of Rocks and the Velocity of Seismic Waves. H. Nagaoka. Phil. Mag., Vol. L, 1900, p. 53.

Of the several experiments made two may be selected. The pan and wire in each case weighed 3 ounces.

In the first experiment the marble had the following dimensions: Length, 12 inches; length between supports, 11 inches; breadth, 1.259 inches; thickness, 0.284 to 0.298 inch.

The figures obtained are as follows:

	Inch.
Load with pan only (taken as zero point)	0.486
with pan plus 4 ounces	.487
8 ounces	.488
12 ounces	.489
16 ounces	.490
20 ounces	.491
24 ounces	.491
28 ounces	.492
32 ounces	.493
36 ounces	.494
40 ounces	.497
44 ounces	.498
48 ounces	.500
52 ounces	.501
56 ounces	.503
60 ounces	.505
60 ounces (after 2 minutes)	.506
64 ounces	.515
64 ounces (after 1½ minutes)	.516
66 ounces	.517
68 ounces	.518
68 ounces (after 1½ minutes)	.520
70 ounces	.521
72 ounces	.522
72 ounces (after 1 minute)	.522
74 ounces	.526
76 ounces	.528
76 ounces (after 1½ minutes)	.531
78 ounces	.533
80 ounces	.534
80 ounces (after 2 minutes, moving fast)	.540
82 ounces	.541
82 ounces (after 1½ minutes)	.543
72 ounces (weight reduced, large permanent set)	.542
84 ounces	.547
86 ounces	.549
86 ounces (after ½ minute; broke)	.554
Total deflection before breaking	.064

In the second experiment the marble lath was longer and at the same time somewhat thicker. Its dimensions were as follows: Length, 16 inches; length between supports, 15 inches; breadth, 1.229 to 1.284 inches; thickness, .347 to .356 inch.

	Inch.
Load with pan only	.343
with pan plus 8 ounces	.349
16 ounces	.368
24 ounces	.389
24 ounces (after 1½ minutes)	.392
28 ounces	.401
32 ounces	.416
32 ounces (after 1½ minutes)	.423
36 ounces	.438
40 ounces	.460
Load with pan only (weight removed, large permanent set)	.412
with pan plus 40 ounces (after 2 minutes)	.471
44 ounces	.492
44 ounces (after a few seconds)	.500
44 ounces (after 1 minute; broke)	.520
Total deflection	.177

Here it will be noticed that when a certain load is reached a distinct movement sets in and is maintained without any further increase of load, the movement growing in amount as the limit of the strength of the rock is approached and producing a permanent set.

Experiments on the determination of the elastic constants of rocks when subjected to twist were also found to be frequently unsatisfactory, owing to the low ultimate shearing values of many rocks.

While a glance at the list of rocks whose elastic constants have been measured by Nagaoka will at once show that most of them are rocks whose elasticity must be of a very imperfect kind, *e. g.*, weathered clay slate, Schalstein, tuff, etc.; the method which he has employed for the determination of Young's modulus gives very low results, even in the case of rocks such as marble and granite, where the elasticity might be supposed to be of a high order, and comparable to that which these rocks have been shown to possess in the case of the types selected for investigation in the present paper. This is shown by the following figures comprising the values obtained by him for each of the marbles and granites contained in his list.

Paleozoic marble:	*E* (Young's modulus).
No. 11A	10,120,000
11B	7,950,000
12A	5,440,000
12B	4,770,000

Granite:	*E* (Young's modulus).
No. 69 (Shodoshima)	6,140,000
68 (Hitachi)	2,853,000
71 (Hitachi)	2,175,000
56 (Hitachi)	1,588,000
52 (Hitachi)	3,265,000

Of these marbles No. 11, if a mean of the two readings be taken, has about the same modulus as the average of those on our list, while No. 12 is very much lower. The highest value given for any granite in Nagaoka's list, viz, No. 69, is somewhat higher than that of the lowest of the granites in our series, that from Stanstead. The other granites examined by Nagaoka have values for E assigned to them which are so low that they are comparable only to that of the sandstone in our series. Of the three sandstones included in Nagaoka's list the Izumi sandstone of the Mesozoic has modulus of 1,322,000, while the other two, which belong to the Diluvium, have values for E of 587,500 and 583,000, respectively.

And so when an attempt is made to calculate the cubic compression D from the values given in Nagaoka's list and obtained by his method, it is found that a negative value is actually obtained in about one-third of the rocks which he has examined. His figures, however, were intended chiefly for the purpose of calculating the velocity of the propagation of earthquake shocks.

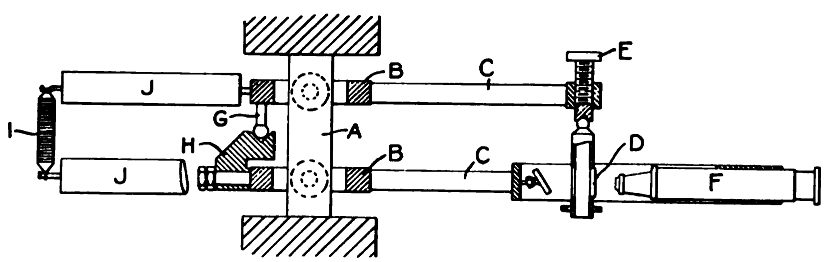

FIG. 1.—Instrument for determining the modulus of a simple strain.

In consequence of the somewhat unsatisfactory results obtained in our preliminary experiments with this method, as well as the facts with regard to Nagaoka's figures just mentioned, it was decided to adopt a somewhat different method and one which avoided both torsion and flexure and depended simply on strain produced by simple compressive stress. This will be termed the "method of simple compression."

Among the possible indirect methods, this seems to be the most satisfactory, as the assumptions necessary in the calculation of compressibility are reduced to a minimum, and the range of stress for which the ratio of stress to strain is practically constant is great. We were able to measure the strains obtained very accurately, by means of an apparatus forming part of the equipment of the testing laboratory of McGill University, for the use of which we are indebted to Dean Bovey.

This is an instrument designed by Professor Ewing, and of which a diagrammatic representation is given in figure 1, in which A is a specimen of the rock

gripped by screws passing through a pair of collars, *B*, which are 1.25 inch apart, to which latter metal rods, *C*, are attached. The upper rod carries a glass plate, *D*, with a fine line scratched upon it, the position of which can be adjusted by a screw, *E*, while the lower rod carries a micrometer microscope, *F*. The upper and lower collars, *B*, are connected by a stud, *G*, the upper one engaging with the conical hole of the swivel piece *H* in the lower, and contact is maintained by a spring, *I*, while the weights of the microscope and projecting arms are balanced by lead cylinders, *J*. A buzzer was attached to the upper lead cylinder which, when operated, caused a slight vibration in the instrument, producing a perfect adjustment as the pressure was applied.

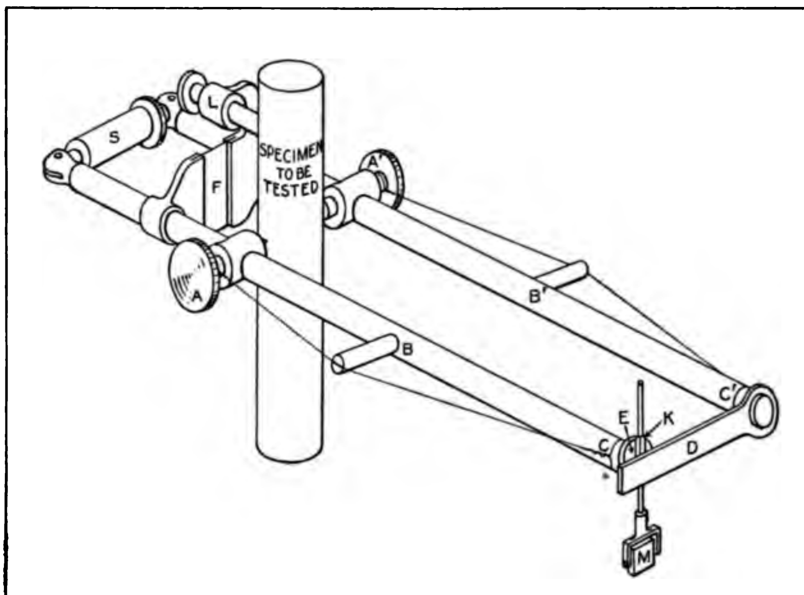

FIG. 2.—Perspective view of lateral extensometer.

The proportions of this instrument were so adjusted that one division on the micrometer scale corresponded to $\frac{1}{250,000}$ of an inch, and before using it the instrument was calibrated by aid of a Whitworth measuring machine and was found to be in correct adjustment. This instrument enabled us to determine the modulus of simple compression with great accuracy.

The linear strain perpendicular to the length of the specimen was measured by an instrument which had been designed by E. G. Coker some time previously for experiments on the lateral strains developed in metals.* Figure 2 is

*See Proceedings Royal Soc., Edinburgh, Session 1904–5. Vol. xxv, pt. vi.

a diagrammatic view of the apparatus, which consists of a pair of brass tubes, *B*, *B'*, provided with set screws, *A*, *A'*, for attachment to the specimen, and connected together by a flexible steel plate, *F*, forming the fulcrum. The ends of the tubes near the fulcrum plate are pressed apart by an adjustable spring *S*, to insure a uniform pressure on the screw points gripping the specimen. On the opposite end of the tubes is a spring finger, *D*, of ebony, pressing against a double knife-edge, *K*, seated in a shallow V notch cut in the end of the other arm. The knife-edge carries an adjustable mirror, *M*, so that if any change in the diameter of the specimen occurs the two tubes move relatively to one another in a horizontal plane and thereby cause the knife-edge mirror to rotate; the rotation of this latter is observed and measured by a telescope and scale placed at a suitable distance.

For convenience in adjustment there is a screw, *L*, for tilting the apparatus about the axis of the gripping screws, and the tubes *B*, *B'* are trussed to prevent vibration. This instrument was calibrated by aid of a Whitworth measuring machine and the scale adjusted so that one division corresponded to one-millionth of an inch.

APPLICATION OF THE METHOD OF SIMPLE COMPRESSION TO THE DETERMINATION OF THE CUBIC COMPRESSIBILITY OF METALS.

The behavior of such metals as wrought iron and steel over a wide range of stress shows that these metals may be considered as almost perfectly elastic. The results of the theory of elastic bodies may therefore be applied in their cases with great confidence.

As a typical example of the behavior of such materials we may consider the deportment of a specimen of wrought iron when subjected to a cycle of compression stresses, commencing at 1,000 pounds and rising to 9,000 pounds, afterwards returning to the original load.

The readings obtained for the longitudinal and lateral strains will show in such a case that equal increments or decrements of load produce strains which are very exactly proportional thereto. This is clearly shown in a plot of these readings, where the ordinates represent the total load and the abscissæ represent strains. In both cases the relation of stress to strain is represented by a straight line returning upon itself. Traces which vary but little from the ideal straight line are given by black Belgian marble, as will be seen on page 25.

Such results afford an arbitrary standard by which can be judged the degree of approximation to perfect elasticity exhibited by other metals and by rocks under similar conditions.

If we now calculate the value of the modulus E for simple compression, since this is the relation of the compression stress p to the strain e, we have

$$p = Ee$$

If we call A the cross-sectional area of the specimen when stressed by a load, P, and x the decrease of length over a measured length, L, gripped between the screw points of the measuring apparatus, we obtain

$$E=\frac{PL}{xA}$$

which, in case of a specimen of wrought iron examined for a range of 8,000 pounds, gave a value of 28,100,000, the units being pounds and inches.

The ratio m of the longitudinal strain to the lateral strain in the same case was 3.65, and using the formula

$$D=\frac{1}{3}\frac{m}{m-2}E$$

we obtain for the modulus of cubical compression (or bulk modulus) D, the value 21,300,000, a constant for the material, the reciprocal of which gives the decrease in volume of 1 cubic inch for 1 pound of pressure.

While certain rocks, such as many of the marbles, have a structure identical with that of wrought iron, most of the rocks constituting the earth's crust are composed of several minerals, and thus resemble cast iron in character, the gray variety of this substance being an aggregate of crystals or individuals of the metal iron (wrought iron), graphite, etc.

It will therefore be of interest to ascertain how a specimen of cast iron behaves under compression stress, and how far its elasticity falls short of that which would be exhibited by a perfectly elastic body.

For this purpose a fine-grained specimen of somewhat hard cast iron was faced and tested. The results of this test are given in the following table, and the stress-strain curves are plotted in figure 3. I represents longitudinal compression and II lateral extension.

The behavior of cast iron, as exhibited by these experimental results, shows a falling away from the theoretical standard of perfect elasticity, but even in the most perfectly elastic bodies there is probably a slight hysteresis effect, so that we are justified in using the results obtained to calculate the modulus of compressibility, if the error introduced thereby is negligible or very small.

It may be pointed out that this method and others of the indirect type have been freely used to obtain values of the bulk modulus for cast iron and metals of like character, and it will be shown that the composite crystalline rocks are very similar to cast iron in their behavior under stress, although generally more perfectly elastic.

Cast Iron.

Size	1.034 × 1.006		1.034	1.006
Area	1.041	1.041	....	....
E	15,000,000	15,000,000	....	....
σ	.25	.25	....	....
D	10,000,000	10,000,000	....	....
C	6,000,000	6,000,000	....	....
Longitudinal compression (multiply readings by 4 for millionths).			Lateral extension—(millionths).	
Load (in pounds).	Side *P.*	Side *U.*	Side *P.*	Side *U.*
1,000	0	0	0	0
2,000	19	20	12	11
3,000	40	37	26	21
4,000	60	58	41	32
5,000	80	78	56	48
6,000	100	100	72	65
7,000	120	120	86	83
8,000	140	143	102	99
9,000	160	160	119	116
8,000	145	143	106	108
7,000	123	125	90	85
6,000	104	110	76	70
5,000	85	90	60	60
4,000	63	63	44	50
3,000	44	40	30	39
2,000	20	21	13	21
1,000	0	0	6	9

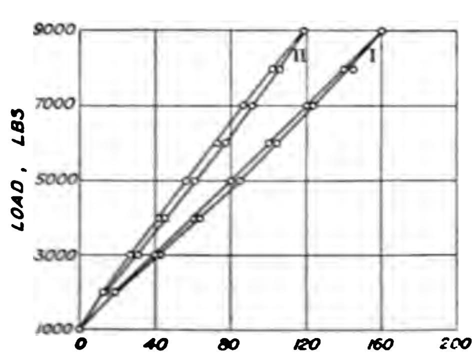

FIG. 3.—Cast iron. Stress-strain curves.

APPLICATION OF THE METHOD OF SIMPLE COMPRESSION TO THE DETERMINATION OF THE COMPRESSIBILITY OF ROCKS.

It has been noted in the case of marble when subjected to bending stress that the strain as exhibited by the deflection of a point of the bar increases with the time, and the strength under shear produced by a torque was also found to be so small that a determination of the strain was very difficult to measure.

These difficulties have been noted by Nagaoka,* who states that:

> Preliminary experiments on granite show that Hooke's law does not hold even for very small flexure and tension, and that the after effect is very considerable from the pressure, when the prism is sufficiently loaded or twisted, the deviation from the direct proportionality between strain and stress was incomparably great as compared with that observed in metal. This must be chiefly due to the low limit of elasticity, so that it is necessary to experiment only within very narrow limits of loading and twisting. These limits are widely different for different specimens of rocks, and the modulus of elasticity, as well as that of rigidity, was always determined with such stresses as will approximately produce strains proportional to them. The deviation from Hooke's law was prominent in certain specimens of sandstone, and it was more marked in tension than in flexure experiments; in certain rocks it is indeed doubtful if anything like a proportionality between stress and strain can be found, even for extremely small change of shape. On releasing these rocks from stress the return toward the former state is extremely small, showing that the elasticity of the rock is of a very inferior order.

These observations of Nagaoka for bending and twisting have been confirmed by our own deflection experiments, as above mentioned.

If, however, the rock be subjected to direct compression, strains in which the time effect is small and the lag of the strain is also small are almost invariably obtained. This is especially the case if before the actual experiment is carried out the material be several times subjected to a range of stresses at least as great as those employed in the experiment itself. This preliminary stressing brings the material to "a state of ease," and is also commonly adopted when the elastic constants of metals are determined.

It is evident, therefore, that this direct compression method may with confidence be applied to the measurement of the cubic compression of rocks, although as mentioned below the accuracy of the result so obtained will differ with different classes of rocks.

If the rock be massive, compact, and crystalline (or glassy) the method can be safely employed and good results will be obtained. If, on the other hand, the rock is schistose, porous, or loosely coherent, the method will from the nature of the case be very much less satisfactory.

The plutonic igneous rocks as a class most nearly resemble the metals in structure, being holocrystalline and massive, and therefore present the

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*Elastic Constants of Rocks and the Velocity of Seismic Waves. H. Nagaoka. Phil. Mag., vol. L, 1900, p. 58.

nearest approach among rocks to perfect elasticity; they are therefore a class of rocks to which this method is especially applicable. It fortunately happens that they are also a class of rocks a knowledge of whose compressibility is of special importance for the elucidation of many geological problems, constituting as they do the greater part of the earth's crust.

A second class of rocks which are comparable with them in their approach to perfect elasticity comprises the marbles and certain limestones.

A series of sixteen typical rocks representative of these two classes were accordingly selected for measurement. Under the first class a number of granites were chosen as representing the acid plutonic rocks and a number of types of the gabbro-essexite series were selected as representing the basic plutonic rocks. In all these cases great care was taken to choose the most homogeneous and massive rocks of each series and to secure test pieces free from all flaws and cracks. As representing the second class a number of typical marbles and limestones, also perfectly massive in character, were selected. For purposes of comparison, or contrast, a sandstone was added to the list as being a rock which, on account of its more or less porous nature could hardly be expected to yield satisfactory results by this method.

An examination of the stress-strain curves of these 16 rocks, omitting the sandstone, shows that on the average they possess a rather more perfect elasticity and exhibit less hysteresis than cast iron. Some of them, as for instance the Baveno granite, the nepheline syenite, the diabase, and the black Belgian marble, show much better curves, approximating in fact to the straight lines given by wrought iron, which may be considered for our present purpose as expressing perfect elasticity.

The close approximation to perfect elasticity is shown by the return of the curve to its initial or starting point, and the amount of the hysteresis is shown by the width of the loop.

The width of this hysteresis (or lag) curve or loop, indicates the amount of the divergence from Hook's law which the material exhibits—this law being that the stress and strain are *directly* proportional. When the curve is narrow, as it is in all cases except the Stanstead granite and the sandstone, the divergence from Hook's law is not great enough to seriously affect the result.

The rocks, therefore, with these exceptions, fulfil the conditions of elasticity necessary to the successful application of the method. In these two cases the results are less certain, owing to the greater hysteresis of the rock.

It might at first sight appear that while the method employed is theoretically perfect as applied to the measurement of the compressibility of vitreous rocks and of very fine grained crystalline rocks, a considerable error might be introduced when the rocks are coarser in grain. In the case of all the common crystalline rocks, the individual grains of which the rock is composed

are anisotropic, that is, they have different moduli of elasticity in different directions. In massive rocks such as those investigated, however, these grains occur in the rock with an absolutely irregular orientation and would in the case of a fine-grained rock mutually compensate for one another in any transverse line along which the expansion of the rock under compression might be measured. If, however, the rock were coarser in grain, fewer individual crystals would be found in any transverse line of section, and there might possibly in this way be a lack of compensation, as the rock in one section might be composed of grains whose axis of greater elasticity approximated on an average more nearly to the direction of measurement than in other sections. If such were really the case, there should be in these coarser-grained rocks an exceptionally great variation in the readings obtained from different specimens of the same rock, as well as from the different sections in the same specimen.

But such is not the case, as will be seen by an examination of the figures in accompanying table. They represent the results obtained from ten measurements of the compressibility of Baveno granite, which is coarse in grain, and ten of Sudbury diabase, which is very fine in grain, together with eight measurements on Tennessee limestone, which is rather coarse grain, and seven on plate glass. They were made in each case on two or more specimens cut from the same mass and the measurements of the expansion were made on several different planes through each, so that in every case the measurement was effected in a different line through the rock, all of these, however, of course being at right angles to the direction of the compressive stress and lying in the medial plane of the column.

Full details concerning each measurement will be found in the tables which set forth the results obtained, under the sections dealing with the several rocks in question. The size of grain and the texture of the rock can also be seen by examining the photomicrographs and color prints of the polished surfaces of the respective rocks.

	Max.	Min.	Diff.
Baveno granite (coarse) 10 trials	4,880,000	4,380,000	500,000
Sudbury diabase (very fine) 10 trials	11,170,000	9,655,000	1,515,000
Plate glass, 13 trials	6,930,000	6,020,000	910,000
Tennessee marble (rather coarse) 7 trials	6,130,000	5,770,000	360,000

It will thus be seen that there is no correspondence between the coarseness of grain and the magnitude of the variations in the readings obtained. The differences in glass, which is an isotropic material in which the elasticity is equal in all directions, are greater than in the Tennessee marble, which is rather coarse in grain, and in Baveno granite, which is the coarsest rock of

the set. The greatest differences obtained are those found in the finest grained rock in the series, viz. the Sudbury diabase.

It is evident, therefore, that the different moduli of elasticity of the constituent grains of a rock do not introduce any perceptible error in measurements made by this method, when a column an inch in diameter is employed, and when the rocks are not coarser in grain than the Baveno granite. In fact, while surrounded on all sides by other grains, no individual grain can expand freely, as it would if subjected to compression unhampered by any surrounding medium, and thus the anisotropic character of the individual grains produces but little effect on the elasticity of the rock as a whole.

These experiments also show that in the case of rocks composed of several minerals it makes no perceptible difference whether the points of attachment of the instrument are embedded in the grains of one mineral or of another.

The chief source of error and the one to which the variations observed are for the most part to be attributed seems to be a mechanical one, viz. the difficulty of getting an ideal contact between these points of attachment and the specimen to be measured, especially in view of the extremely small dimensions of the movement to be measured.

The question of the influence of temperature on the elasticity and compressibility of rocks is of course one which has an important bearing on certain problems of geophysics. The only investigation of this subject, so far as can be ascertained, consists of a few preliminary experiments by Nagaoka and Kasakabe.* In these the torsion method was employed, and the experiments were carried out on a single rock, viz. sandstone. This rock, as has already been mentioned, being porous and stratified in character, is a material whose elastic properties are far from ideal. The results are summed up by the authors in the following words:

> Preliminary experiments with sandstone show that the modulus of elasticity is much affected by the variation of temperature, *i. e.*, about 0.5 per cent per degree. It does not, however, necessarily diminish with the increase of temperature where the temperature is low, *i. e.*, it is maximum about 9° C.

As has been shown however, the values for the elastic constants obtained by this torsion and bending method have yielded results which can not in all cases be correct and which differ very considerably from those obtained by the more direct and simple method which has been employed in the present paper. These results bearing on the variation of elasticity induced by changes of temperature, especially in view of the fact that they are stated by the

*Modulus of Elasticity of Rocks and Velocities of Seismic Waves. Publications of the Earthquake Investigation Committee, No. 17. Tokyo, 1904, p. 43.

investigators to be "preliminary," can as yet hardly be taken as of general application to all rocks, even if correct for the specimen of sandstone examined.

In our own investigations the laboratory was maintained at a temperature of from 63° to 68° F. (17.2° to 20° C.), and a thorough investigation into the effect of temperature was not undertaken, as this would be very difficult to carry out when employing the method of direct compression used, the difficulty consisting in heating the specimen itself without in any way affecting the measuring apparatus attached to it.

It seemed, however, possible to ascertain whether any serious change in the elastic constants of the massive crystalline rocks employed in the present investigation would result from a moderate change of temperature. For purpose of trial the rock selected was the Sudbury diabase, a typical fine-grained plutonic rock. A column of it was placed by Mr. McKergow in a small testing machine having a capacity of 50 tons, and the temperature of the room in which the machine was set up having been lowered to + 10° F., a cycle of compression readings were taken in the usual way adopted when Young's modulus is to be determined. The temperature of the room was then raised by about 10° and another cycle of readings were taken. It was then raised another 10° and a third series of readings were obtained, and so through successive stages of 10° until the normal temperature of the room (about 65° F.) was reached. The initial reading of the instrument before the application of pressure was of course different in each case, owing to the expansion of the rock which followed from heating. These initial points were plotted on a line, and the results obtained when the specimen was subjected to a certain maximum load, together with the increase of temperature at each stage, were plotted on a second line. If the compression was greater at 65° than at 10° for the same load these two lines should have diverged, but as a matter of fact they were practically parallel. The differences between the readings given by the same load at different temperatures were no greater than those obtained by different measurements under the same load at the same temperature. The conclusion therefore seems to be indicated that a change of temperature made no perceptible difference within the range of temperatures employed, although a difference of 0.5 per cent for each degree centigrade, which was Nagaoka's result, would mean a difference of about 25 per cent in range of temperature employed by Mr. McKergow.

While, therefore, this experiment can not be considered as supplying accurate information concerning the effect produced by a rise in temperature on the elastic constants of rocks, for the instruments themselves are in some measure affected by the same changes of temperature, they serve to show that in the case of the massive crystalline rocks the influence of temperature is probably not very great. The subject is one which requires further investigation.

THE METHOD OF MEASUREMENT.

In carrying out the measurements, prisms of the rock approximately 1 inch square and 3 inches long were usually employed (see fig. 4). These were cut and ground with smooth faces, but were not polished. In these two small round holes were drilled in the medial line of each vertical face for the purpose of attaching the instrument, when Young's modulus was to be measured. These holes were made by means of a small diamond drill and were perfectly round and smooth. They were each 0.05 to 0.08 inch in diameter and 0.125 inch deep and 1.25 inches apart, lying at equal distances above and below the center of the prisms. These holes were champfered at the outer end, as shown in figure 4, and were found to afford the most perfect attachment which could be secured for the points of the instrument. By means of these prisms two sets of measurements of the vertical compression could be made with each prism, by attaching the instrument first to one pair of opposite faces and then to the other.

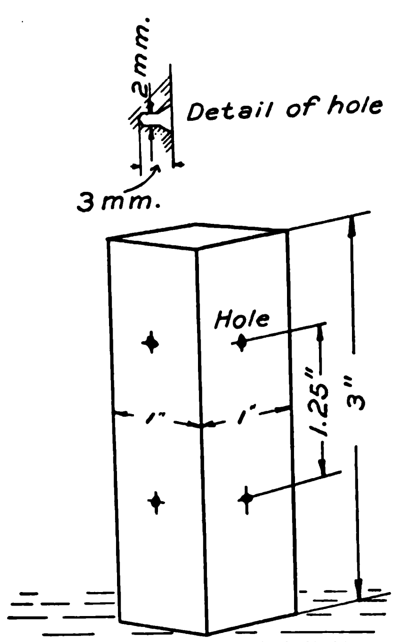


FIG. 4.—Square test specimen of rock.

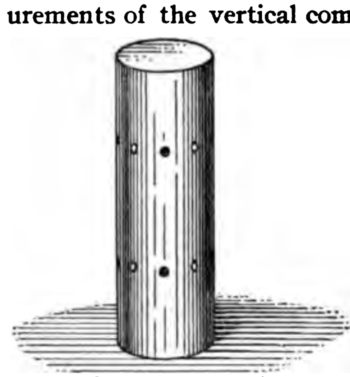


FIG. 5.—Round test specimen, showing position of holes.

In some cases round columns were used (see fig. 5). These were approximately 1 inch in diameter and 3 inches in length. With these it was possible to make four sets of measurements in compression with each column, by drilling eight pairs of holes, as above described, whose planes intersected at angles of 45° instead of 90° as in the square prisms.

It was of course necessary in every case, whether prisms or columns were employed, to exercise great care to have the ends of the test pieces very

carefully faced and absolutely parallel to one another. Before the actual measurements were made, the rock in every case was brought to a "state of ease" in the manner already described.

The pressure was applied in most cases by a 100-ton Wickstead testing machine, which was so carefully adjusted that it was sensitive to a load of 4 pounds.

The specimen, having been placed in the press and reduced to a state of ease, was then after careful adjustment submitted to loads increasing in successive stages of 1,000 pounds until the limit of safety had been reached, when the load was reduced successively by the same amounts, accurate readings being taken at each increment and decrement of load. The maximum load employed in the case of most rocks was 9,000 pounds, equivalent to from 9,000 pounds to about 11,500 pounds per square inch, according to whether a square or round prism was employed. In the case, however, of some of the stronger rocks a load of as much as 15,000 pounds per square inch was employed.

In the determination of the lateral strain, which was made upon the same set of columns as those used for measuring the vertical compression, care was taken that the theoretical conditions were realized, and that the material was free to expand laterally, as otherwise the values obtained for the lateral extension would be inaccurate. In all cases, therefore, the measuring apparatus was set as nearly as possible upon the central section of the test piece, and the ends of the specimen, after being ground smooth, were coated with a thin film of oil, so that the polished pressure plates of the machine would have as little tendency as possible to prevent freedom of lateral expansion.

In a number of cases accurate measurements were taken during the successive cycles of loading and unloading to which the specimen was subjected in order to bring it to a state of rest. These are recorded in the case of the Baveno granite and the Stanstead granite and served to show how the hysteresis of the rock may be reduced to a minimum by subjecting the test piece to this process. The measurements of each cycle usually occupied from 10 to 15 minutes.

It was at first conjectured that in the case of rocks composed of several minerals differences of reading might result from the attachment of the extensometer to different portions of the rock, the points of the instrument being fixed in some cases in grains of one mineral and in other cases in grains of another. It was found, however, as has already been mentioned, that the measurements on two sets of prism faces made in the manner above described, or on the four planes intersecting the vertical columns, where these had been provided with eight pairs of holes, showed that in the case of the rocks examined the differences between the several measurements on the same prism seem to be unaffected by the circumstance above referred

to. The differences between the measurements thus made on rocks composed of several minerals were no greater than those found in the case of the limestones, which were composed of the single mineral calcite, or on glass.

In the case of the majority of the rocks investigated, a number of prisms or columns cut from the same block of rock were measured in order to ascertain whether different test pieces would give identical readings. It was found as a result of these investigations that the differences between the different specimens were no greater than those which were obtained by measuring the same specimens with the instrument attached at different places. In the case, however, of the Quincy granite, test pieces from two different blocks of the rock were prepared, and it was found that while the several measurements made on each test piece agreed among themselves, there was a distinct divergence in the elastic constants of the two specimens of the rock. This was probably due to a difference in composition, as the two rocks differed somewhat in appearance.

In the case of the green gabbro from New Glasgow, the results obtained by measurements made upon different parts of the same prism were discordant, for reasons which will be pointed out and which were dependent upon the structure of the rock.

Fifty-five specimens of rock, nineteen of glass, and two of iron were employed in this investigation and every conceivable precaution was taken to insure the attainment of accurate results. The rocks in all cases were air dry, having been allowed to remain in the laboratory for several weeks after they had been cut, before the measurements were made.

In the accompanying tables the following elastic constants are given:

E = Young's modulus, *i.e.*, the quotient of the longitudinal stress by the longitudinal compression.

σ = Poisson's Ratio; this is the reciprocal of m.

D = Modulus of Cubic Compression $= \frac{1}{3}\left(\frac{m}{m-2}\right)E$. The reciprocal of this gives the decrease in volume of a cubic inch of the material for a pressure of 1 pound per square inch applied on every side.

C = Modulus of Shear $= \frac{1}{2}\left(\frac{m}{m+1}\right)E$, which is the quotient of torsional stress to torsional strain.

m = The ratio of longitudinal compression to lateral extension per unit of length.

E and m are measured directly; the other values are calculated from them.

These values in the case of each rock are given in the respective tables, expressed in inch and pound units, and the results are summarized in a general table on page 69.

The measurements were made in these units on account of the fact of the testing machine employed was graduated to read pounds.

For purposes of comparison, however, this latter table has been recalculated in C. G. S. units, and the results are set forth in the second table to be found on page 69.

In the case of metal, Poisson's ratio is generally arrived at by stretching the bar and determining the value of the longitudinal extension divided by the lateral contraction. In case of rocks the tensile strength being low and the materials being brittle, it is more convenient and more accurate to make the determination by compressing a short bar or column, and determining the value of the longitudinal compression divided by the lateral expansion. This gives the value designated as m, of which Poisson's ratio is the reciprocal. Theoretically one method is as accurate as the other. In actual practice it might be supposed that the short compression columns in question would not expand quite so much at the ends as in the middle because of the friction against the compression plates. In order, however, to cause these to slip as easily as possible over the ends of the column, the surface of the rock in contact with them was always made very smooth and also was slightly oiled. It was found that, these precautions being observed, the expansion at the ends of the column was practically as great as at the center, where the measurement was taken, the differences being so small that no serious discrepancy was introduced.

In the tables the first transverse line designates the specimen employed as a, b, c, or d. The second line gives the diameter of the specimen, which is often slightly different in the two directions. The length of the column in all cases was about 3 inches, but this is not stated in the table, as the compression is not measured on the total length of the column, but on the length of that portion of it which lies between the points of attachment of the instrument.

The third line gives the area, which is approximately 1 square inch in the case of a square prism and three-quarters of a square inch in the case of a round column.

In the four succeeding lines the four elastic constants E, σ, D, and C, are given, as determined by each measurement.

Another transverse line contains the letters U or P, which designate the two diameters of the column when two measurements were made on the same square prism, these two directions being always at right angles to one another. In the case of round columns, on which measurements were frequently made in several planes, these are designated as "first holes," "second holes," etc.

In each table there follows the values obtained for successive loadings of 1,000 pounds in the case of each specimen, first for compression, when the figures multiplied by four give millionths of an inch, and then for lateral expansion, given directly in millionths of an inch. These afford the data for calculating the constants and for plotting the curves which accompany every table.

In the figures for the constants of iron and of one or two of the rocks, which are the result of measurements which were made at the beginning of the investigation, a slight correction has been made, owing to the inaccurate calibration of the extensometer, which will explain a certain discrepancy which will appear if the figures are recalculated.

THE ELASTIC CONSTANTS OF ROCKS COMPOSED OF A SINGLE MINERAL.

MARBLES AND LIMESTONES.

BLACK BELGIAN MARBLE, BELGIUM.

This rock is known in trade by the name of "Belgian black" or "Noir fin." It is an extremely fine grained black marble which takes a very high polish and is used very extensively in interior decoration. It has a splintery fracture, breaking almost like glass.

When thin sections are examined under the microscope the rock is found to be so fine in grain that a high power is necessary to resolve it. It is composed of minute calcite grains from 0.02 mm. to 0.002 mm. in diameter and of irregular shape, between and around which are occasional minute films and spots of a black color.

In this very fine grained and even groundmass are embedded a very few larger forms of clear white calcite, some of them rodlike, others circular in shape, and others possessing more complicated outlines. These are evidently of organic origin, representating small fragments of fossils. They are very sparsely scattered through the rock. The rock also contains occasional minute grains or crystals of iron pyrites.

Fragments of this rock dissolve readily in cold dilute hydrochloric acid, giving off a fetid odor and leaving a considerable amount of a light flocculent residue, black in color and apparently consisting of some form of bituminous matter. In the residue there are also a few minute grains of pyrite.

Plate I A is a color-process photograph of a polished surface of this marble and Plate I B is a photomicrograph of a thin section of the rock, taken in ordinary light and magnified 27 diameters.

A square prism of the rock of the usual dimensions was employed to measure the elastic constants, and the results are set forth in the table found on page 25.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (× 27 DIAM.-ORDINARY LIGHT)

BLACK BELGIAN MARBLE, ("NOIR FIN").

Black Belgian Marble.

Size	.96 × .96	.96
Area	.922	...
E	11,070,000	....
σ	.278	....
D	8,303,000	....
C	4,330,000	....
Load (in pounds).	Longitudinal compression (multiply readings by 4 for millionths).	Lateral extension (millionths).
1,000	0	0
2,000	25	24
3,000	53	51
4,000	84	76
5,000	116	103
6,000	147	129
7,000	178	155
8,000	210	182
9,000	240	209
8,000	211	183
7,000	180	157
6,000	148	131
5,000	118	104
4,000	88	78
3,000	57	51
2,000	30	23
1,000	1	4

The elastic constants were found to be as follows:

$E = 10{,}070{,}000$; $\sigma = 0.278$; $D = 8{,}303{,}000$; $C = 4{,}330{,}000$.

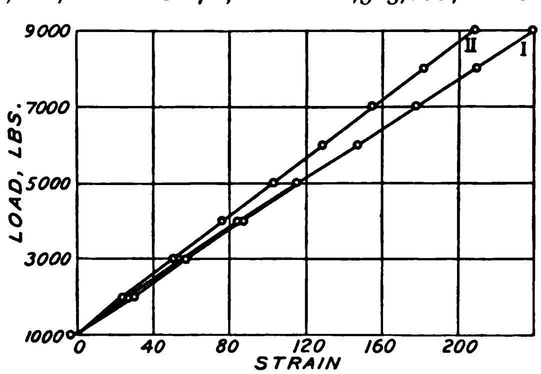


FIG. 6.—Black Belgian Marble. Stress-strain curves.

A plot of the readings is given in figure 6, from which it is clearly seen that the rock is practically free from hysteresis, and that within the range of pressures employed its elasticity is almost perfect. I represents longitudinal compression and II lateral extension.

WHITE MARBLE, CARRARA, ITALY.

A white, very fine grained saccharoidal marble. Under the microscope it is seen to consist of a mosaic of calcite grains. In this mosaic some grains are larger than others, but there is no great difference in their relative sizes and the average grain of the rock is uniform throughout. The average diameter of calcite crystals closely approximates 0.2 mm. The grains come against one another along sharp and usually straight lines. There is no trace of foliation in the rock, nor is there any trace of flattening or elongation of the grains in any one direction. The rock is perfectly massive. Between crossed nicols the calcite individuals extinguish uniformly and show no signs of pressure. Some of them show a few twin lamellæ.

A color-process photograph of a polished surface of the rock employed is shown in Plate II A and a photomicrograph of a thin section of the rock, taken in ordinary light and magnified 27 diameters, is shown in Plate II B.

Three specimens of the rock were used in measuring the elastic constants, two square prisms (*a* and *b*) and a round column (*c*). Two sets of measurements were made on both *b* and *c*, the instrument being as usual affixed to the specimen in two positions at right angles to one another in each specimen. In this way five complete sets of measurements were made. The results are set forth in the table on page 27.

The means of the results obtained for the respective elastic constants are as follows:

$$E = 8{,}046{,}000; \quad \sigma = 0.2744; \quad D = 5{,}946{,}000; \quad C = 3{,}154{,}000.$$

The difference between the highest and lowest determinations of D is 420,000 pounds.

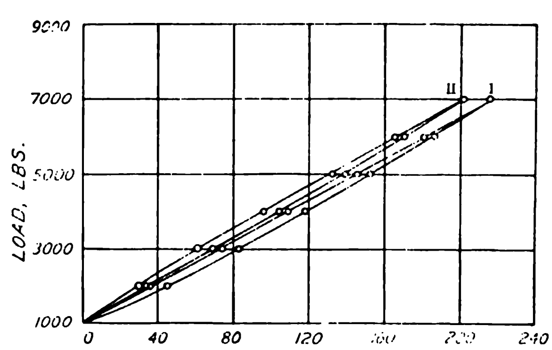
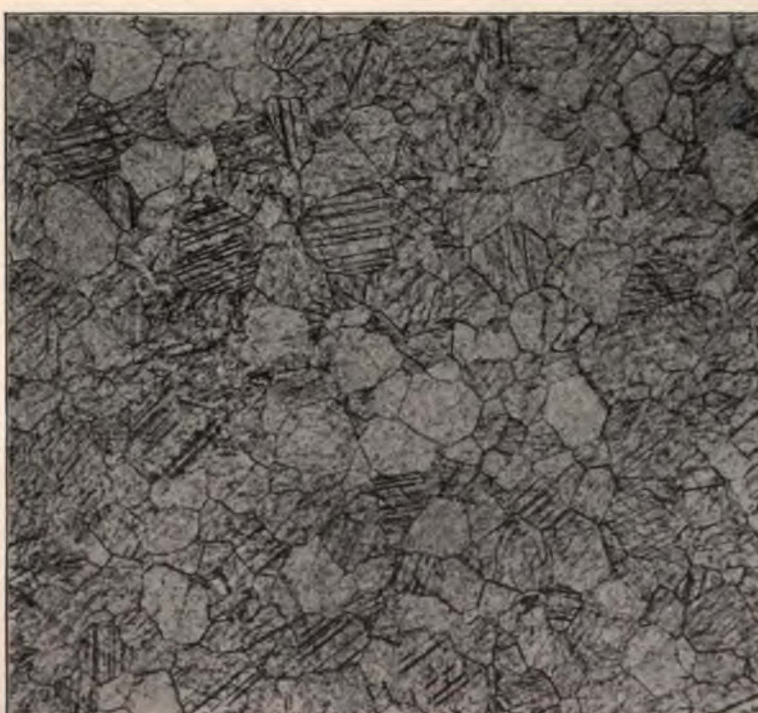


FIG. 7.—Carrara marble, specimen *a*. Stress-strain curves.

Figure 7 shows the results obtained from specimen *a* in graphic form. I represents longitudinal compression and II lateral extension. The hysteresis is greater than in the case of Belgian black or Tennessee marble, but is about the same in amount as that shown by the Vermont marble and the Trenton limestone from Montreal.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)



B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

CARRARA MARBLE.

Carrara Marble.

No. . . .	*a*	*b*	*b*	*c*	*c*
Size . .	1.032 × 1.035	1.017 × 1.016	. . . .	.985	.985
Area . .	1.07	1.033	1.033	.762	.762
E	8,120,000	7,800,000	8,055,000	8,210,000	8,045,000
σ	.281	.274	.273	.275	.269
D	6,170,000	5,750,000	5,920,000	6,100,000	5,790,000
C	3,170,000	3,060,000	3,160,000	3,210,000	3,170,000

LONGITUDINAL COMPRESSION.—MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*
1,000.	0	0	0	0	0
2,000.	35	40	35	50	50
3,000.	75	80	80	100	100
4,000	110	120	120	150	155
5,000	145	160	160	200	205
6,000.	180	205	200	250	255
7,000.	216	240	235	. . . .	. . . .
8,000	. . . .	275	270	. . . .	. . . .
9,000.	. . . .	310	300	. . . .	. . . .
8,000.	. . . .	290	295	. . . .	. . . .
7,000.	216	247	260	. . . .	. . . .
6,000.	185	210	215	250	255
5,000.	152	171	170	210	210
4,000.	118	125	130	160	165
3,000	82	85	90	105	110
2,000.	44	45	30	53	55
1,000.	0	5	−5	0	4

LATERAL EXTENSION.—MILLIONTHS.

No.	*a*	*b*	*b*	*c*
Size.	1.032	1.016	1.017	.985
Load (in pounds).		Side *P.*	Side *U.*	
1,000.	0	0	0	0
2,000.	30	35	37	45
3,000.	62	75	73	95
4,000.	95	110	108	130
5,000	130	145	141	175
6,000	165	183	177	217
7,000	200	. . . .	. . . .	. . . .
8,000.	. . . .	. . . .	. . . .	. . . .
9,000.	. . . .	. . . .	. . . .	. . . .
8,000.	. . . .	. . . .	. . . .	. . . .
7 000.	200	. . . .	. . . .	. . . .
6,000.	170	183	177	217
5,000.	154	150	180	180
4,000.	105	120	112	137
3,000.	70	83	75	100
2,000.	35	40	35	50
1,000.	3.	4	2	4

WHITE MARBLE, VERMONT, UNITED STATES.

This is a pure white marble indistinguishable from the Carrara marble in a hand specimen. Under the microscope also it resembles this rock very closely. The grains show, however, a somewhat greater variation in relative size and there is a tendency to a flattening in one direction, giving a very faint foliation to the rock. On this account only a single specimen was used, since the foliation in question, although barely perceptible, might affect the elasticity of rock, and it was therefore considered safer to rely upon the Carrara marble in measuring the elastic constants of this class of rocks. In the prism of Vermont marble employed, the foliation lay in the direction of the longer axis of the prism. It is probable that this foliation would not be found in all Vermont marbles, but happened to be present in the specimen procured for examination.

A photomicrograph of a thin section of the rock, in this case taken between crossed nicols and magnified 31 diameters, is shown in Plate III. A color-process photograph was not prepared, since the rock in such a photograph would be identical in appearance with the Carrara marble, of which such a photograph has already been given.

Vermont Marble.

Size	1.017 × 1.012	
Area	1.029	
E	7,592,000	
σ	.263	
D	5,341,000	
C	3,000,000	
Load (in pounds).	Longitudinal compression. (multiply readings by 4 for millionths).	Lateral extension (millionths).
1,000	0	0
2,000	40	30
3,000	80	64
4,000	120	100
5,000	157	135
6,000	200	172
7,000	....	....
8,000	....	....
9,000	....	....
8,000	....	....
7,000	....	....
6,000	200	172
5,000	165	140
4,000	125	108
3,000	90	75
2,000	51	39
1,000	5	1

VERMONT MARBLE.

PHOTOMICROGRAPH OF THIN SECTION, (X 31 DIAM.-NICOLS CROSSED)

A square prism of the marble was employed in measuring the elastic constants, and the detailed results are given in the accompanying table, and are represented in graphic form in figure 8. I represents longitudinal compression and II lateral extension.

The following are the values obtained:

$E = 7{,}592{,}000$; $\sigma = 0.263$; $D = 5{,}341{,}000$; $C = 3{,}000{,}000$.

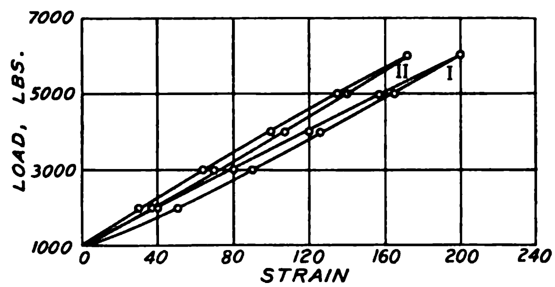


FIG. 8.—Vermont marble. Stress-strain curves.

MARBLE, TENNESSEE, UNITED STATES.

This is a marble known in trade as "Pink Tennessee," and is largely used for decorative work. It has a brownish pink color and when polished shows a somewhat mottled surface.

Under the microscope it is seen to consist of rather large irregular-shaped and often distinctly rounded individuals of calcite, which are fitted closely together along sharp and in some cases crenulated lines. These individuals are almost invariably traversed by narrow lamellæ, due to polysynthetic twinning, and are occasionally twisted, so that they show an undulatory extinction. Between these large calcite individuals there are frequently present masses of what is apparently a tabulate coral, showing sheaves of tubes which in cross section are approximately circular in outline. The calcite individuals are often embedded in this coralline material, as if they had been devéloped by its recrystallization; in other cases, however, their appearance suggests a derivation from crinoidal fragments. All the tubes of the coral, as well as the interspaces of the tubes, if any existed, are now filled with calcite, so that the substance of the rock is continuous, resulting in a compact marble. Fragments of the rock dissolve readily in cold dilute hydrochloric acid, leaving only a very trifling residue, which has the color of the rock itself.

A color-process photograph of a polished surface of the rock is shown in Plate IV A, and a photomicrograph of a thin section of the rock, taken in ordinary light and magnified 27 diameters, is shown in Plate IV B. In this photomicrograph a fragment of the coralline material is seen in the center of the field, while the border is formed chiefly of individual calcite grains.

Tennessee Marble.

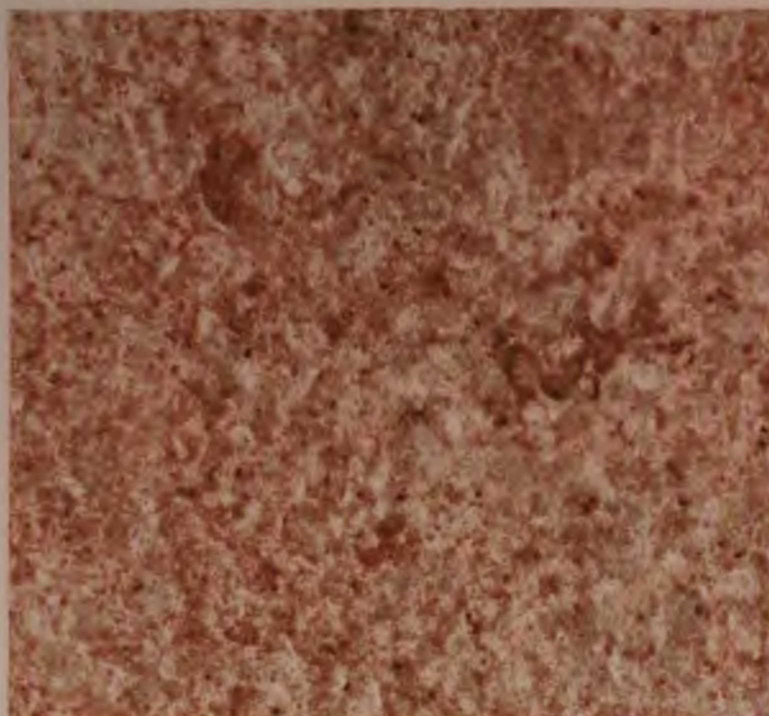
No.	*a*	*a*	*b*	*b*	*c*	*c*	*c*
Size	1.037 × .996	....	1.004 × .911	....	1 × .97	....	....
Area	1.033	1.033	.915	.915	.97	.97	.97
E	9,140,000	8,960,000	9,120,000	9,260,000	8,760,000	8,900,000	8,900,000
e	.251	.267	.251	.2415	.247	.2435	.258
D	6,080,000	5,970,000	6,070,000	5,970,000	5,780,000	5,770,000	6,130,000
C	3,655,000	3,595,000	3,650,000	3,725,000	3,510,000	3,575,000	3,540,000

Longitudinal Compression.—Multiply Readings by 4 for Millionths.

Load (in pounds).	Side *U.*	Side *P.*	Side *P.*	Side *U.*	Side *U.*	Side *P.*	Side *P.*
1,000	0	0	0	0	0	0	0
2,000	20	35	40	30	35	30	25
3,000	65	70	75	60	75	70	60
4,000	100	100	110	100	110	105	95
5,000	130	130	150	140	145	140	130
6,000	165	170	190	185	184	180	170
7,000	200	205	230	220	....	215	210
8,000	230	230	270	270	....	250	250
9,000	265	270	300	295	....	290	290
8,000	230	....	275	....	....	275	..
7,000	200	....	250	....	....	255	..
6,000	170	....	220	....	184	190	..
5,000	135	...	180	....	170	165	.
4,000	100	...	140	....	140	120	..
3,000	65	...	110	....	100	90	
2,000	30	..	45	....	60	60	...
1,000	5	4	10	5	5	5	4

Lateral Extension.—Millionths.

No.	[illegible]	*a*	*b*	*b*	*c*	*c*
Size	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Load (in pounds).	Side [illegible]	Side [illegible]	Side [illegible]	Side [illegible]	Side [illegible]	Side [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]					
[illegible]	[illegible]					
[illegible]	[illegible]					
[illegible]	[illegible]					
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]					
[illegible]	[illegible]					
[illegible]	[illegible]					
[illegible]	[illegible]					
[illegible]	[illegible]					
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]					
[illegible]	[illegible]					
3,000	[illegible]					
2,000	[illegible]					
1,000	−15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

MARBLE, TENNESSEE, ("PINK TENNESSEE").

Three square prisms of the rock were employed in measuring the elastic constants, and on these seven sets of measurements of vertical compression and six of lateral extension were made, as shown in the table on page 30.

The averages of the results obtained are as follows:

$E = 9{,}006{,}000$; $\sigma = 0.2513$; $D = 5{,}967{,}000$; $C = 3{,}607{,}000$.

The difference between the highest and lowest values obtained for D is 360,000. As will be seen by consulting figure 9 the rock is almost free from hysteresis. In this figure I represents the longitudinal compression and II lateral extension.

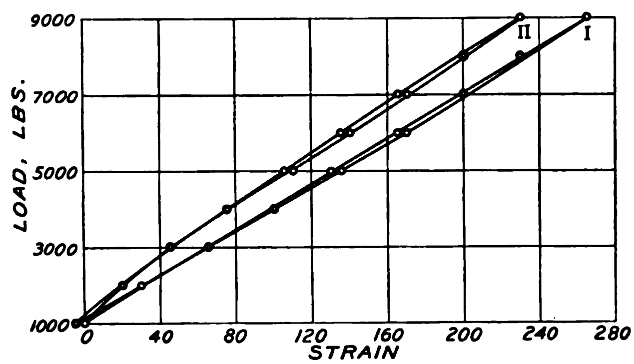


FIG. 9.—Tennessee Marble. Stress-strain curves.

FOSSILIFEROUS LIMESTONE (TRENTON FORMATION), MILE END QUARRY, MONTREAL, CANADA.

This is a typical fossiliferous limestone of the Trenton formation (Ordovician). It was taken from a massive bed 2 feet in thickness known as the "Lower Bed" at the quarry from which the greater part of the building stone for the city of Montreal is obtained. The rock is dark gray in color, and is compact and solid in character.

Under the microscope it is seen to be composed of fragments of fossils which are in some cases angular and in others more or less rounded. They are chiefly bits of Monticulipora and of Crinoids and show the structure of these organisms perfectly. These fragments lie embedded in clear transparent calcite, occurring as large individuals which form a continuous mosaic, giving rise in this way to a perfectly compact rock.

A color-process photograph of a polished surface of this rock is given in Plate V A. A photomicrograph of a thin section of the rock, taken in ordinary

light and magnified 27 diameters, is shown in Plate V B. In the photomicrograph a fragment of a Monticuliporid is seen in the center of the field, while the darker areas about the periphery of the field are Crinoid fragments, each with a secondary enlargement, consisting of pure calcite, surrounding it.

Trenton Limestone, Montreal, Canada.

No.	*a*	*a*	*b*	*b*	*a*	*b*
Size	.97	....	.984	....	.97	.984
Area	.749	.749	.762	.762	....	....
E	9,280,000	9,490,000	9,120,000	8,930,000	....	....
σ	.25	.2562	.2545	.2482	....	....
D	6,180,000	6,480,000	6,190,000	5,820,000	....	....
C	3,710,000	3,775,000	3,645,000	3,415,000	....	....
Longitudinal compression.—Multiply readings by 4 for millionths.					Lateral extension (millionths).	
Load (in pounds).	Side *U*.	Side *P*.	Side *U*.	Side *P*.		
1,000	0	0	0	0	0	0
2,000	50	50	50	50	30	34
3,000	90	90	85	90	60	70
4,000	140	135	135	130	95	100
5,000	180	175	180	170	135	140
6,000	225	220	225	230	175	180
7,000	....	....	....	....	....	....
8,000	....	....	....	....	....	....
9,000	....	....	....	....	....	....
8,000	....	....	....	....	....	....
7,000	....	....	....	....	....	....
6,000	225	220	225	230	175	180
5,000	185	180	190	185	140	147
4,000	149	143	140	140	100	106
3,000	95	95	90	95	65	73
2,000	55	54	54	55	32	35
1,000	4	0	4	5	2	2

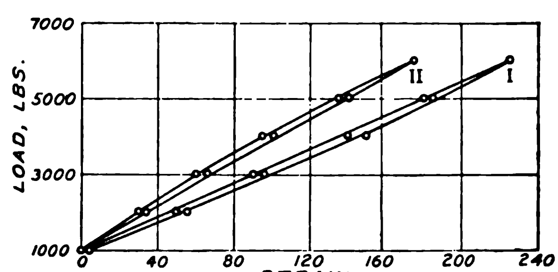


FIG. 10.—Trenton limestone. Stress-strain curves.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

TRENTON LIMESTONE, MONTREAL, CANADA.

The elastic constants of the rock were measured on two round columns, four measurements of vertical compression and two of lateral extension being made. The results are given in the table on page 32.

The averages of the results obtained are as follows:

$$E = 9,205,000; \quad \sigma = 0.2522; \quad D = 6,167,500; \quad C = 3,636,000.$$

The difference between the highest and lowest readings for D is 660,000. The results of the measurement of specimen *a* are shown in graphic form in figure 10. I represents longitudinal compression and II lateral extension.

THE ELASTIC CONSTANTS OF ROCKS COMPOSED OF MORE THAN ONE MINERAL.

ACID PLUTONIC ROCKS.

GRANITE, BAVENO, ITALY.

This well-known granite is pale pink in color, and although coarser in grain than the rocks just described, is a granite of medium grain and is very uniform in character. It resembles the Lily Lake granite closely in appearance, although it is a little finer in grain.

It is composed essentially of quartz and orthoclase, with very small proportion of biotite; the biotite is in places somewhat altered to chlorite, and the orthoclase is in places somewhat turbid from the presence of kaolin, but the rock may nevertheless be characterized as a very fresh one.

It has a typical hypidiomorphic granular structure. The orthoclase often shows faint microperthitic intergrowths, and some plagioclase is present as an accessory constituent. The quartz usually shows a very faint undulatory extinction, although this is in some cases quite distinct.

A color-process photograph of a polished surface of the rock is shown in Plate VI A, and a photomicrograph of a thin section of the rock is shown in Plate VI B. The latter was taken between crossed nicols in polarized light and is magnified 30 diameters. The crack seen crossing the rock in the photomicrograph was developed during the grinding of the thin section and does not indicate any shattering or lack of continuity in the substance of the rock itself.

Four round columns, *b*, *c*, *d* and *e*, were employed in the measurement of the elastic constants of this granite. On specimen *b* a double set of measurements was made in each of the planes U and P, which planes were at right angles to one another. In the case of column *c* two sets of measurements were made in two planes, also at right angles to one another (referred to as "first

holes" and "second holes"). In column *d* four sets of holes were drilled, so as to carry out measurements on four diametral planes, making angles of 45° with one another. The first set of holes, however, were defective, so measurements were made on the second, third, and fourth sets only. On column *e* a single set of measurements was completed when the column broke.

In this way ten measurements of vertical compression and six of lateral extension were obtained. The results are presented in the table on page 35. The averages of the results obtained are as follows:

$$E = 6{,}833{,}000; \quad \sigma = 0.2528; \quad D = 4{,}604{,}000; \quad C = 2{,}724{,}800.$$

The difference between the highest and lowest results obtained for *D* amounts to 500,000.

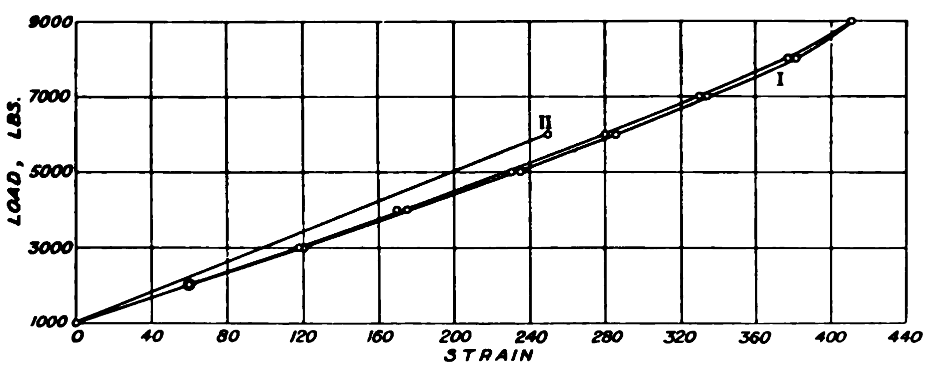


FIG. 11.—Baveno Granite. Stress-strain curves.

As mentioned in the chapter on the "Application of the Method of Simple Compression to the Case of Rocks," in order to obtain consistent and reliable results, the specimen of the rock—and for that matter the same is true of the specimen of any metal if its elastic constants are to be determined—must first be bought to "state of ease" by loading and unloading it several times in succession, employing each time a pressure equal to the maximum load to which the specimen is to be subjected, when the measurements are subsequently made.

As a matter of interest in the case of the Baveno granite (as well as in that of the Stanstead granite referred to later), readings were taken during the first four cycles of compression, when this state of ease was being induced, and the results are presented graphically in figure 12.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

GRANITE, BAVENO, ITALY.

Granite, Baveno, Italy.

No....	*b*	*b*	*b*	*b*	*c*	*c*	*d*	*d*	*d*	*e*
Size..	.978	....	....	....	.978	....	.978	....	...	.978
Area..	.75	....	....	....	.75	....	.75	....	....	.75
Side..	*U.*	*U.*	P_1	P_2	1st holes	2d holes	2d holes	3d holes	4th holes	....
E.....	6,620,000	6,730,000	7,430,000	6,950,000	6,950,000	6,840,000	6,840,000	6,620,000	6,730,000	6,620,000
σ.....	.2483	.2525	.2465	.2505	.2505	.247	.265	.257	.261	.2495
D.....	4,380,000	4,530,000	4,880,000	4,645,000	4,645,000	4,470,000	4,850,000	4,540,000	4,680,000	4,420,000
C.....	2,650,000	2,682,000	2,980,000	2,780,000	2,780,000	2,730,000	2,700,000	2,631,000	2,670,000	2,645,000

LONGITUDINAL COMPRESSION—MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).										
1,000	0	0	0	0	0	0	0	0	0	0
2,000	....	....	60	70	....	....	....	....	....	80
3,000	....	....	118	130	....	....	....	....	....	145
4,000	....	....	175	180	....	....	....	....	....	210
5,000	....	....	230	240	....	....	....	....	....	260
6,000	315	310	280	300	300	305	305	315	310	315
7,000	....	....	330	342	....	....	....	....	....	368
8,000	....	....	380	400	....	....	....	....	....	419
9,000	....	....	410	450	....	....	....	....	....	465
8,000	....	....	385	410	....	....	....	....	....	420
7,000	....	....	335	390	....	....	....	....	....	375
6,000	315	310	285	312	300	305	305	315	310	325
5,000	....	....	235	254	....	....	....	....	....	280
4,000	....	....	170	204	....	....	....	....	....	225
3,000	....	....	120	145	....	....	....	....	....	160
2,000	....	....	60	85	....	....	....	....	....	90
1,000	2	5	3	18	2	4	4	10	8	10

LATERAL EXTENSION—MILLIONTHS.

No	*b*		*b*	*c*	*d*	*e*
Size	.978		.978	.978	.978	.978
Area..........	....		....	....	....	....
Side..........	P_1 and P_2		*U*	....	....	....
1,000..........	0	0	0	0	0	0
2,000..........	....	....	....	....	....	....
3,000..........	....	....	....	....	....	....
4,000..........	....	....	....	....	....	....
5,000..........	....	....	....	....	....	....
6,000..........	216	235	245	235	253	246
7,000..........	....	....	....	....	....	....
8,000..........	....	....	....	....	....	....
9,000..........	....	....	....	....	....	....
8,000..........	....	....	....	....	....	....
7,000..........	....	....	....	....	....	....
6,000..........	216	235	245	235	253	246
5,000..........	....	....	....	....	....	....
4,000..........	....	....	....	....	....	....
3,000..........	....	....	....	....	....	....
2,000..........	....	....	....	....	....	....
1,000..........	1	4	5	7	5	10

The curves represent the readings for longitudinal compression, and, as will be seen, after the first cycle of compression the rock did not return quite to its original position, but this imperfection in elasticity becomes progressively smaller in the subsequent loadings till in the fourth compression cycle the return is almost perfect and the hysteresis very small.

Figure 11 shows the curves obtained by plotting the values secured from the measurement of the elastic constants of specimen *b* after the state of ease had been induced, and if the curve for longitudinal compression in this be compared with that shown in figure 12 the great improvement in the elasticity of the rock will at once be seen. In figure 11, I represents longitudinal compression and II lateral extension.

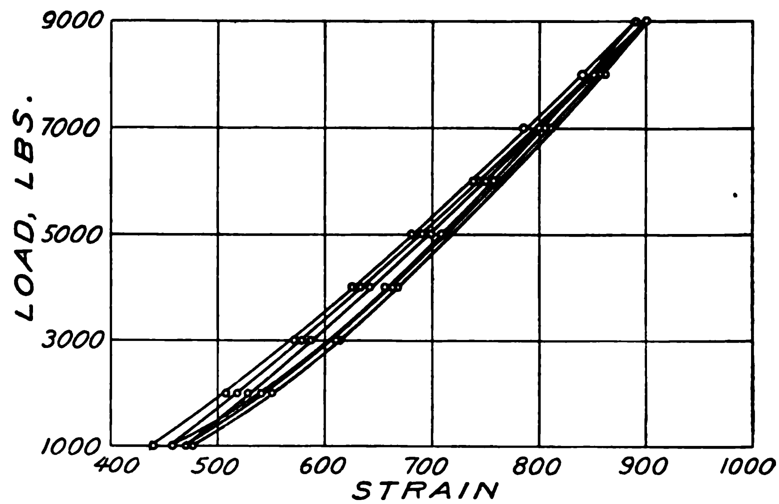


FIG. 12.—Stress-strain curves obtained in the first four cycles of compression from a column of Baveno granite, showing the progress toward a state of ease.

GRANITE, PETERHEAD, SCOTLAND.

A pink granite which is almost indistinguishable from the Lily Lake granite in hand specimens or polished blocks. In the thin sections also the resemblance is very close. The description given of the Lily Lake granite would also apply to this rock, except that the Peterhead granite contains rather more plagioclase and less biotite. The microperthite also is more turbid, indicating greater alteration. The quartz and microperthite which make up the greater part of the rock have evidently crystallized out at about the same time, since they have equally good crystalline outlines and impress their form upon each other with about equal frequency. The quartz usually shows pronounced undulatory extinction.

Owing to its practical identity with the Lily Lake granite, in appearance and composition, it has been considered unnecessary to give either a photograph of the polished surface of the rock or a photomicrograph of a thin section.

Those given for the Lily Lake granite may be considered as representing this rock also. Two square prisms of the rock were prepared and on these five sets of measurements of vertical compression and two of lateral expansion were made. These are given in the accompanying table, and the curves

Granite, Peterhead, Scotland.

No......	*b*	*b*	*b*	*a*	*a*	*b*	*a*
Size.....	1.006 × 1	....	....	.998 × 1.056	....	1.000	.998
Area....	1.006	1.006	1.006	1.052	1.052	....	....
E.......	8,020,000	8,280,000	8,375,000	8,400,000	8,400,000	....	....
σ.......	.204	.212	.215	.214	.213	....	....
D.......	4,520,000	4,790,000	4,860,000	4,900,000	4,890,000	....	....
C	3,330,000	3,415,000	3,450,000	3,400,000	3,400,000	....	....
Longitudinal compression (multiply readings by 4 for millionths).						Lateral extension (millionths).	
Load (in pounds).	Side. *U.*	Side. *U.*	Side. *P.*	Side. *P.*	Side. *P.*		
1,000..	0	0	0	0	0	0	0
2,000..	40	40	40	40	....	25	25
3,000..	85	80	85	80	....	50	49
4,000..	125	120	125	120	....	75	74
5,000..	165	160	160	160	....	100	101
6,000..	200	195	195	200	....	125	125
7,000..	240	230	225	230	....	150	149
8,000..	270	260	260	270	....	178	175
9,000..	310	300	296	310	309	203	200
8,000..	270	....	....	....	....	180	170
7,000..	245	....	....	....	....	155	150
6,000..	205	....	....	....	....	130	125
5,000..	170	....	....	....	....	105	100
4,000..	135	....	....	....	....	78	75
3,000..	95	....	....	....	....	54	49
2,000..	50	....	....	....	....	26	20
1,000..	5	4	−4	5	4	−5	−2

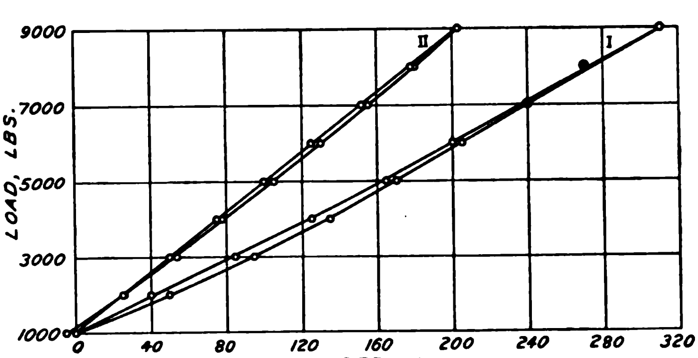


FIG. 13.—Peterhead granite. Stress-strain curves.

given by b are shown in figure 13. Of these curves, I represents longitudinal compression and II lateral extension.

The averages of the results obtained are as follows:

$$E = 8,295,000; \quad \sigma = 0.2112; \quad D = 4,792,000; \quad C = 3,399,000.$$

The difference beween the highest and lowest values obtained for D amounts to 380,000, or if one abnormally low determination be omitted the difference is 110,000.

GRANITE, LILY LAKE, PROVINCE OF NEW BRUNSWICK, CANADA.

A typical rather coarse-grained pink granite. Under the microscope it is seen to present the usual hypidiomorphic structure of this rock, and to be composed of biotite, microperthite, and quartz as essential constituents. A small amount of plagioclase occurs as an accessory constituent. There are also a few minute crystals of a highly doubly refracting mineral which has also a high index of refraction, and apparently crystallizes in square prisms. This is probably zircon or possibly monazite.

The feldspars and quartz preponderate largely. The microperthite, which is the most abundant constituent in the rock, is composed of a minute intergrowth of two feldspars, in neither of which can twinning be detected. One is, in all probability, orthoclase and the other albite. The former is more or less turbid from the presence of alteration products, such as are commonly found in this mineral species, while the latter is clear and fresh. The quartz shows marked undulatory extinction as in the case of the Westerly granite. The biotite is fresh and deep brown in color.

This rock is, as stated above, a typical granite, rather coarse in grain, and which has undergone but very little alteration.

A color-process photograph of a polished surface of the rock is seen in Plate VII A and a photomicrograph of a thin section magnified 30 diameters and taken between crossed nicols in polarized light, is shown in Plate VII B.

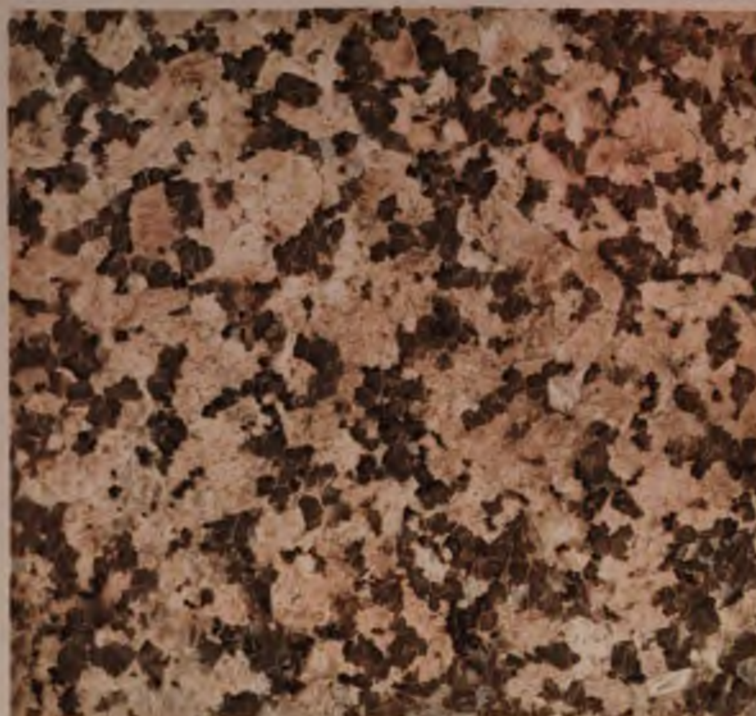
Two square prisms of the rock were prepared and their elastic constants determined. The results are given in the table on page 39.

The stress-strain curves given by specimen c are shown in figure 14. In this figure I represents longitudinal compression and II lateral extension.

The means of the results obtained are as follows:

$$E = 8,165,000; \quad \sigma = 0.1982; \quad D = 4,517,500; \quad C = 3,380,000.$$

The difference between the two determinations of D is only 105,000.



A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

GRANITE, LILY LAKE, CANADA.

Granite, Lily Lake, Province of New Brunswick, Canada.

No.	a	c	a	c
Size	.965 × .955	.995 × 1.05	....	....
Area	.922	1.045	....	....
E	8,230,000	8,100,000	....	....
σ	.2	.1965	....	....
D	4,570,000	4,465,000	....	....
C	3,370,000	3,390,000	....	....
	Longitudinal compression (multiply readings by 4 for millionths).		Lateral extension (millionths).	
Load (in pounds).				
1,000	0	0	0	0
2,000	43	40	21	15
3,000	84	80	39	41
4,000	125	120	59	62
5,000	161	155	81	84
6,000	201	190	100	106
7,000	238	225	119	130
8,000	274	260	141	152
9,000	308	295	159	185
8,000	277	268	140	155
7,000	245	234	118	135
6,000	208	200	99	110
5,000	170	170	80	90
4,000	133	132	60	70
3,000	93	92	39	50
2,000	50	50	20	25
1,000	0	−4	0	0

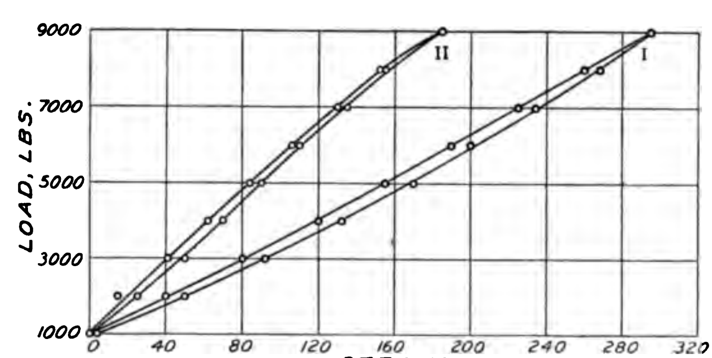


FIG. 14.—Lily Lake Granite. Stress-strain curves.

GRANITE, WESTERLY, RHODE ISLAND, UNITED STATES.

This rock is a fresh, very fine grained, massive, pale pink granite, being much finer in grain than the other granites referred to in this paper.

Under the microscope it is seen to be composed essentially of biotite, microcline, orthoclase, and quartz. In addition to these constituents a small percentage of plagioclase and a few grains of magnetite are present as accessory constituents, together with a little chlorite and muscovite as alteration products.

The feldspars form the greater part of the rock, microcline being by far the most abundant of these. It shows in a striking manner the characteristic cross-hatched twinning of this species, and is usually quite fresh. The orthoclase in untwinned individuals is frequently distinctly turbid from the development of kaolin, and in a few places muscovite in larger individuals can be seen inclosed in it, apparently developing as a secondary product at its expense.

The quartz, which is next in abundance, usually shows marked undulatory extinction, and some grains have been so strained that they fall into areas with distinctly different optical orientations. The quartz, instead of occupying corners between the feldspar individuals, usually occurs as subangular or more or less rounded grains associated with the feldspar, and apparently more nearly contemporaneous with this mineral in its crystallization than is usually the case. The rock often shows a tendency to granophyric structure, small rounded grains or vermiform inclusions of quartz being sometimes seen in the microcline. The structure otherwise is of the normal granite type. The biotite is very subordinate in amount and is more or less changed into chlorite.

Although these decomposition products are present, the rock can not be considered as one which has undergone much alteration. It has, as a matter of fact, undergone very little, and is to be classed as a distinctly fresh rock—much fresher than granites usually are.

A color-process photograph of the rock is seen in Plate VIII A and a photomicrograph of a thin section taken between crossed nicols in polarized light and magnified 30 diameters is shown in Plate VIII B.

Four test pieces were used in measuring the elastic constants, viz, two square prisms, *a* and *b*, and two round columns, *c* and *d*. Two sets of determinations were made on each of the first three specimens, the instruments being attached to different pairs of sides in each case, and four sets of determinations were made on specimen *d* in planes making angles at 45° with one another. Ten determinations of vertical compression and three of lateral extension were thus made the results of which are given in the following table:

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

GRANITE, WESTERLY, RHODE ISLAND.

Granite, Westerly, Rhode Island, U. S. A.

No. . .	*a*	*a*	*b*	*b*	*c*	*c*	*d*	*d*	*d*	*d*
Size . .	1.008 × 1.002		.981 × .929	....	.975	....	.975	....	....	....
Area .	1.01	....	.91	....	.75	....	.75	....	....	....
Side . .	*U.*	*P.*	*U.*	*P.*	1st holes	2d holes	1st holes	2d holes	3d holes	4th holes
E.	7,180,000	7,090,000	7,625,000	7,745,000	7,670,000	7,335,000	7,575,000	7,335,000	7,170,000	7,250,000
σ. . . .	.21	.1985	.241	.214	....	....	.225	.223	.2225	.222
D.	4,110,000	3,950,000	4,925,000	4,515,000	....	....	4,600,000	4,420,000	4,320,000	4,340,000
C.	2,970,000	2,960,000	3,070,000	3,185,000	....	....	3,090,000	2,980,000	2,940,000	2,961,000

LONGITUDINAL COMPRESSION—MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	1st holes	2d holes	1st holes	2d holes	3d holes	4th holes
1,000	0	0	0	0	0	0	0	0	0	0
2,000	50	40	50	....	....	....	....	....	....	....
3,000	95	80	100	155	....	....	143	145	145	144
4,000	145	120	150	....	....	....	....	....	....	....
5,000	180	160	190	....	....	....	....	....	....	....
6,000	225	210	235	310	....	....	298	301	302	303
7,000	265	260	275	....	....	....	....	....	....	....
8,000	305	310	315	....	....	....	....	....	....	....
9,000	345	349	361	355	435	455	440	455	465	460
8,000	310	....	320	....	....	....	....	....	....	....
7,000	270	....	280	....	....	....	....	....	....	....
6,000	235	....	240	312	....	....	300	305	305	305
5,000	200	....	195	....	....	....	....	....	....	....
4,000	160	....	....	....	....	....	....	....	....	....
3,000	110	....	....	157	....	....	145	149	150	149
2,000	60	....	....	....	....	....	....	....	....	....
1,000	5	....	....	3	....	....	3	10	15	12

LATERAL EXTENSION—MILLIONTHS.

No.	*a*	*b*	*d*
Size	1.002	.929	.975
Load (in pounds).			
1,000	0	0	0
2,000	25	30	30
3,000	50	50	80
4,000	75	85	120
5,000	100	125	175
6,000	130	177	209
7,000	160	....	....
8,000	190	....	....
9,000	220	....	....
8,000	195	....	....
7,000	165	....	....
6,000	140	177	209
5,000	115	135	160
4,000	90	100	130
3,000	60	70	90
2,000	30	30	45
1,000	0	0	5

The stress-strain curves obtained from specimen a are given in figure 15. In this figure, I represents longitudinal compression and II lateral extension.

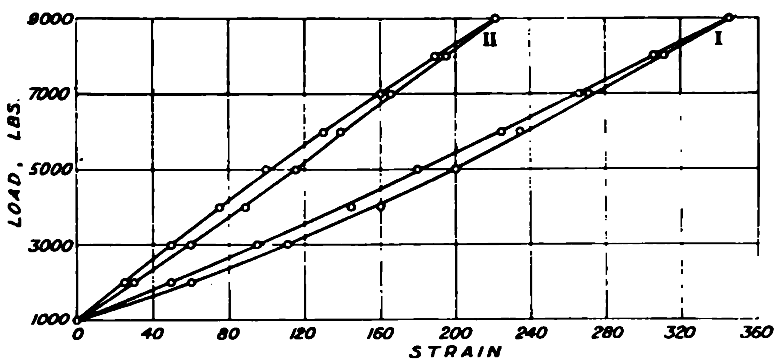


FIG. 15.—Westerly Granite. Stress-strain curves.

The averages of the values obtained are as follows:

$E = 7,394,500$; $\sigma = 0.2195$; $D = 4,397,500$; $C = 3,019,700$.

The differences between the highest and lowest values in the four determinations of D on specimen d was only 280,000. Of the other columns a gave on an average somewhat lower, and b somewhat higher results.

GRANITE, QUINCY, MASSACHUSETTS, UNITED STATES.

The rock is a rather coarse grained gray granite composed very largely of microperthite and quartz. The iron-magnesia constituents are represented by a very deep green, almost black, hornblende, associated with which there appears to be a smaller amount of a very dark colored pyroxene. These dark constituents belong to the alkali-rich varieties of their respective families and are so opaque that it is difficult to determine their precise character. They are also very irregular in shape, occupying corners between the feldspar grains and often penetrated by crystals of the microperthite, showing that they separated out later than the feldspar. The quartz shows strong undulatory extinction. The rock is fresh and unaltered.

A color-process photograph of a polished surface of the Quincy granite is shown in Plate IX A, and a photomicrograph of a thin section taken between crossed nicols in polarized light and magnified 30 diameters is reproduced in Plate IX B.

Two large specimens of the rock, which differed slightly from one another in appearance, were secured and examined. Three square prisms, a, c and d, were prepared from one specimen and one square prism, b, from the other

A. PHOTOGRAPH OF POLISHED SURFACE. (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

GRANITE, QUINCY, MASSACHUSETTS.

Granite, Quincy, Massachusetts, United States.

	First specimen.						Second specimen.	
No....	*a*	*a*	*c*	*c*	*d*	*d*	*b*	*b*
Size ..	.989 × 1.071	......	1.063 × .955	......	1.011 × .954	......	.945 × .892	....
Area..	1.06	1.06	1.01	1.01	.965	.965	.843	.843
E.....	6,560,000	6,840,000	6,630,000	6,630,000	6,820,000	7,000,000	8,135,000	8,360,000
σ.....	.185	.1925	.21	.21	.244	.25	.1915	.204
D.....	3,470,000	3,710,000	3,810,000	3,810,000	4,440,000	4,666,000	4,390,000	4,720,000
C.....	2,765,000	2,865,000	2,760,000	2,760,000	2,740,000	2,800,000	3,410,000	3,480,000

LONGITUDINAL COMPRESSION—MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000	0	0	0	0	0	0	0	0
2,000	60	50	56	....	45	....	....	50
3,000	110	95	109	....	100	....	....	100
4,000	160	145	161	....	150	....	....	142
5,000	200	185	206	....	198	....	....	193
6,000	240	230	259	....	246	....	....	225
7,000	280	270	291	....	290	....	....	260
8,000	320	310	334	....	330	....	....	310
9,000	360	345	374	374	380	370	365	355
8,000	321	315	325	....	340	....	....	314
7,000	282	280	290	....	298	....	....	274
6,000	245	235	260	....	250	....	....	235
5,000	203	195	210	....	210	....	....	190
4,000	162	140	165	....	165	....	....	145
3,000	112	90	110	....	115	....	....	100
2,000	63	40	60	....	60	....	....	50
1,000	4	5	5	0	5	5	3	5

LATERAL EXTENSION—MILLIONTHS.

	First specimen.			Second specimen.	
No..........	*a*	*c*	*d*	*b*	*b*
Size..........	.989	.955	1.011	.892	.945
1,000..........	0	0	0	0	0
2,000..........	....	25	35	24	20
3,000..........	....	55	70	51	45
4,000..........	....	90	110	76	70
5,000..........	....	115	150	101	100
6,000..........	....	145	190	125	125
7,000..........	....	175	230	157	155
8,000..........	....	200	270	174	186
9,000..........	210	240	300	206	219
8,000..........	....	205	275	175	192
7,000..........	....	180	240	153	164
6,000..........	....	155	195	127	134
5,000..........	....	120	155	103	105
4,000..........	....	95	120	77	88
3,000..........	....	60	75	50	50
2,000..........	....	30	40	25	25
1,000..........	5	5	10	2	3

specimen, which was rather darker in color. Two series of compression determinations were made on each of these prisms. Eight series of measurements were thus made of vertical compression and five of lateral extension.

In the second specimen of the rock, D was found to have a rather higher value than in the case of the first specimen, although prism d, cut from the first specimen, approaches this value closely. The duplicate determinations made on each of the prisms agree very closely with one another. The results of the measurements are given in the table on page 44.

The average of the values obtained in the case of the first specimen are as follows:

$E = 6{,}747{,}000$; $\sigma = 0.2152$; $D = 3{,}984{,}000$; $C = 2{,}781{,}600$.

The average of those obtained from the second specimen are as follows:

$E = 8{,}247{,}500$; $\sigma = 0.1977$; $D = 4{,}555{,}000$; $C = 3{,}445{,}000$.

In this case, as has been mentioned, the two specimens represent different varieties of the Quincy granite. The stress strain curve given by specimen b is shown in figure 16. In this figure, I represents longitudinal compression and II lateral extension.

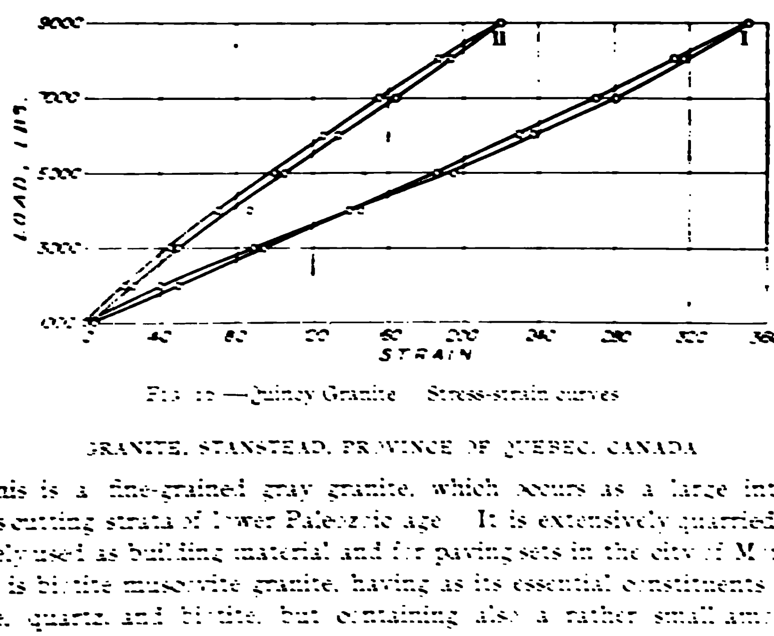


Fig. 16.—Quincy Granite. Stress-strain curves.

GRANITE, STANSTEAD, PROVINCE OF QUEBEC, CANADA

This is a fine-grained gray granite, which occurs as a large intrusive mass cutting strata of lower Paleozoic age. It is extensively quarried and is largely used as building material and for paving sets in the city of Montreal.

It is biotite muscovite granite, having as its essential constituents orthoclase, quartz and biotite, but containing also a rather small amount of muscovite and epidote, both of which occur as skeleton crystals of considerable dimensions, for the most part growing in the feldspar and apparently of secondary origin. The rock is very fresh, being almost entirely free from the usual decomposition products. In addition to orthoclase, the rock con-

tains a considerable percentage of microcline and of a plagioclase of the soda-lime series. The mica is relatively more abundant than in the other granites described in the present paper. The quartz shows marked undulatory extinction and in some cases even an incipient granulation. The size of the grain of this rock is intermediate between that of the Westerly and the other granites, which latter are themselves about equally coarse.

The elastic constants were measured on three square prisms, four sets of measurements of vertical compression and three of lateral extension being made. The results are given in the following table:

Granite, Stanstead, Province of Quebec, Canada.

No.....	*a*	*b*	*b*	*d*	*a*	*b*	*d*
Size....	.954 × .95	1.015×1.0009	1.015 × 1.00098	1.0083 × .957	.954	1.015	.957
Area....	.906	1.016	1.016	.965	....	....	....
Side....	....	*U.*	*P.*	....	....	....	....
E	6,000,000	5,030,000	5,540,000	6,170,000	....	....	....
σ.......	.253	.251	.282	.248	....	....	....
D.......	4,040,000	3,360,000	4,250,000	4,110,000	....	....	....
C.......	2,395,000	2,015,000	2,155,000	2,470,000	....	....	....
Longitudinal compression (multiply readings by 4 for millionths).					Lateral extension (millionths).		
Load (in pounds).							
1,000	0	0	0	0	0	0	0
2,000	70	58	75	60	29	30	40
3,000	135	122	145	125	61	70	80
4,000	200	190	200	180	105	125	119
5,000	250	250	245	230	149	180	155
6,000	310	310	305	280	200	230	195
7,000	360	370	350	325	246	280	235
8,000	415	430	395	375	295	335	275
9,000	460	490	445	420	355	400	320
8,000	430	440	....	385	320	340	295
7,000	385	380	....	340	275	290	265
6,000	330	315	....	295	230	250	230
5,000	280	255	....	250	189	190	200
4,000	220	195	....	200	140	130	160
3,000	160	125	....	140	90	70	110
2,000	90	60	....	80	45	20	50
1,000	10	10	5	2	−5	−6	15

The averages of the results obtained are as follows:

$$E = 5{,}685{,}000; \quad \sigma = 0.2585; \quad D = 3{,}940{,}000; \quad C = 2{,}258{,}700.$$

This rock, as will be seen, has a low modulus of elasticity, and like other rocks of which this is true, the lateral extension varies considerably in different specimens and the rock does not come readily to a state of ease. This is seen

from figure 17, which shows the results obtained in the first three cycles of compression made upon a column of the rock. The hysteresis shown is much greater than in the case of any of the other crystalline rocks examined, and even after repeated stressing this hysteresis, although reduced, does not disappear, as is seen from the curve of the results obtained from column *a* given in figure 18. In this figure I represents longitudinal compression and II lateral extension. The variation in the results obtained for *D* accordingly is high, amounting to 890,000.

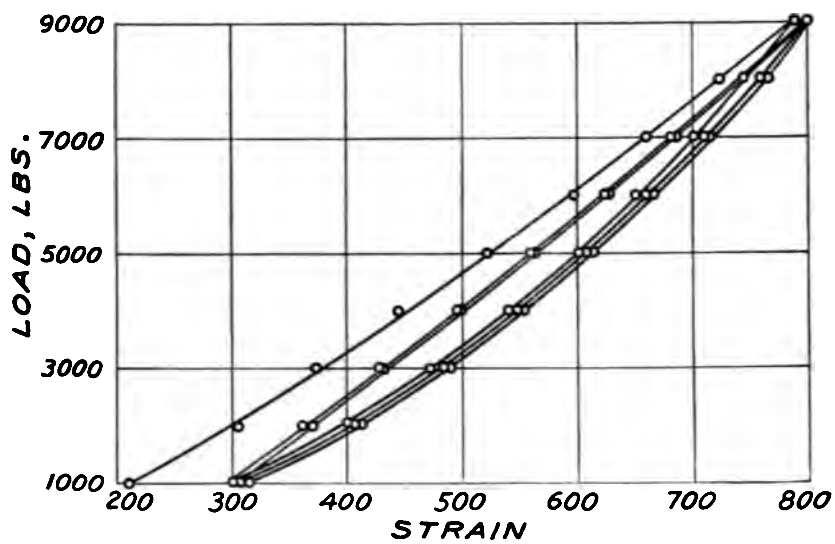


FIG. 17.—Stress-strain curves obtained in the first three cycles of compression, from a column of Stanstead Granite, showing its imperfect elasticity.

On account of its defective elasticity the result of the measurement of the compressibility of this rock is less satisfactory than that of the other granites, from which it differs considerably in the value obtained for *D*, although the values obtained in the case of the other granites agree pretty closely among themselves. The cause of this defective elasticity in the Stanstead granite is not clear, although it may be connected with a lack of strength in the rock, which in its turn may be connected with the presence in the rock of so large an amount of mica.

It is a weak rock compared with other granites or with the essexite from Mount Johnson, as shown by the results of a series of tests carried out in the Testing Laboratory of McGill University, and given in the table on page 47.

A color process photograph of a polished surface of the rock is shown in Plate X A, and the photomicrograph of a thin section of it in Plate X B. This

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

GRANITE, STANSTEAD, CANADA.

Table showing comparative strength of Stanstead Granite.

Specimen.		Dimensions.	Area.	Weight.	Actual load at initial failure.	Load per sq. in. at initial failure.	Max. load.	Max. load per sq inch.
		Inches.	*Sq. In.*	*Lbs.*	*Lbs.*	*Lbs.*	*Lbs.*	*Lbs.*
Granite,	A	2.56 by 2.55 by 2.66	6.528	1.5915	113,000	17,310	124,300	18,040
St. Philip, Quebec..	B	2.47 by 2.46 by 2.52	6.076	1.441	115,000	18,926	142,800	23,500
Essexite,	A	2.47 by 2.68 by 2.54	6.619	1.700	138,000	20,849	148,700	22,465
Mt. Johnson, Quebec	B	2.48 by 2.56 by 2.56	6.368	1.636	141,000	22,141	167,700	26,334
Granite,	A	2.61 by 2.50 by 2.52	6.525	1.565	92,700	....	92,700	14,206
Stanstead, Quebec..	B	2.56 by 2.55 by 2.47	6.528	1.511	88,300	....	89,300	13,526

latter is taken in ordinary light and magnified 27 diameters. The fact that this photomicrograph is taken in ordinary light, while those of other granites just described are taken between crossed nicols, gives this rock an appearance of being coarser in grain than it really is, owing to the boundaries of the colorless constituents being ill-defined. The size of the grain may be seen, however, by comparing the dimensions of the iron-magnesia constituents of the rocks or still better by comparing the grain of the several rocks as shown in the photographs of the polished surfaces.

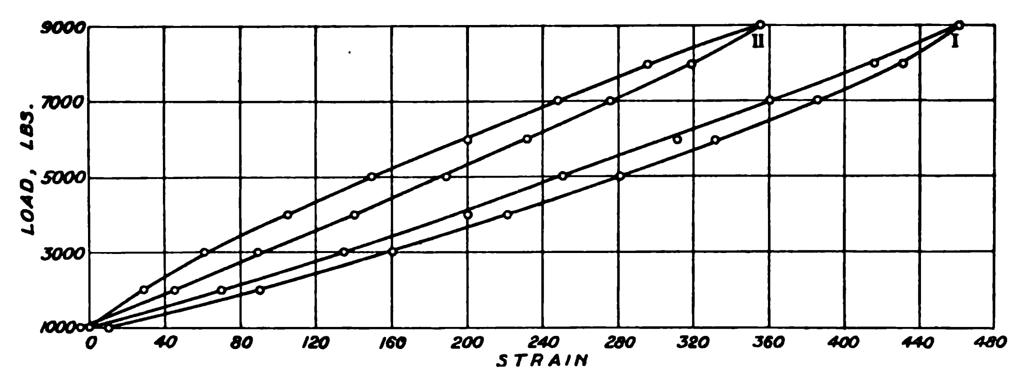


FIG. 18.—Stanstead Granite. Stress-strain curves.

NEPHELINE SYENITE.

NEPHELINE SYENITE, CORPORATION QUARRY, MONTREAL, CANADA.

This is a typical nepheline syenite which forms a portion of Mount Royal, one of the Monteregian Hills and which cuts an earlier intrusion of essexite like that to be described later from Mount Johnson.

It is a hard and tough rock used as road metal on the streets of the city of Montreal. It is rather light gray in color and often shows locally a more

or less distinct parallelism of the constituent minerals, owing to movements during the final stages of the consolidation of the rock, representing in fact a sort of fluidal structure. Traces of this are seen in the specimen from which the colored photograph accompanying this description was taken, but the prism of the rock on which the elastic constants were measured, while otherwise identical with the specimen photographed, showed no traces whatsoever of the fluidal structure in question, but was absolutely massive.

Under the microscope the rock is seen to be composed chiefly of light-colored "salic" constituents of which feldspar is by far the most abundant. This is chiefly orthoclase, but this mineral is much intergrown with plagioclase, which is also present in not inconsiderable amount. These feldspars have for the most part a lath-shaped development, and it is on account of the more or less parallel arrangement of these laths and of the hornblende crystals that the

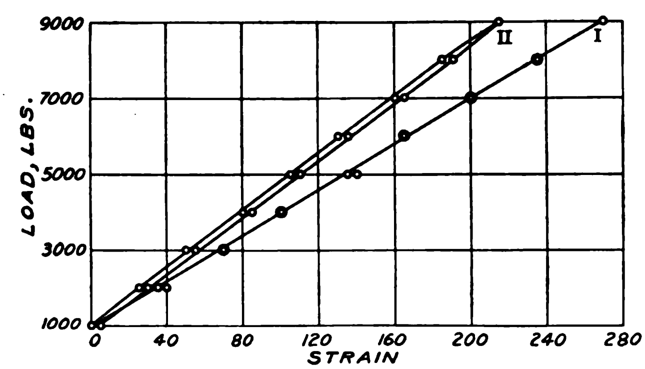


FIG. 19.—Nepheline Syenite. Stress-strain curves.

fluidal structure above mentioned results. Associated with the feldspar is nepheline in rather small amount, and also nosean often in well-defined individuals. These occur in some cases as inclusions in the feldspar. In other cases they lie in the corners between the latter. The dark ("femic") constituents are represented chiefly by a greenish-brown alkali hornblende, with which biotite is associated in much smaller amount. This hornblende has a tendency to an acicular development. There are also present in small amounts, as accessory constituents, sphene magnetite and pyrite.

The rock is fresh, there being no signs of decomposition. Although the nepheline and the nosean are in most cases somewhat altered, the changes which have overtaken them are quite independent of surface decay.

A color-process photograph of a polished surface is seen in Plate XI A, and a photomicrograph taken between crossed nicols and magnified 30 diameters is shown in Plate XI B.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

NEPHELINE SYENITE, MONTREAL, CANADA.

Two sets of measurements for the elastic constants were made on a single square prism of the rock, using first one set of faces and then the other. The results are set forth in the following table:

Nepheline Syenite, Montreal, Canada.

No.	*a*	*a*	*a*	*a*
Size	1 × 1.003	....	1	1.003
Area	1.003	1.003	....	....
E	9,230,000	9,045,000	....	....
σ	.249	.263	....	....
D	6,125,000	6,350,000	....	....
C	3,695,000	3,575,000	....	....
Longitudinal compression (multiply readings by 4 for millionths).			Lateral extension (millionths).	
Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000	0	0	0	0
2,000	35	35	25	25
3,000	70	65	50	51
4,000	100	105	80	80
5,000	135	135	105	110
6,000	165	170	130	145
7,000	200	205	160	175
8,000	235	245	185	200
9,000	270	275	216	230
8,000	234	240	190	200
7,000	200	210	165	180
6,000	165	170	135	155
5,000	140	140	110	125
4,000	100	105	85	95
3,000	70	70	55	65
2,000	35	35	30	35
1,000	5	2	5	5

The stress-strain curves obtained in the first set of measurements are seen in fig. 19 (p. 48), in which I represents longitudinal compression and II shows lateral extension. An examination of these will show that the rock exhibits very little hysteresis, the values for longitudinal compression giving a straight line, as in the case of wrought iron and other metals.

The averages of the results obtained are as follows:

$$E = 9{,}137{,}500; \quad \sigma = 0.256; \quad D = 6{,}237{,}500; \quad C = 3{,}635{,}000.$$

The differences between the two determinations for the value of D amounted to only 225,000 pounds. As will be observed, the value of D for this rock is much higher than that for any of the granites.

BASIC PLUTONIC ROCKS.

ANORTHOSITE, NEW GLASGOW, PROVINCE OF QUEBEC, CANADA.

This rock is from the great Morin anorthosite intrusion which occupies an area of 990 square miles on the border of the Laurentian protaxis, some 30 miles north of the city of Montreal.* The specimen is from the margin of the intrusion, where the mass has undergone extensive movement of the nature of rockflow, which movement has been brought about by pressure exerted upon the earth's crust in this district. The flow has taken place through a granulation of the larger individuals of the original rock, combined with a movement of this granulated material under the influence of the pressure, giving rise to a rude banding in the rock. This granulation has not, however, been accompanied by any loss of strength, for the rock is a hard and exceedingly tough one, being used as paving sets in some of the streets in the city of Montreal, where there is an especially heavy traffic.

Most of the Morin intrusion consists almost exclusively of plagioclase feldspar, which has the composition of labradorite, with only a very small portion of iron-magnesia constituents, and hence the rock is properly termed "anorthosite."

The specimen used for the determination of the elastic constants of the rock was cut from a paving set which was richer than usual in the iron-magnesia constituents and which consequently might be more properly referred to as gabbro, although it is merely a part of the anorthosite locally richer in these darker constituents. It has a rudely streaked structure, as seen in the accompanying color-process photograph of a polished specimen, Plate XII A This structure crossed the vertical face of the test piece diagonally, so that if there be a variation in the values of the elastic constants dependent on the direction of the streaking, the readings attained will represent a mean, or at any rate an intermediate value.

Under the microscope the rock is seen to be composed chiefly of plagioclase, associated with which is a pale green augite, a deep green hornblende, with a few grains of ilmenite, and an occasional individual of hypersthene, now altered to serpentine, and of pyrite.

The plagioclase forms a mosaic of well-twinned grains, through which are distributed the other constituents in little irregular-shaped grains of rounded or subrounded outline Of these the augite in the most abundant. With the exception of the alteration which has overtaken the few hypersthene grains

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*Adams, F. D Report on the Geology of a Portion of the Laurentian Area lying to the North of the Island of Montreal. Annual Report of the Geological Survey of Canada, Part J, vol. VIII, 1890, p. 111.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

ANORTHOSITE, NEW GLASGOW, CANADA

the rock is absolutely fresh. The structure is allotriomorphic, and there is a tendency to a parallel arrangement among the grains of the darker constituents.

A photomicrograph of a thin section of the rock taken between crossed nicols in polarized light and magnified 30 diameters is shown in Plate XII B.

The elastic constants were determined on a square prism of the rock, and as the rock is very strong, the loading was carried up to 15,000 pounds instead of 9,000 pounds, as in the other rocks.

The figures obtained are set forth as in the following table:

Anorthosite, New Glasgow, Province of Quebec, Canada.

Size	.99 × .99	
Area	.981	
E	11,960,000	
σ	.262	
D	8,368,000	
C	4,750,000	
Load (in pounds).	Longitudinal compression (multiply readings by 4 for millionths).	Lateral extension (millionths).
1,000	0	0
3,000	51	41
5,000	101	81
7,000	157	124
9,000	212	168
11,000	264	214
13,000	318	260
15,000	373	309
13,000	328	266
11,000	278	222
9,000	227	179
7,000	173	135
5,000	116	90
3,000	58	44
1,000	3	0

The averages of the values found are as follows:

$E = 11{,}960{,}000$; $\sigma = 0.262$; $D = 8{,}368{,}000$; $C = 4{,}750{,}000$.

The stress-strain curves of the rock are shown in figure 20, in which I represents longitudinal compression and II lateral extension.

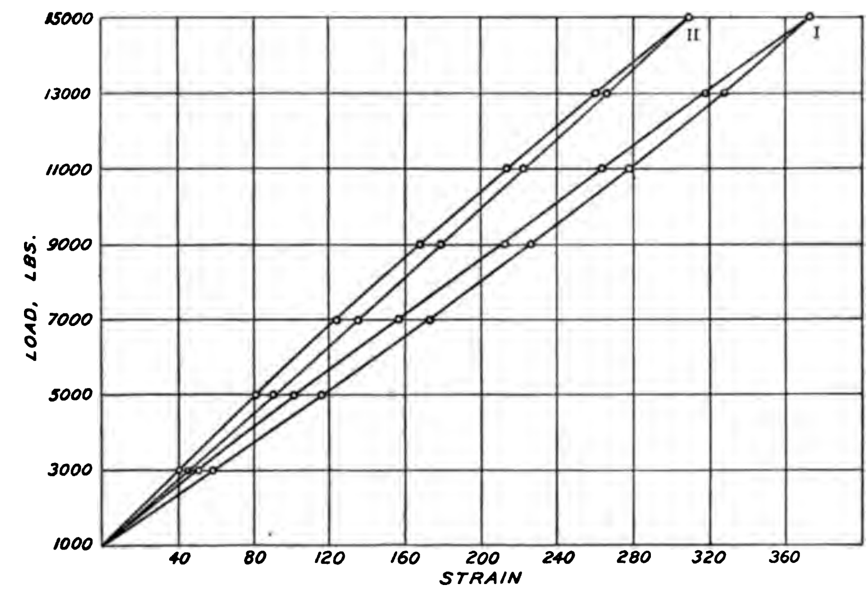


FIG. 20.—New Glasgow Anorthosite. Stress-strain curves.

ESSEXITE, MOUNT JOHNSON, PROVINCE OF QUEBEC, CANADA.

This is a rather coarse grained essexite from a quarry on the slope of Mount Johnson, which is a typical butte arising from the Paleozoic plain to the south of the city of Montreal and forming one of the Monteregian Hills.* The rock is massive and uniform in character and dark gray in color, and is extensively used as a building stone and also for monuments.

The iron-magnesia constituents are represented by a violet augite, a deep brown hornblende, and a biotite also very deep brown in color, the first mentioned being the most abundant and all three being frequently intimately intergrown. The light-colored constituents are plagioclase and nepheline, the former being more abundant than the latter, which often occurs as inclusions in the feldspar. Although polysynthetic twinning is frequently seen in the feldspar, a considerable proportion of it is untwinned. A separation by Thoulet's solution, however, shows that the feldspar is all plagioclase, there being no orthoclase in the rock. Magnetite in the form of small grains and apatite in rather large, well-defined crystals are present in considerable amount as accessory constituents. The rock is perfectly fresh. The constituents of the rock, and more especially the feldspar, have a tendency

*Adams, F. D. The Monteregian Hills, a Canadian Petrographical Province. Journal of Geology, April-May, 1903.

Essexite, Mount Johnson, Province of Quebec, Canada.

No.	a	a	a	b	c
Size	.975 × .992	.975 × .992	.975 × .992	.9025 × .9825	.971 × 1.007
Area	.966	.966	.966	.886	.978
E	9,580,000	9,580,000	9,580,000	9,565,000	10,430,000
σ	.2663	.2663	.2663	.2363	.2563
D	6,840,000	6,840,000	6,840,000	6,060,000	7,170,000
C	3,781,000	3,781,000	3 781,000	3,860,000	4,160,000
Longitudinal Compression—Multiply Readings by 4 for Millionths.					
Load (in pounds).					
1,000	0	−1	−4	0	0
2,000	29	30	30	30	28
3,000	60	68	63	60	50
4,000	95	100	100	100	90
5,000	131	135	133	140	115
6,000	168	169	165	180	150
7,000	200	200	196	215	180
8,000	238	240	236	260	210
9,000	270	270	266	295	245
8,000	236	235	235	260	210
7,000	200	200	200	220	180
6,000	169	170	165	185	150
5,000	135	135	132	140	115
4,000	100	100	100	110	90
3,000	68	65	66	70	50
2,000	30	30	31	25	28
1,000	−1	−4	−5	−5	0

Lateral Extension—Millionths.

No.	a	b	c
Size	.975	.9825	.971
Load (in pounds).			
1,000	0	0	0
2,000	27	25	....
3,000	35	45	....
4,000	82	70	....
5,000	110	95	....
6,000	136	130	....
7,000	161	150	....
8,000	188	180	....
9,000	225	220	195
8,000	195	180	....
7,000	172	155	....
6,000	146	135	....
5,000	118	100	....
4,000	90	75	....
3,000	49	45	....
2,000	30	20	....
1,000	0	5	3

to assume a more lath-shaped development than in the case of the granites. The laths running as they do in all directions through the rock, probably have a tendency to bind the rock more firmly together than when the feldspar has a more equi-dimensional development, as in the granites. The rock has a hypidiomorphic structure, and, like the granites described in this paper, is perfectly massive.

A color-process photograph of a polished surface of this rock is shown in Plate XIII A, and a photomicrograph of a thin section taken between crossed nicols in polarized light and magnified 30 diameters is to be seen in Plate XIII B.

Three square prisms of the rock were used, and five determinations of vertical compression with three of lateral extension were made. The results are given in the table on page 53.

The averages of the results obtained are as follows:

$E = 9,746,00$; $\sigma = 0.2583$; $D = 6,750,000$; $C = 3,872,600$.

The results obtained for the three measurements on prism *a* were practically identical. The figures obtained for the compressibility of *c* are little higher and those for *b* are considerably lower. The difference between the highest and the lowest values obtained for *D* amounts to 1,110,000 pounds, but the difference, if the results of the single measurement on *b* be omitted from consideration, amounts to only 330,000 pounds.

The stress-strain curves plotted from the measurements obtained from the prism *a* are given in figure 21, and show that the elasticity of the rock is of a very high order. In this figure I represents vertical compression and II lateral extension.

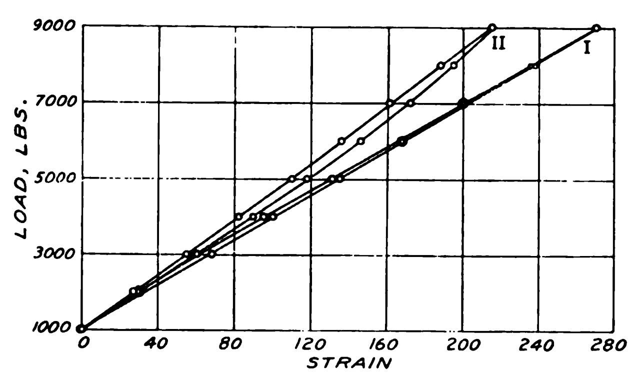
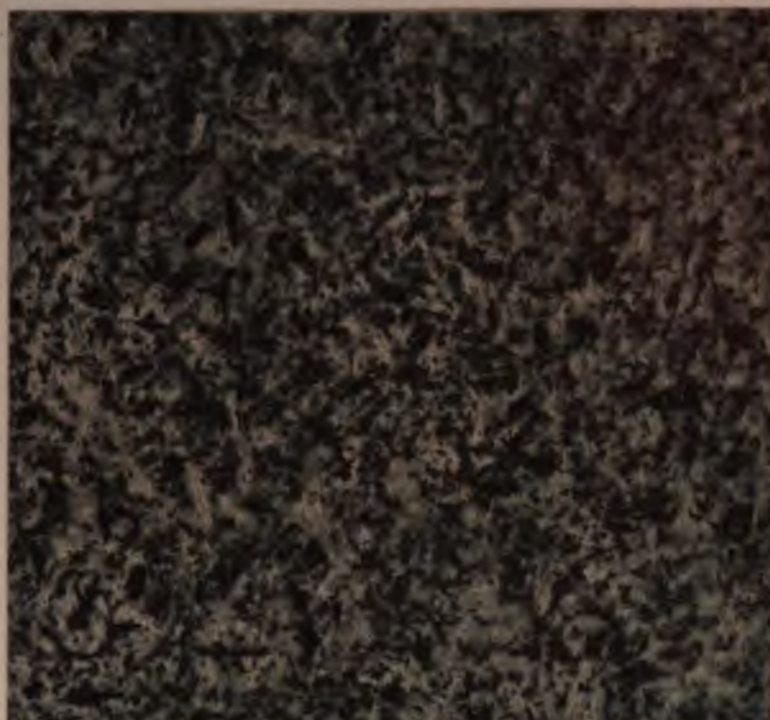
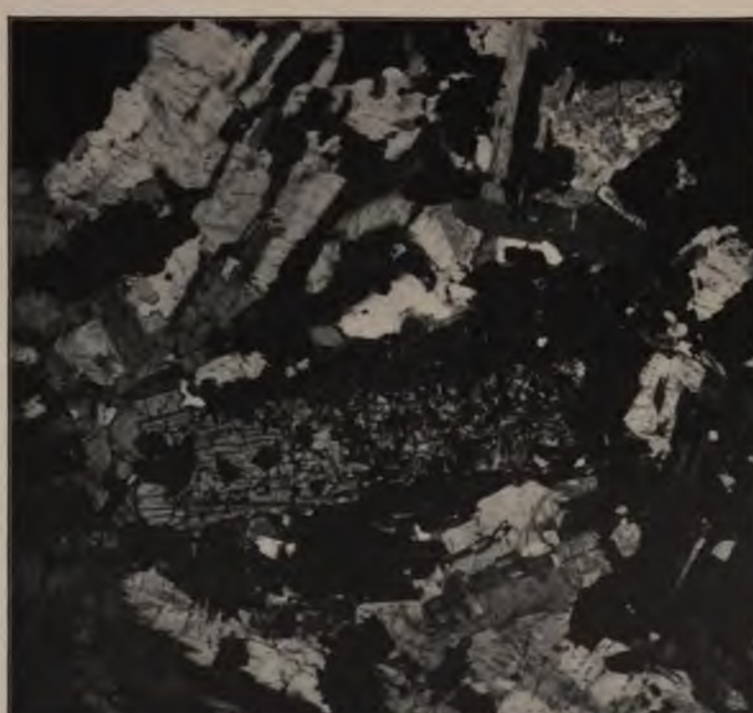


FIG. 21.—Mount Johnson Essexite. Stress-strain curves



A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)



B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

ESSEXITE, MOUNT JOHNSON, CANADA.

GREEN GABBRO, NEW GLASGOW, PROVINCE OF QUEBEC, CANADA.

This rock forms a large dyke* cutting the anorthosite from the locality described above. It is a rock which is darker in color than the anorthosite, owing to a much higher content of iron-magnesia constituents, but which, like that rock, is quarried and used for paving sets.

Under the microscope this rock is seen to differ entirely in structure from the other igneous rocks examined. It is composed of a very pale green augite, a rhombic pyroxene of the same color, and plagioclase, the two former minerals being present in about equal amount, and the plagioclase not forming more than about one-quarter of the rock; there is also present a small amount of a pale green spinel.

The rock is seen to have been crushed in a most extraordinary manner and to present a most striking cataclastic structure. The plagioclase occurs in groups of individuals which are well twinned, and are frequently very much bent and twisted—one individual being bent through an angle of 65°. The mineral is also filled with very minute rounded inclusions, which give to it a green color. These plagioclase grains, quite irregular in form, lie embedded in a mass of little irregular-shaped grains of augite and rhombic pyroxene. These vary somewhat in size. The two pyroxenes are sometimes intimately intermixed and at other times separated into groups of grains of their respective species, which are distinguished from one another by the different values of their double refraction and by the fact that one has parallel and the other inclined extinction. The spinel is associated with this minutely granulated pyroxene.

The original structure of the rock has been entirely broken down, and it now presents an assemblage of grains of the minerals varying in size and differing in arrangement from place to place in the slide. The pyroxenes are granulated, the plagioclase twisted, and the whole presents a perfect cataclastic appearance, differing entirely in this respect from that of the anorthosite just described. This cataclastic structure is combined in some specimens of the rock with a more or less distinct parallel arrangement of the constituent minerals, although this is not very distinct in the specimen shown in the color-process photograph of a polished surface (Plate XIV A).

To this irregularity in structure may be attributed the irregularities in the elastic deportment of the rock.

A photomicrograph of a thin section of the rock taken between crossed nicols in polarized light and magnified 30 diameters is given in Plate XIV B.

It is found that satisfactory measurements of the elastic constants could not be made in the case of this rock, the same specimen giving a great variation

*Adams F D. Op. cit., p. 121.

in values for Poisson's ratio, when measured in different directions. A similar variation is also obtained with different specimens of the rock. The rock in fact, is not uniform and isotropic, so that, as has been mentioned, it is not one which is suitable for the application of the method employed in this paper, if accurate results are required.

The figures obtained from the measurement of two specimens are given in the following table:

Green Gabbro, New Glasgow, Province of Quebec, Canada.

No.	b	c	b	c
Size	.956 · .968	1 × 1.022	.968	1
Area	.925	1.022	....	....
E	12,300,000	19,000,000	....	....
σ	.1985	.24	....	....
D	6,810,000	12,300,000	....	....
C	5,130,000	7,600,000	....	....
	Longitudinal compression (multiply readings by 4 for millionths).		Lateral extension (millionths).	
Load (in pounds).				
1,000	0	0	0	0
2,000	25	14	17	11
3,000	50	29	33	21
4,000	75	47	50	32
5,000	105	59	67	46
6,000	135	79	83	61
7,000	160	94	....	....
8,000	190	100	....	....
9,000	220	120	....	....
8,000	190	105	....	....
7,000	165	95	....	....
6,000	140	83	83	61
5,000	110	60	65	45
4,000	85	50	55	35
3,000	60	30	35	25
2,000	35	15	20	15
1,000	9	0	5	2

As will be seen, D in one case is 6,810,000 and in the other 12,300,000. In figure 22 the stress-strain curves obtained by plotting the results of the measurement of prism b are given, and show a considerable permanent set, but comparatively little hysteresis. In this figure I represents longitudinal compression and II lateral extension. Prism c gives an equally good curve. It is quite probable that both are correct for their respective specimens.

A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 30 DIAM.-NICOLS CROSSED)

GREEN GABBRO, NEW GLASGOW, CANADA.

In the table giving a summary of results (see page 69), the values given for this rock represent the mean of these highly divergent readings and should be used only in the light of the explanation given above.

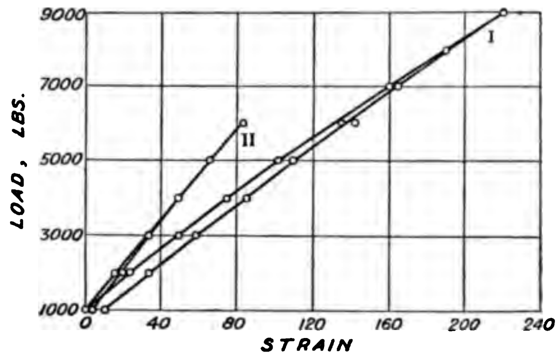


FIG. 22.—Green Gabbro, New Glasgow. Stress-strain curves.

OLIVINE DIABASE, NEAR SUDBURY, PROVINCE OF ONTARIO, CANADA.

This is a very typical fresh olivine diabase, which occurs in the form of a large dyke, cutting rocks of Huronian age just northwest of the Murray Mine near Sudbury. It is one of a number of similar diabase dykes, which occur in this district of great nickel-bearing gabbro intrusions. It is rather coarse in grain for a diabase, but nevertheless much finer in grain than any of the granites described in this paper, except that from Westerly, Rhode Island, these two rocks being approximately equal in coarseness of grain, although differing entirely in structure. The rock is composed of violet-brown augite, pale green olivine, colorless plagioclase, and opaque black iron ore. There is also a very small amount of accessory biotite, a few minute acicular crystals of apatite, and an occasional minute grain of pyrite. The augite presents the usual microscopical characters of this species, and is very fresh, scarcely a trace of decomposition being anywhere discernible in it. The olivine, which crystallized before the augite, and therefore often occurs as inclusions in it, while for the most part fresh, is in many places partially altered to a deep green serpentine. It is much less abundant than the augite. The plagioclase occurs in the usual sharp, well-defined, lath-like form, always showing polysynthetic twinning according to the albite law, which in the same individual is often combined with twinning according to the pericline or Carlsbad law. It is fresh and brilliantly polarizing. The iron ore, which is black and opaque, is abundant, occurring in well-defined more or less angular grains.

The rock is perfectly massive and possesses a typical "ophitic" or "diabase" structure, the plagioclase having the form of well-defined laths

penetrating the augite and even the iron ore, but not the olivine so far as can be observed. Many little seams apparently of the nature of joints traverse the rock, and care had to be exercised to secure prisms of the rock free from these, on which to determine the elastic constants.

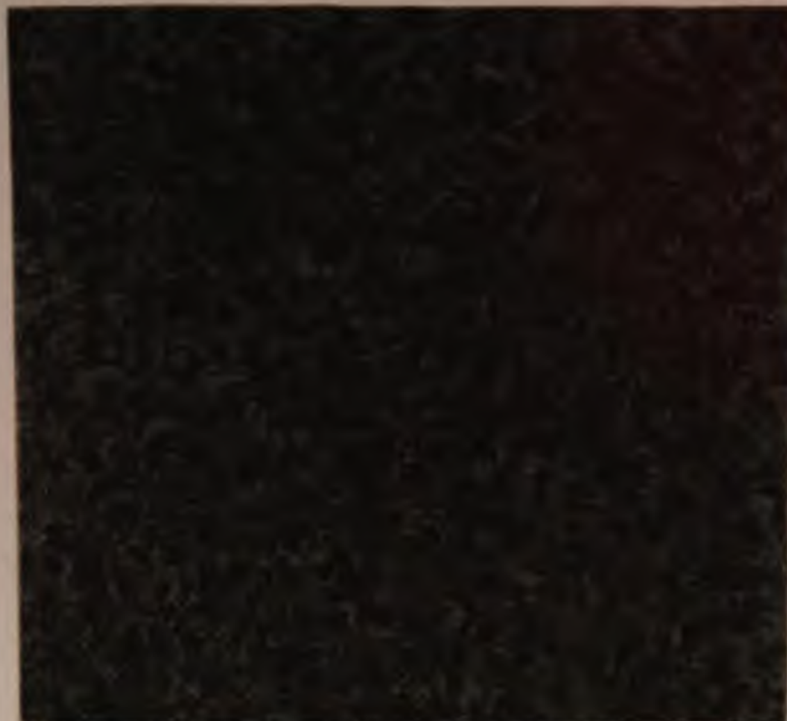
A color-process photograph of a polished surface of the rock is shown in Plate XV A, and a photomicrograph of a thin section of the rock taken in ordinary light and magnified 27 diameters is seen in Plate XV B.

Four test pieces were used in determining the elastic constants of the rock, viz, three round columns and one nearly square prism. They are designated as *a*, *b*, *c*, and *d*. The three round columns were cut out of a block of the diabase by means of an annular diamond drill. For these we are indebted to Dr. Logan Waller Page, of the Agricultural Department at Washington. Two measurements were made on each of these in planes at right angles to one another, in each case, while four measurements were made on the prism *d*, using two pairs of faces. In this way ten sets of measurements were made for the elastic constants of this diabase.

The values obtained are given in the following tables:

Olivine Diabase, Sudbury, Ontario, Canada.

No.	*d*	*d*	*d*	*d*	*d*	*d*	*d*
Size	1.000 × .864	....	....	....	1.00	1.00	1.00
Area	.864	.864	.864	.864	....	....	....
E	13,150,000	13,330,000	13,450,000	12,860,000	....	....	....
σ	276	.285	.287	.279	....	....	....
D	9,810,000	10,340,000	10,500,000	9,655,000	....	....	....
C	5,170,000	5,200,000	5,230,000	5,020,000	....	....	....
Longitudinal compression (multiply readings by 4 for millionths).					Lateral extension (millionths).		
Load (in pounds).	Side $U.$	Side U_1	Side U_2	Side U_3	Side U_2	Side U_1	Side $U.$
1,000	0	0	0	0	0	0	0
2,000	25	25	25	26	25	25	25
3,000	50	51	52	58	50	50	49
4,000	75	76	80	85	75	75	74
5,000	102	103	105	110	100	100	98
6,000	127	130	135	140	124	125	124
7,000	155	155	152	169	148	150	146
8,000	185	187	182	195	173	170	168
9,000	220	217	215	225	200	198	194
8,000	190	190	185	200	175	....	168
7,000	157	157	155	175	152	....	147
6,000	130	134	134	145	128	....	125
5,000	105	105	105	110	103	....	99
4,000	76	80	80	80	78	....	75
3,000	50	54	50	51	52	....	50
2,000	25	27	25	25	28	....	25
1,000	0	3	0	0	0	0	0



A. PHOTOGRAPH OF POLISHED SURFACE, (NATURAL SIZE)

B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

OLIVINE DIABASE, SUDBURY, CANADA.

Olivine Diabase, Sudbury, Province of Ontario, Canada—Continued.

No.	*a*	*a*	*b*	*b*	*c*	*c*
Size.....	.981	.981	.983	.983	.983	.983
Area ...	.756	.756	.758	.758	.758	.758
E.......	13,250,000	13,780,000	14,020,000	14,320,000	14,020,000	14,320,000
σ.......	.2865	.281	.291	.277	.291	.283
D	10,340,000	10,460,000	11,170,000	10,720,000	11,170,000	11,000,000
C	5,160,000	5,380,000	5,430,000	5,620,000	5,430,000	5,580,000

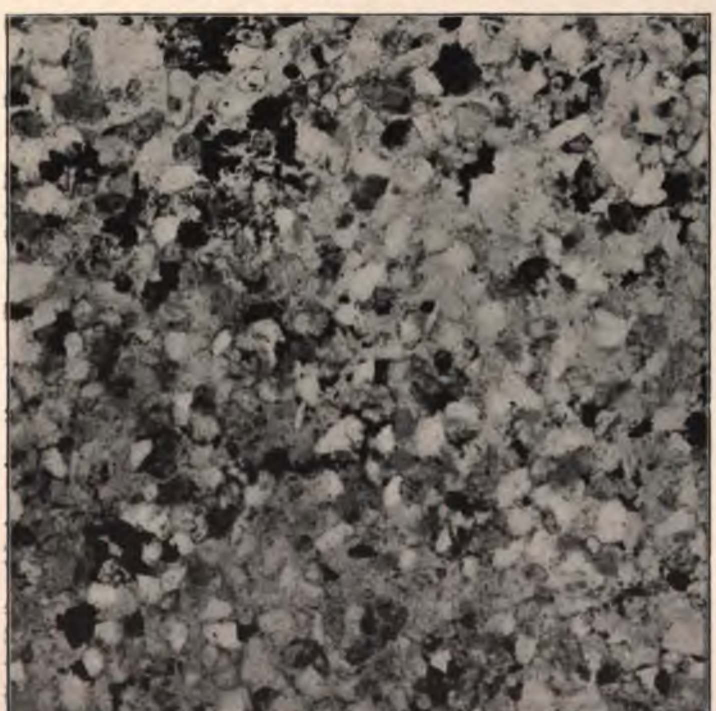
LONGITUDINAL COMPRESSION —MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000..	0	0	0	0	0	0
2,000..	30	30	30	25	30	30
3,000..	60	60	60	55	60	60
4,000..	90	90	90	95	90	85
5,000..	125	120	115	110	120	115
6,000..	155	150	145	140	150	145
7,000..	185	180	175	170	180	175
8,000..	225	215	210	200	210	205
9,000..	250	240	235	230	235	230
8,000..	220	210	210	200	210	205
7,000..	190	185	170	175	180	175
6,000..	165	155	145	140	150	145
5,000..	130	125	115	115	120	115
4,000..	105	100	85	95	90	85
3,000..	75	60	55	55	60	60
2,000..	45	25	30	25	30	30
1,000..	15	0	0	0	0	0

LATERAL EXTENSION —MILLIONTHS.

No......	*a*	*a*	*b*	*b*	*c*	*c*
Size.....	.981	.981	.983	.983	.983	.983
Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000..	0	0	0	0	0	0
2,000..	28	28	28	25	21	27
3,000..	54	51	54	49	49	53
4,000..	83	78	82	73	78	79
5,000..	111	103	110	100	105	107
6,000..	140	130	138	122	131	130
7,000..	169	156	164	149	160	154
8,000..	198	183	191	172	185	183
9,000..	225	210	215	200	215	205
8,000..	200	185	192	174	195	180
7,000..	172	155	170	150	170	166
6,000..	144	135	140	125	145	130
5,000..	115	110	118	100	115	110
4,000..	85	78	88	75	85	80
3,000..	52	54	56	50	55	55
2,000..	22	26	30	25	25	26
1,000..	0	0	0	0	0	0

A. PHOTOGRAPH OF FLAT SURFACE, (NATURAL SIZE)



B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

SANDSTONE, CLEVELAND, OHIO.

As will be seen, the values obtained for D in this rock are considerably higher than those yielded by any other rock of the series examined. In the six independent measurements carried out on the first three specimens, the difference between the highest and lowest values for D amounted to 830,000 pounds, while on the four measurements made on specimen d there is a rather greater difference amounting to 845,000 pounds.

The averages of the determinations made on each of these columns are as follows:

	E	D	σ	C
a	13,515,000	10,400,000	0.2838	5,270,000
b	14,170,000	10,945,000	0.2840	5,525,000
c	14,170,000	11,085,000	0.2870	5,505,000
d	13,197,750	10,076,000	0.2812	5,155,000
Average......	13,763,187	10,626,500	0.2840	5,363,750

The stress-strain curves given by a specimen this rock are shown in figure 23. As will be seen from these curves, in its approach to perfect elasticity the rock is comparable to plate glass.

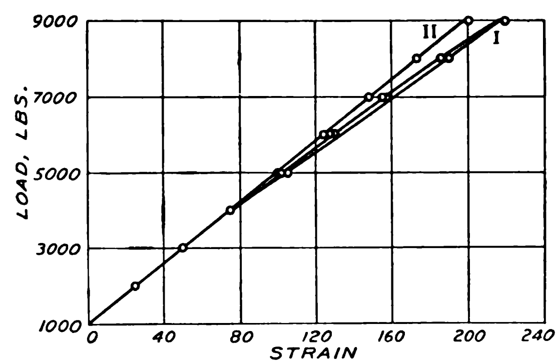


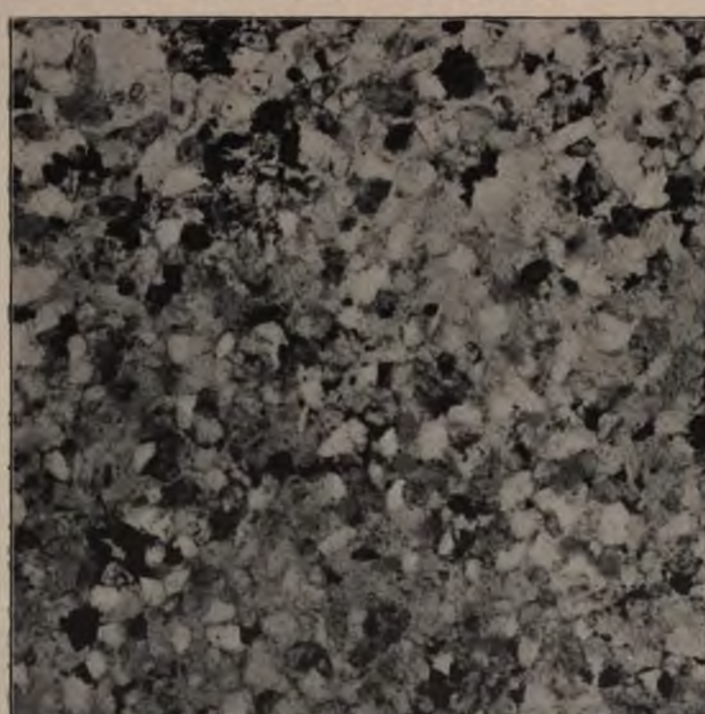
FIG. 23.—Sudbury Diabase. Stress-strain curves.

SANDSTONE, CLEVELAND, OHIO, UNITED STATES.

This is a fine and even grained yellowish sandstone used very extensively for building purposes. The bedding is marked by a slight variation in color in different beds. The prism of the rock used in determining its elastic constants was cut from a single bed of uniform character and color, and was taken in the plane of the bedding. A color-process photograph of a smooth surface of the rock is shown in Plate XVI A.

Under the microscope it is seen to be a typical highly feldspathic sandstone. The constituent minerals are present in grains which are approximately

A. PHOTOGRAPH OF FLAT SURFACE, (NATURAL SIZE)



B. PHOTOMICROGRAPH OF THIN SECTION, (X 27 DIAM.-ORDINARY LIGHT)

SANDSTONE, CLEVELAND, OHIO.

uniform in size and of rudely rounded or subangular outline. The quartz grains are clear and fresh; the feldspar individuals, which are abundant, on the other hand, are for the most part in an advanced stage of alteration, being always turbid and in most cases quite opaque, from the presence of alteration products. Some few grains of comparatively unaltered plagioclase are, however, present, and scattered through the rock there is a considerable amount of hydrated oxide of iron, which often lies between the grains, forming a cement. The rock, however, also contains a not inconsiderable amount of calcite, which causes it to effervesce slightly when treated with dilute hydrochloric acid, and which is also seen to lie between the clastic grains also forming a cement, often in the form of individuals of a size comparable to those of the other minerals.

The rock, however, is not a crystalline rock, but a typical clastic one. There is not a continuous crystalline web or mosaic, but a mass of rounded or subangular grains which are in part cemented together as above described, but in part are separated by minute open spaces. It is to be expected, therefore, that the rock will show serious defects in elasticity, as proves to be the case when attempt is made to determine its elastic constants. A photomicrograph of the rock taken in ordinary light and multiplied 27 diameters is shown in Plate XVI B.

A square prism of the rock was employed, and it was found to be dangerous to submit it to a load of over 4,000 pounds, the crushing weight of the rock being much lower than that of the other rocks, which are crystalline in texture.

The figures obtained are given in the following table:

Sandstone, Cleveland, Ohio, United States.

Size	1.000 × 1.025	1.000
Area	1.025	....
E	2,290,000	....
σ	.29	....
D	1,816,000	....
C	888,000	....
Load (in pounds).	Longitudinal compression (multiply readings by 4 for millionths).	Lateral extension (millionths).
1,000	0	0
2,000	152	110
3,000	288	241
4,000	426	396
3,000	309	305
2,000	175	178
1,000	4	0

The stress-strain curves are shown in figure 24.

As will be seen, the rock displays a marked hysteresis and is not therefore an ideal material for the application of this method of determining compressibility.

The results obtained are as follows:

$$E = 2{,}290{,}000; \quad \sigma = 0.29; \quad D = 1{,}816{,}000; \quad C = 888{,}000.$$

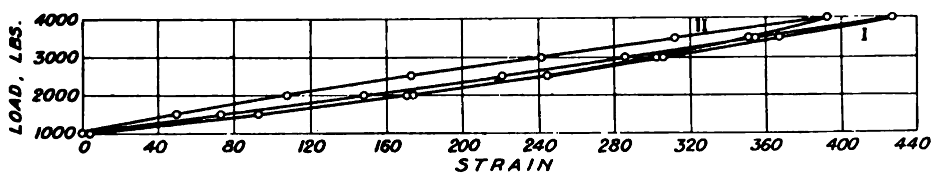


Fig. 24.—Sandstone. Stress-strain curves.

THE ELASTIC CONSTANTS OF GLASS.

As in geophysical speculations, the earth in respect to its rigidity and compressibility is often compared to a globe of glass, it seemed advisable to determine as accurately as possible the elastic constants of glass, for the purpose of comparing them with the results obtained in the case of the various rocks considered in this paper, employing the same methods and carrying out the work under exactly the same conditions. This material lends itself excellently to this method of measuring these constants, provided the glass is free from all irregularities in its substance and is isotropic in character. The first difficulty experienced was that of obtaining such a glass. At the outset it was thought that thick glass rods such as are used for various purposes in the chemical and physical laboratory might be employed, but although several lots of the purest variety of this material were procured, the glass constituting it was found in all cases to contain minute air bubbles, and when examined between crossed nicols in polarized light, showed brilliant colors—red, yellow, and blue. This indicated a state of marked tension in the glass, evidently due to the rod having been drawn when the glass was in a viscous state, which was also shown by the circular arrangement of the little bubbles in the rod, following the direction of its surface. Short lengths of this rod, moreover, when tested in compression, so soon as the maximum load had been exceeded, instead of splitting from top to bottom, broke as if composed of a series of rudely concentric shells. All attempts on the part of the various glass makers to whom this glass was submitted for a thorough annealing, failed to remove or in fact to reduce to any considerable extent this anisotropic condition.

The figures obtained from one of these glass rods approximately an inch in diameter are given in the following table:

Glass Rod.

Size	.985	.97
Area	.774	....
E	8,075,000	....
σ	.2	....
D	4,485,000	....
C	3,361,000	....
Load (in pounds).	Longitudinal compression (multiply readings by 4 for millionths).	Lateral extension (millionths).
1,000	0	0
2,000	55	32
3,000	95	61
4,000	145	95
5,000	200	123
6,000	250	155
7,000	300	....
8,000	350	....
9,000	....	....
8,000	350	....
7,000	305	....
6,000	250	155
5,000	205	125
4,000	160	100
3,000	115	65
2,000	65	35
1,000	5	2

That the tension in this glass seriously affected the results obtained—as might be expected—is clearly seen in the value for D being much too low, as will be shown later.

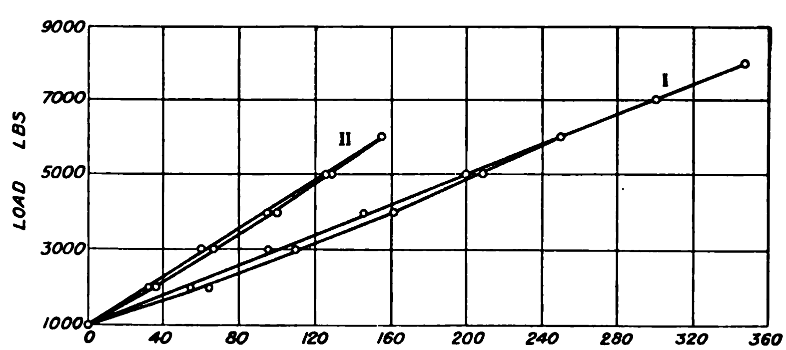


FIG. 25. Glass Rod. Stress-strain curves.

The stress-strain curves plotted from these values are shown in figure 25. As will be seen, the material exhibits a distinct hysteresis.

After a prolonged search for isotropic glass in masses of sufficient size to measure the elastic constants, it was found that plate glass answered the requirements. A piece of one-inch plate glass made in Great Britain was accordingly secured and was cut into strips an inch wide, and these again into three-inch lengths. The square prisms thus produced were then properly faced and polished. The glass was found to be absolutely free from all flaws and impurities and when examined between crossed nicols the prisms, although an inch thick, showed in one direction at right angles to vertical axis absolute blackness throughout a complete revolution, while in the other direction at right-angles to this there was during a revolution an alternation of blackness with a pale grayish illumination. This change was so slight that, considering the thickness of the glass and the sensitiveness of the test, the material may

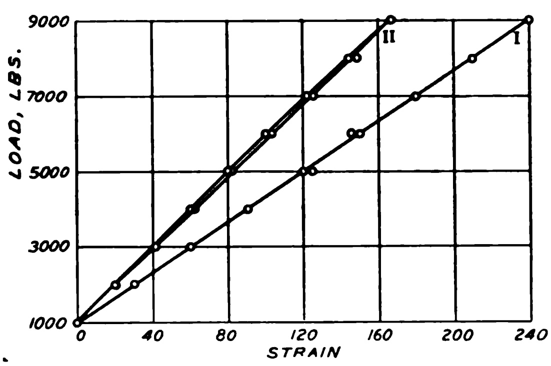


FIG. 26.—Plate Glass. Stress-strain curves.

be considered to be practically free from internal tension and to be isotropic in character.

In order to get a good average and to eliminate chance errors as far as possible, seven of these prisms were taken, and two complete sets of determinations were made on each of them, using in every case different pairs of faces. Fourteen determinations were thus made of each of the elastic constants. The figures obtained are set forth in the table on page 65.

In this table a complete series of values obtained from each specimen are given in double rows. When the average of all these results is taken, the values obtained for the several constants of plate glass are as follows:

$E = 10{,}500{,}000$; $\sigma = 0.2273$; $D = 6{,}448{,}000$; $C = 4{,}290{,}000$.

The stress-strain curves given by one of the prisms is shown in figure 26. In this figure I represents longitudinal compression and II lateral extension.

Plate Glass.

No. . .	*a*	*b*	*c*	*d*	*e*	*f*	*g*
Size. . .	.9855×1.0205	.9865×1.0055	.981×1.0135	1.016×1.008	1.0215×.9955	1.022×1.0025	1.025×.994
Area. .	1.0057	.992	.994	1.024	1.017	1.024	1.016
E.	10,350,000 10,590,000	10,950,000 10,500,000	10,480,000 10,350,000	10,380,000 10,380,000	10,450,000 10,930,000	10,380,000 10,600,000	10,450,000 10,230,000
σ. . . .	.2281 .228	.236 .2341	.226 .235	.233 .227	.221 .23	.229 .225	.216 .215
D.	6,370,000 6,480,000	6,930,000 6,580,000	6,460,000 6,520,000	6,480,000 6,350,000	6,380,000 6,760,000	6,370,000 6,430,000	6,140,000 6,020,000
C.	4,220,000 4,310,000	4,440,000 4,250 000	4,280,000 4,190,000	4,210,000 4,230,000	4,280,000 4,440,000	4,220,000 4,330,000	4,300,000 4,360,000

LONGITUDINAL COMPRESSION—MULTIPLY READINGS BY 4 FOR MILLIONTHS.

Load (in pounds).	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*	Side *U.*	Side *P.*
1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2,000	30	30	25	30	30	30	30	30	30	30	28	25	30	30
3,000	60	55	55	60	60	60	60	60	60	55	55	55	60	60
4,000	90	85	85	90	90	90	90	90	90	85	85	85	85	90
5,000	120	115	115	120	120	120	120	115	115	110	115	110	115	125
6,000	145	145	145	145	150	150	150	145	145	135	145	140	145	145
7,000	175	175	175	175	180	180	180	175	175	170	175	170	180	175
8,000	210	205	205	210	210	210	205	205	205	195	200	195	205	210
9,000	240	235	230	240	240	243	235	235	235	225	235	230	235	240
8,000	210	205	205	210	210	210	205	205	205	195	205	195	205	210
7,000	180	180	175	180	180	180	180	175	175	170	175	170	175	180
6,000	150	145	145	150	150	155	150	145	145	135	145	140	145	150
5,000	120	115	115	120	120	120	120	115	115	110	115	115	115	120
4,000	90	85	85	90	90	90	90	90	85	85	85	85	85	90
3,000	60	55	55	60	60	60	60	55	55	60	60	60	60	60
2,000	30	30	25	30	30	30	30	30	30	30	30	30	30	30
1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0

LATERAL EXTENSION—MILLIONTHS.

1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2,000	22	22	21	22	19	22	20	19	19	19	20	19	21	20
3,000	44	45	42	45	38	46	42	39	39	40	41	39	42	42
4 000	68	68	66	68	60	69	63	60	60	59	65	60	60	62
5,000	89	89	88	89	81	92	86	81	82	80	89	79	80	83
6,000	111	111	109	111	103	115	112	105	102	100	109	100	100	103
7,000	132	133	129	133	124	138	134	126	124	122	130	122	122	122
8,000	153	156	151	156	149	160	157	148	145	143	155	145	144	144
9,000	173	175	172	179	170	180	178	169	169	165	176	165	167	165
8,000	155	155	152	157	149	163	159	149	149	145	155	147	149	147
7,000	135	133	130	135	125	140	135	129	126	125	130	125	125	125
6,000	115	112	110	115	103	115	100	106	105	105	112	105	103	106
5,000	93	90	90	90	80	90	89	85	88	88	90	85	82	89
4,000	70	71	70	70	60	65	64	62	62	62	69	65	62	69
3,000	49	45	45	50	40	42	40	42	41	41	45	43	42	49
2,000	22	23	22	25	21	20	20	21	21	21	22	22	22	28
1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Determinations of the cubic compressibility of glass, D, have been made by other observers using various methods. The results go to show that different varieties of glass vary considerably in their compressibility. These determinations may be tabulated as follows:*

Everett..........	5,074,600 to 6,379,400 (C. G. S. = 3.5 to 4.4 $\times$ 10^{11}).
Amagat—Common glass.......	6,745,000 (.000002181 per atmosphere).
Amagat—Crystal glass........	6,112,300 (.000002405 per atmosphere).
Tait.........................	5,657,700 (.0000026 per atmosphere).

As will be seen, the figures obtained for plate glass in the present investigation lie a little above the average of the various values here given, and are nearly those of the highest value obtained by Everett.

SUMMARY OF RESULTS.

The table on page 69 gives a summary of the average values obtained for E, σ, C and D in the case of all rocks examined in this investigation. With these are placed, for purposes of comparison, the results obtained for these constants in the case of wrought iron, cast iron and glass. In the second table on page 69 these values are again presented, recalculated into C. G. S. units.

The rocks fall naturally into three groups, differing from one another in compressibility, but the several members of each group agreeing fairly closely among themselves.

These three groups show a corresponding difference in composition.

The first group consists of the marbles and limestones. These have an average value for D of 6,345,000. One of these, however, the Black Belgian marble which is very much finer in grain than the others and breaks almost like a piece of glass, has a very much higher value for D than that possessed by the other rocks which among themselves are nearly identical. If we omit this Belgian marble, the average of D for the other limestones and marbles, is 5,855,000.

The second group comprises the granites. These again show a close agreement of values among themselves, except in the case of the Stanstead granite, which rock, as already mentioned, shows a defective elasticity. The average value of D for the granites is 4,399,000.

The third group embraces the basic intrusives (gabbro, anorthosite, essexite, and diabase). These show greater differences, but have an average value for D of 8,825,000. The nepheline syenite, although higher in silica and therefore properly speaking an acid rock; in its freedom from quartz, and its richness in feldspar (although the feldspar is largely orthoclase instead of plagioclase), in mineralogical composition belongs with these basic rocks rather than with the granites. It also approaches the essexite most nearly in its compressibility.

*See Everett, Illustrations of the C. G. S. System of Units with tables of Physical Constants. MacMillan & Co., 1902, pp. 60 to 64. The figures there expressed in various units have been here recalculated into inch-pound values.

If the nepheline syenite be included with the basic rocks, an average value of D is obtained of 8,308,000.

This omits from consideration the sandstone, it being a rock of an entirely different class from the others, and furthermore one which shows so much hysteresis that the application of this method to it is less satisfactory than in the case of the other rocks of the series.

These results may be presented as follows:

	Average of D.
Marbles and limestone	6,345,000
Granites	4,399,000
Basic intrusives	8,308,000

The cause of the much greater compressibility of granite as compared with the marbles and basic intrusives is not clear, but would seem to be connected with the presence of quartz. The only determination of the cubic compressibility of quartz, so far as can be ascertained, is one by Voigt,* the value obtained being 5,504,190 pounds (387×10^6 grams per sq. cm.). This compressibility, as will be seen, is much greater than that found in the case of either the limestones or the basic intrusives, and while not in itself sufficiently great to account for the high compressibility of the granites, goes to show that in the quartz we have a mineral which is more compressible than the ordinary rock making minerals which form the chief constituents in the rocks of the series examined.

The marbles and the limestones of the earth's crust are confined to its most superficial portion, resulting as they do from the process of sedimentation. There is every reason to believe, however, that what we may term the substructure of the earth's crust is composed of acid and basic plutonic igneous rocks. These make up the lowest part of the crust to which we have access and are found coming up from the still greater depths.

The cubic compressibility D of the earth's crust must lie between the values given above for the granites and the basic intrusives, approaching one or other of these values according to the relative proportion in it of one or other of these classes of rocks.

If we take the average of the values obtained from these two classes of rocks as represented by the seven granites and the five basic intrusives (including the nepheline syenite) the values obtained for D of 6,353,500.

This, as will be seen, differs but little from the value of D obtained for plate glass which is 6,448,000.

If, therefore, the earth's crust be composed of granite and basic igneous rocks in approximately equal proportions, its compressibility will be that of glass. If it be composed almost exclusively of granite, the earth's crust will be more

*Quoted in Becker: Experiments on Schistosity and Slaty Cleavage, Bulletin 241, U. S. Geol. Survey, p. 32.

compressible than glass, and if the basic rocks preponderate very largely it will be less compressible than this substance.

It is, however, in any case much more compressible than steel, which has a value for D of from 26,098,000 to 27,547,000 (18 to 19×10^{11}, C. G. S.).*

The compression to which the rocks were subjected in this investigation ranged from 6,000 to 17,340 pounds to the square inch. Most of the rocks, however, were subjected to a load of from 9,000 to 15,000 pounds per square inch, and their bulk compression was determined for these loads as maxima. Higher pressures could not be employed without running the risk of breaking the specimen and at the same time of destroying the measuring apparatus. One apparatus was in fact so destroyed.

The question arises as to whether under still higher pressures, if rupture could be avoided, the ratio of load to compression would be maintained. Judging from the deportment of much stronger substances such as steel, when similarly tested, it is inferred that this ratio of bulk compression will remain constant for very much higher pressures, or until deformation sets in and the rock begins to flow.

With regard to the accuracy of the results obtained by this method as compared with those obtainable by any method in which cubic compression is actually produced and measured, it may be observed that by far the best method of this kind hitherto suggested seems to be that proposed by Richards and Stull.† We have endeavored to make use of this method in order to obtain results for purposes of comparison with those given in the present paper but have not hitherto succeeded in overcoming certain experimental difficulties. The experimental errors in this method, though apparently small, still exist, and in applying it to rocks, which are much less compressible than the substances examined by Richards and Stull, these errors become proportionately more serious. Moreover, higher pressures than those used in the method employed in the present paper could scarcely be employed in this direct method, while difficulties dependent on the possible lack of absolute continuity in the substance of the rock and the danger of minute air-filled spaces would probably present themselves in the case of most rocks. It seems that, all things being considered, the indirect method here employed is probably as accurate as any direct method which can be used. The attempt to apply Richards and Stull's method to rocks is still being continued, however, and it is hoped that satisfactory results may be eventually obtained by its use.

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*Illustrations of the C. G. S. System of Units, with Tables of Physical Constants. MacMillan & Co., 1902, p. 60.

†New Method of Determining Compressibility. Published by the Carnegie Institution of Washington, December, 1903.

Elastic Constants of Rocks.

SUMMARY OF RESULTS (AVERAGE) EXPRESSED IN INCH-POUND UNITS.

Specimen.	E	σ	C	$D=\frac{1}{3}\left(\frac{m}{m-2}\right)E$
Wrought iron	28,100,000	0.2800	11,000,000	21,300,000
Cast iron	15,000,000	0.2500	6,000,000	10,000,000
Black Belgian marble	11,070,000	0.2780	4,330,000	8,303,000
Carrara marble	8,046,000	0.2744	3,154,000	5,946,000
Vermont marble	7,592,000	0.2630	3,000,000	5,341,000
Tennessee marble	9,006,000	0.2513	3,607,000	5,967,000
Montreal limestone	9,205,000	0.2522	3,636,000	6,167,500
Baveno granite	6,833,000	0.2528	2,724,800	4,604,000
Peterhead granite	8,295,000	0.2112	3,399,000	4,792,000
Lily Lake granite	8,165,000	0.1982	3,380,000	4,517,500
Westerly granite	7,394,500	0.2195	3,019,700	4,397,500
Quincy granite (1)	6,747,000	0.2152	2,781,600	3,984,000
Quincy granite (2)	8,247,500	0.1977	3,445,000	4,555,000
Stanstead granite	5,685,000	0.2585	2,258,700	3,940,000
Nepheline syenite	9,137,500	0.2560	3,635,000	6,237,500
New Glasgow anorthosite	11,960,000	0.2620	4,750,000	8,368,000
Mount Johnson essexite	9,746,000	0.2583	3,872,600	6,750,000
New Glasgow gabbro*	15,650,000	0.2192	6,365,000	9,555,000
Sudbury diabase	13,763,000	0.2840	5,364,000	10,626,500
Ohio sandstone	2,290,000	0.2900	888,000	1,816,000
Plate glass	10,500,000	0.2273	4,290,000	6,448,000

SUMMARY OF RESULTS (AVERAGE) EXPRESSED IN C. G. S. UNITS.

Specimen.	E	σ	C	D
Wrought iron	19.37×10^{11}	0.2800	7.590×10^{11}	14.680×10^{11}
Cast iron	10.34×10^{11}	0.2500	4.132×10^{11}	6.897×10^{11}
Black Belgian marble	7.24×10^{11}	0.2780	2.982×10^{11}	5.736×10^{11}
Carrara marble	5.54×10^{11}	0.2744	2.171×10^{11}	4.090×10^{11}
Vermont marble	5.24×10^{11}	0.2630	2.069×10^{11}	3.680×10^{11}
Tennessee marble	6.21×10^{11}	0.2513	2.482×10^{11}	4.115×10^{11}
Montreal limestone	6.35×10^{11}	0.2522	2.504×10^{11}	4.250×10^{11}
Baveno granite	4.71×10^{11}	0.2528	1.875×10^{11}	3.179×10^{11}
Peterhead granite	5.71×10^{11}	0.2112	2.340×10^{11}	3.300×10^{11}
Lily Lake granite	5.63×10^{11}	0.1982	2.330×10^{11}	3.103×10^{11}
Westerly granite	5.09×10^{11}	0.2195	2.080×10^{11}	3.029×10^{11}
Quincy granite (1)	4.64×10^{11}	0.2152	1.916×10^{11}	2.750×10^{11}
Quincy granite (2)	5.68×10^{11}	0.1977	2.373×10^{11}	3.140×10^{11}
Stanstead granite	3.92×10^{11}	0.2585	1.556×10^{11}	2.718×10^{11}
Nepheline syenite	6.29×10^{11}	0.2560	2.505×10^{11}	4.290×10^{11}
New Glasgow anorthosite	8.25×10^{11}	0.2620	3.275×10^{11}	5.760×10^{11}
Mount Johnson essexite	6.71×10^{11}	0.2583	2.670×10^{11}	4.650×10^{11}
New Glasgow gabbro*	10.80×10^{11}	0.2192	4.380×10^{11}	6.589×10^{11}
Sudbury diabase	9.49×10^{11}	0.2840	3.700×10^{11}	7.329×10^{11}
Ohio sandstone	1.58×10^{11}	0.2900	$.612 \times 10^{11}$	1.250×10^{11}
Plate glass	7.24×10^{11}	0.2273	2.960×10^{11}	4.439×10^{11}

*See page 57

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RHYTHMICAL PULSATION IN SCYPHOMEDUSÆ

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RHYTHMICAL PULSATION IN ANIMALS.

1. PULSATION OF JELLYFISHES, ARMS OF LEPAS, HEART OF SALPA AND OF LOGGERHEAD TURTLE.

I. CONCLUSIONS NEW TO SCIENCE.

1. If we cut off the marginal sense-organs of the scyphomedusa *Cassiopea*, the disk * becomes paralyzed and does not pulsate in sea-water. The disk will pulsate in sea-water, however, if we make either a single ring or a series of concentric broken-ring-like cuts through the muscular tissue of the sub-umbrella. Then upon momentarily stimulating the disk in any manner, it suddenly springs into rapid, rhythmical pulsation so regular and sustained as to recall the movement of clockwork.

Pulsation will not start unless the disk be momentarily stimulated, as by a mechanical or electrical shock or by a single touch with a crystal of K_2SO_4, but once started it continues indefinitely in normal sea-water without further external stimulation.

The waves of pulsation all arise from the stimulated point, and the labyrinth of sub-umbrella tissue around this center must form a closed circuit. It is not necessary that the cuts through the sub-umbrella tissue of the disk be concentric circles, for any shape will pulsate which allows contraction waves to travel through tissue forming a closed circuit from the stimulated center and back to this center. When each wave returns to the center it is reinforced and again sent out through the circuit; and thus the center sustains the pulsation.

NOTE.—It is a pleasure to express my gratitude to those who have aided me in the prosecution of this research. To Prof. H. S. Jennings for his kindness in sending to me lists of the coefficient *i* for the making of isotonic solutions; to Dr. Leon J. Cole and Dr. Charles Zeleny for important suggestions and criticisms; to Mr. Davenport Hooker for collecting *Gonionemus* and *Dactylometra*, and to Prof. H. F. Perkins for aid in collecting *Cassiopea* at Tortugas; to Professors Ulrich Dahlgren and Edward L. Mark for instruction and aid.

* In this paper the term "disk" will be used to designate Medusæ from which the marginal sense-organs have been excised; while the term "Medusa" will designate the normal perfect animal.

The pulsating labyrinth may be simplified after the rhythmic movement has started, by cutting parts of it away, or cuts may be made in such manner as to increase its complexity. Any cut which breaks the circuit, however, stops the wave of pulsation, and continuous movement can not again be started.

The rate of pulsation of the disk is fully twice as fast as that of the normal perfect Medusa. This rate remains constant in the pulsating disk, and when pulsation ceases the movement stops *instantly*, never gradually. The rate of pulsation in disks deprived of marginal sense-organs depends not upon the area of the tissue forming the circuit, but only upon the length of the circuit. Short circuits pulsate more rapidly than do long ones. In this respect it differs from the control normally exercised by the marginal sense-organs; for small pieces of tissue with a marginal sense-organ attached pulsate slower than large ones. Moreover, when a sense-organ is present, tissue of any shape will pulsate even if its shape does not form a closed circuit.

The disks of *Aurelia* and *Dactylometra*, if cut as described above, will pulsate as does the disk of *Cassiopea*.

These experiments show that the rhythmical pulsation in Medusæ must arise from a definite center or centers, but this center may be established at any point in the muscular layer of the sub-umbrella. Once established it remains at a fixed point, while the disk continues to pulsate. Sustained pulsation *in disks* occurs only in tissue forming a complete circuit, and depends upon an electric transmission of energy, and the pulsation is self-sustaining (*i.e.*, sustained by internal stimuli) once it be started by an *external*, *momentary stimulus*.*

2. If normal perfect Medusæ be lifted out of water and then thrown back, the rate and amplitude of their pulsation suddenly increases. Pulsating disks react in a similar manner, but in their case the *amplitude* only increases, the rate remaining practically constant. The presence of marginal sense-organs is therefore not necessary for the display of "excitement."

3. The stimulus which causes pulsation is transmitted by the diffuse nervous or epithelial elements of the sub-umbrella. Newly regenerated sub-umbrella tissue, which lacks muscular elements and can not itself contract, will still serve as a bridge to transmit the stimulus which causes contraction in muscular tissue attached to but

* Professor W. T. Porter (1897) found that any part of the ventricle of the mammalian heart (heart of the dog) will beat for hours if supplied with defibrinated blood through its nutrient artery. Isolated portions of the heart of the hag-fish continue to beat rhythmically for hours even in the absence of nutrition. (See A. J. Carlson, 1905, Amer. Journ. Physiol., p. 220.)

beyond the bridge. In this connection, Carlson has demonstrated that the stimulus which causes the pulsation of the heart of *Limulus* is nervous in nature.

4. The paralyzed disk of *Cassiopea* is stimulated into temporary pulsation by all salts of potassium, sodium, lithium, barium, iodine, bromine, platinum, weak acids (hydrogen), ammonia, and glycerin. Magnesium, calcium, strontium, urea, and dextrose do not stimulate the disk, and produce no contraction.

5. The *sodium chloride* of the sea-water is the chief stimulant to pulsation in *Cassiopea*, while *magnesium* is the chief restrainer of pulsation, and counteracts the influence of the sodium chloride. Thus *Cassiopea* will pulsate in a pure $\frac{5}{8}$n NaCl solution for more than half an hour, but usually comes to rest in less than two minutes in a solution containing the amounts and proportions of NaCl and magnesium found in sea-water.

I find also that the heart of *Salpa democratica*, the branchial arms of *Lepas*, and the heart of the embryo loggerhead turtle pulsate actively in solutions containing only NaCl, K, and Ca, *magnesium* being absent. Magnesium *inhibits* pulsation in all of these cases, as it does also in *Cassiopea*.

The general rôle of NaCl, K, and Ca in all of the above cases is to combine to form a powerful stimulant producing an abnormally energetic pulsation, which, however, can not continue indefinitely; and *magnesium* is necessary to control and reduce this stimulus so that the pulsating organ is merely upon the threshold of stimulation.

A Ringer's solution is an optimum combination of NaCl, K, and Ca, and is only a stimulant, not an inorganic food, as has been commonly assumed. The organism must in time become exhausted under the influence of this stimulant unless a certain proportion of magnesium be present to restrain its action. Indeed, Ringer's solution probably acts by withdrawing magnesium ions by osmosis, and replacing them by a stimulant composed of salts of Na, K, and Ca. Magnesium is therefore a most important element in controlling and sustaining pulsation. If magnesium be precipitated in the pulsating *Cassiopea*, the NaCl, K, and Ca immediately produce a violent pulsation which soon passes into sustained tetanus, and all movement ceases in cramp-like contraction.*

* Loeb, J. 1906; Journ. Biological Chemistry, vol. 1, p. 331; finds that in the hydromedusa *Polyorchis* the mouth and tentacles are permanently contracted in any solution which lacks magnesium; and that magnesium serves to relax the muscles of the bell, thus counteracting the tetanus caused by other constituents of the sea-water and guaranteeing the relaxation after a systole.

The *calcium* of the sea-water *assists* the NaCl to resist the retarding effects of magnesium. Thus *Cassiopea* will pulsate from half an hour to an hour in a solution containing the amounts and proportions of NaCl, *magnesium*, and *calcium* found in sea-water, but usually ceases to pulsate in less than two minutes in a solution containing only the NaCl and *magnesium*.

Unlike calcium, *potassium* does *not* assist the NaCl to overcome the stupefying influence of the *magnesium*.* Thus *Cassiopea* ceases to pulsate almost as quickly in a solution containing NaCl, *magnesium*, and *potassium* of sea-water as it does in a solution containing only the NaCl and *magnesium*.

The potassium of sea-water serves, however, to stimulate pulsation in *connection with both* calcium and NaCl. Thus *Cassiopea* pulsates only from 20 to 120 minutes and at about a normal rate in NaCl + K_2SO_4, or in NaCl + KCl, whereas it pulsates for more than three hours at fully twice its normal rate in NaCl + K_2SO_4 + $CaSO_4$, or NaCl + KCl + $CaCl_2$.

We see, then, that the NaCl, K, and Ca of the sea-water unite in stimulating pulsation and in resisting the stupefying effect of the Mg. All four salts conjointly produce, in sea-water, an indifferent, or balanced, fluid which neither stimulates nor stupefies the disk of *Cassiopea*, and permits a recurring internal stimulus to produce rhythmic movement.

6. *Cassiopea* does not pulsate when its marginal sense-organs are removed, simply because the sea-water does not stimulate it. If stimulated in sea-water, in any manner, it readily pulsates. This is also true of *Gonionemus*, and Loeb's statement that both the K and Ca of sea-water inhibit pulsation is not supported; for the center of *Gonionemus* will pulsate actively, though temporarily, in sea-water whenever it is touched by a crystal of any *potassium salt*, or otherwise stimulated.†

On the other hand, the disks of *Aurelia* and *Dactylometra* begin to pulsate in sea-water in a few minutes, as soon as they recover from the shock of the operation resulting in the loss of their marginal sense-organs. Unlike *Cassiopea* and *Gonionemus*, both *Aurelia* and *Dactylometra* are weakly stimulated by the sea-water *as a whole* and pulsate almost immediately after the removal of their margins.

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*The general anesthetic effect of magnesium has been well known since the researches of Tullberg, 1892; Archiv. Zool. Exper. et Gen., Tome x, p. 11.

† As a matter of fact, the disk of *Gonionemus* is often seen to give isolated pulsations, at irregular intervals, in sea-water without apparent external stimulation. (See Yerkes, 1902.)

The disk of *Cassiopea* usually pulsates spontaneously in an irregular manner, immediately after the removal of its marginal sense-organs, if it be placed in a solution containing NaCl, NaCl + KCl, NaCl + $CaCl_2$, or NaCl + KCl + $CaCl_2$ in the amounts and proportions found in sea-water; but it will not pulsate in any solution which contains *magnesium*.

7. The central disk of *Cassiopea*, if set into pulsation, will pulsate longer than an hour in a solution resembling sea-water but lacking calcium, whereas the normal perfect Medusa, or parts of the margin containing sense-organs, cease to pulsate in this solution in less than six minutes. The marginal sense-organs can not send forth stimuli producing contractions unless they be *constantly supplied* with calcium from the sea-water, whereas the sub-umbrella tissue of the disk itself is relatively independent of the calcium of the sea-water.

On the other hand, both the disk and the perfect Medusa will pulsate in sea-water saturated with $CaSO_4$.

8. The normal *Cassiopea* Medusa will pulsate fully three times as long in a solution of Na_2SO_4 containing the same amount of sodium as is found in sea-water as it will in a solution of Na_2SO_4 isotonic with sea-water. This indicates that the amount and proportion of sodium in the sea-water is more important to pulsation than is its osmotic property.

9. The contractions of the heart of the loggerhead turtle are conducted and maintained exclusively by the thin peripheral muscular part of the wall of the heart, the thick cavernated tissue of the heart being passive. Moreover, the outer muscular part of the heart's wall is a better electrical conductor than is the cavernated tissue. A similar condition is seen in *Cassiopea*, where the thin sub-umbrella tissue of the disk is the only part which conducts and maintains the stimulus for pulsation, and is a better electrical conductor than is the thick gelatinous substance of the disk.

10. The chief results of the paper are the discovery of a new method of restoring pulsation in paralyzed Medusæ, and also that *magnesium* plays a most important rôle in restraining, controlling, and prolonging pulsation in animal organisms.

In *Cassiopea* the ectodermal, epithelial, or diffuse nervous elements of the sub-umbrella transmit the stimulus which produces rhythmical contraction.

Rhythmical pulsation can be maintained only when a stimulus and an inhibitor counteract one another, and cause the organism to be upon the threshold of stimulation; thus permitting weak *internal* stimuli to promote periodic contractions.

MINOR CONCLUSIONS.

There are certain minor conclusions, mainly confirmations or amplifications of the excellent work of Romanes upon Scyphomedusæ.

In *Cassiopea* the sub-umbrella and mouth-arms are the only parts which respond to mechanical or chemical stimuli. The ex-umbrella is wholly insensitive.

There is no essential difference in kind between the physiological action of the sense-organs, in pulsation, and that of any other part of the sub-umbrella.

Cassiopea will live for more than a month in absolute darkness. Its plant cells then degenerate, but the Medusa does not suffer; hence its vitality is not dependent upon the commensal plant cells within its tissues.

Starved Medusæ will shrink to about one-sixteenth their initial volume and still survive. They will live in brackish water containing 75 per cent fresh water better than they will if we maintain the amounts and proportions of calcium and potassium, merely reducing the amounts of NaCl and magnesium of the sea-water.

The fluids of the gastro-vascular space and of the body of the Medusa are only slightly alkaline, while the sea-water at Tortugas is decidedly alkaline.

The sense-organs tend to send out contraction stimuli at various rates, but the fastest working sense-organ controls the Medusa.

Excitement of the disk forces the sense-organs to maintain a higher rate of pulsation than they are capable of maintaining if cut off, and it is evident, from other experiments, that the disk reacts reciprocally upon the sense-organs, stimulating them into activity.

Small pieces of the disk, enervated by sense-organs, pulsate slower than large ones.

Small *young* Medusæ pulsate faster than large *old* ones.

The sub-umbrella surface of the disk exercises a reflex control over both sense-organs and mouth-arms.

Repeated stimulation of any one part of the disk finally tires the stimulated place so that it ceases to respond. Other parts of the disk still respond as readily as did the tired place in the first instance.

Having stated the principal conclusions, we will now proceed to give a detailed account of the experiments upon *Cassiopea*, *Lepas*, *Salpa*, and the loggerhead turtle.

Aboral views of Cassiopea xamachana Bigelow. From life. Natural size.

Above, rare, small variety. This bears a close superficial resemblance to the common Cassiopea ndrosia of the Fiji Islands. (See Agassiz and Mayer, 1899, Bull. Mus. Comp. Zool. at Harvard Coll., vol. 32, p. 175, pl. 14.)

Below, the common variety.

II. PULSATION OF CASSIOPEA IN SEA-WATER.

INTRODUCTION—NORMAL MOVEMENTS.

The rhizostomous Scyphomedusa *Cassiopea xamachana* (plates I, II), is very abundant during spring and summer in the salt-water moat of Fort Jefferson, at Tortugas, Florida. It was described by Bigelow (1892, 1900) from a salt-water lagoon in Jamaica, and also under the name of *Cassiopea frondosa* by Fewkes (1883), who found it at the Tortugas.

The Medusæ are usually found gathered in clusters upon the weedy bottom of the moat in water about four feet deep. They lie with the aboral side of the disk pressed downward upon the bottom, and with the 8 mouth-arms, with their numerous suctorial mouths, spread out above. A sucker-like concavity on the aboral side of the disk allows the Medusa to adhere with considerable strength to the bottom or sides of an aquarium, and the tenacity of its hold is still further enhanced by the rhythmical movement of the disk, which beats with considerable regularity, thus tending to hold the bell firmly against its fastening, and also to drive a current of water out over the mouth-arms.

If moved from its normal position and placed in the water with its disk uppermost and arms downward, the rhythmical beating of the disk causes it to swim upward, but if the water be of considerable depth it soon topples over and thus swims downward to the bottom or reaches the side of the aquarium. If, however, it should reach the surface, the concavity at the center of the aboral side of the disk often serves to permit the surface tension to hold the Medusa upon the surface, where it may float for a long time, pulsating normally with the concavity relatively dry, although lower than the general surface of the water.

The Medusa pulsates with a regular rhythmical movement, pauses or irregularities in the rhythm being exceptional. Occasionally, however, its rate suddenly increases, with or without apparent cause, and the pulsation may become so active as to cause the Medusa to break away from its anchorage and glide over the bottom. A regular unexcited movement is, however, often maintained for hours at a time, and in general this rate of pulsation is faster in small than in large Medusæ, as will appear from table I, on page 8.

The relatively rapid rate of small Medusæ is probably due to their being young and possessed of more vitality than are the large, old animals; for not only do small Medusæ regenerate lost parts more readily, but we also find that specimens which have become reduced in size through starvation pulsate at a slower rate than young and

well-fed Medusæ of the same size. Thus one *Cassiopea* was starved for three months, and the diameter of its disk shrank from 78 to 21 millimeters, while at the same time its rate of pulsation declined from about 40 to 16 per minute. It is also interesting to observe that if we cut off the margins of the disks of Medusæ of various sizes, the severed rims of the small Medusæ pulsate at a more rapid rate than do those of the large Medusæ, although in both cases this rate is slower than that of the uninjured Medusa.

TABLE 1.—*Relation between the rates of pulsation and the diameters of the disks in Medusæ of Cassiopea xamachana.*

Diameter of Medusa in millimeters.	No. of pulsations per minute.	Diameter of Medusa in millimeters.	No. of pulsations per minute.	Diameter of Medusa in millimeters.	No. of pulsations per minute.
13	78	28	39-55	62	29
15	82-86	28.5	16-23	63	35
16	94-111	30	53-63	82	40-50
18	68	31	55-61	84	28-39
20	36	32	58-62	90	16-28
20.5	45-65	36	42	102	20-21
22	60	42	51-54	107	23-24
23	44-71	46	45-46	118	7
23.5	40	47	43-36	124	12-16
26	43-52	50	27	136	9-12
27	41-56	57	36-37	153	7

EXCITEMENT.

As we have said, the pulsating Medusæ occasionally exhibit a sudden increase in their rate and amplitude of pulsation without apparent cause. This can, however, be invariably brought about as a response to any stimulus, such as a water current, a mechanical shock, or the introduction of some irritating chemical into the water. When lifted wholly or partially out of water, and replaced, the Medusæ pulsate at about twice their normal rate for two or three minutes, and the amplitude of their pulsations is also increased. Even small fragments of the disk containing a marginal sense-organ will usually display this excitement, although the duration of the period of excitement is shorter for small than for large pieces, and their rate of pulsation slower.

However, the presence of marginal sense-organs is not necessary for this "excitement," for, as we shall soon show, we have succeeded in causing disks deprived of marginal sense-organs to pulsate constantly and regularly in sea-water; and if such disks be pinched or lifted out of water or otherwise disturbed the *amplitude* of their pulsations becomes suddenly increased, while the *rate* remains practically constant. In normal uninjured Medusæ both rate and amplitude increase,

but as we shall see, disks without sense-organs pulsate at the maximum rate at which their tissue is capable of transmitting the wave of pulsation, and they can therefore exhibit "excitement" only by an increase in amplitude.

It is worthy of note that if the forceps used to stimulate the Medusa be made to seize upon only a small area of tissue, the Medusa will not respond, but on bringing a larger area between the forceps the response is sudden and violent. In this connection it will be recalled that Romanes showed that the bell of *Sarsia*, when deprived of its margin, will respond to mechanical shocks by pulsations, each stimulus usually giving rise to one or two pulsations, and this is also true of the paralyzed disk of *Cassiopea*. We must conclude that the presence of marginal sense-organs is not necessary for the display of that sudden increase in activity which we have called "excitement," and that this response may come from many or all parts of the undifferentiated tissue of the sub-umbrella.*

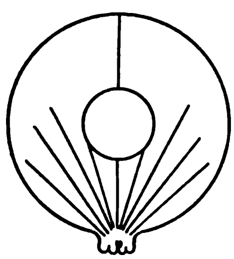


Fig. 1.

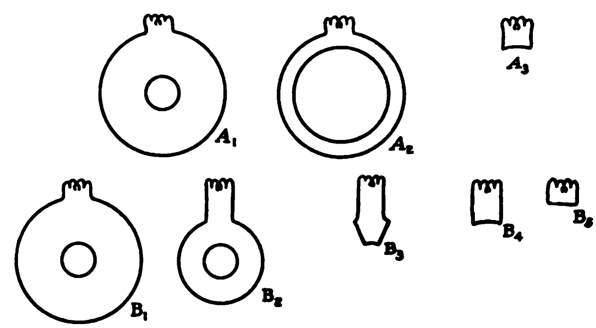


Fig. 2.

Romanes showed that in *Aurelia* annular cuts separating the margin from the center of the disk caused the rhythm to become slower, and he was led to suspect (1885, p. 163) that a stimulus of an afferent character emanates from all parts of the sensory surfaces of the sub-umbrella to the marginal sense-organs, although of this he had no direct proof. I think we can prove that this is the case in *Cassiopea*, for if we cut off all but one marginal sense-organ, and then make cuts through the sub-umbrella tissue (fig. 1) radiating outward from the sense-organ and therefore not interfering with any stimulus which may travel by the shortest path from any point in the disk to the sense-organ, the final rate of pulsation, after the excitement due to the operation has sub-

* It is interesting to observe that Bancroft and Esterly (1903) find that while contractions normally originate from the ganglionated ends of the heart of *Ciona*, they *may* originate from any other region.

sided, will remain the same as it was before the radiating cuts were made. Moreover, its excited rate, due to being lifted out of water and dropped back, remains the same as it was before the cuts were made. On the other hand, cuts designed to successively reduce the area of the sub-umbrella tissue enervated by a sense-organ (such as are shown in fig. 2, A and B) usually cause the normal rate of pulsation to decline. The *excited rates*, however, are less influenced by reduction of area, small pieces sometimes pulsating almost as rapidly as large ones, but the *duration* of the excitement displayed by small pieces is much reduced. For example, in the A series of figure 2—

	Area.	Normal rate per minute.	Excited rate per minute.
A_1	280	17	32
A_2	54	12	40
A_3	1	6	11

In the B series the relative areas and rates were as follows:

	Area.	Normal rate per minute.	Excited rate per minute.
B_1	271	27	50
B_2	153	14-20	50
B_3	5	17	48
B_4	2	14	19
B_5	1	14-20	35

The above results are quite similar to those of Romanes upon *Aurelia*, and are opposed to the conclusion of Eimer that severed portions of the disk pulsate at rates approximately proportionate to their respective areas.

It is interesting to observe that if we stimulate a Medusa into prolonged and active pulsation at an "excited" rate and then cut out the marginal sense-organs, each sense-organ, together with the piece of tissue attached to it, instantly subsides into a *slow* rate of pulsation, never faster than the average unexcited rate of the entire Medusa. Moreover, these pieces with sense-organs attached can not immediately be stimulated into a display of excitement, although after an interval of time they will readily respond and exhibit an excited rate commensurable with that of the perfect Medusa. As we have seen, the display of "excitement" is a function of the undifferentiated tissue of the sub-umbrella, and it appears that the excited rate of the Medusa may be maintained by the influence of the general sub-umbrella tissue

upon the sense-organs even after the sense-organs have become too exhausted to themselves maintain an "excited" rate. Moreover, if we stimulate the sub-umbrella surface by touching it repeatedly with a crystal of K_2SO_4 the disk responds by active contractions and forces the sense-organs to respond at the same rate. Then after the stimulus is withdrawn the sense-organs are found to have been exhausted by the contractions of the disk and can not again resume pulsation until after a long interval of rest.

Direct evidence showing that the sub-umbrella may exert a controlling influence on all parts of the sensory tissues of the Medusa is also afforded by the following experiment: If we cut off the basal plate with the 8 mouth-arms, the mouth-arms remain normally expanded in sea-water. If now we place the mouth-arms in a solution which resembles sea-water, but lacks potassium, the arms contract into a close bunch, and will *not again expand* as long as they remain in the solution. If, however, we place a perfect Medusa in the solution it exhibits periods of active pulsation alternating with periods of rest. Immediately *after* it comes to rest its mouth-arms contract into a close bunch, but they *always expand* again as soon as the Medusa resumes pulsation. It will be remembered that Romanes showed that removal of the margin of the bell in *Sarsia* caused the manubrium to elongate and lose its muscular tonus. He also found that in *Sarsia* stimulation of the sub-umbrella caused the manubrium to contract, and that the manubrium of *Tiaropsis indicans* would apply its mouth to any stimulated part of the sub-umbrella, provided the stimulus could travel radially inward from the stimulated spot to the manubrium. Otherwise the manubrium executed ill-directed or wandering movements.

We will soon show that any difference between the physiological action of the marginal sense-organs and that of the general sensory tissue of the sub-umbrella is one of degree, not of kind.

CONTROL OVER PULSATION EXERCISED BY THE MARGINAL SENSE-ORGANS.

Romanes found that the potency of the marginal sense-organ attached to a segment of the disk has more to do with its rate of pulsation than has the size of the segment; nevertheless small segments usually pulsate slower than large ones.

In *Cassiopea xamachana* there are 13 to 23 marginal sense-organs, and I find that the average rate of the perfect Medusa is apt to be the same as the rate of its *most rapidly* working sense-organ. As Romanes saw in *Aurelia*, the sense-organs tend to initiate stimuli at various

rates, but the fastest controls all the others and forces them to beat in unison with it. To test this, I took a *Cassiopea* having 19 marginal sense-organs and a normal unexcited rate of 12 to 16 pulsations per minute. I then made 19 radial cuts midway between the 19 sense-organs, so as to divide the disk into 19 practically equal sectors, each enervated by a single sense-organ. These radial cuts through the sub-umbrella completely separated the sectors one from another in so far as the transmission of nervous impulses were concerned (fig. 3). Under these conditions one of the sectors pulsated 18 times per minute; 2 pulsated 17 times; 2 pulsated 16 times; 1 pulsated 15 times; 3 pulsated 9 times; 1 pulsated 8 times; 4 pulsated 7 times; 2 pulsated 6 times; 2 pulsated 5 times, and 1 failed to pulsate.

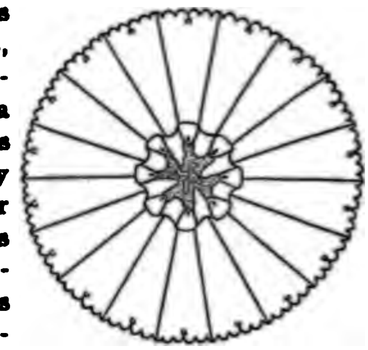


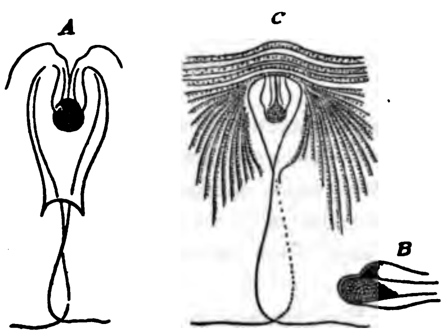
Fig. 3.

The sense-organs gradually change their rates, so that at the end of an hour or two the fastest may sink to second or third place, etc. Quite often one or more of the sense-organs either failed to send out pulsations or did so at very infrequent intervals. These sense-organs appeared normal, however, and if stimulated by being thrown into sea-water containing 1 per cent excess of K_2So_4 they initiated pulsations at a rapid rate.

As Romanes and Eimer showed, if we cut off all but one of the marginal sense-organs this one will maintain a rhythmical pulsation of the disk, whereas if this last sense-organ be removed the disk at once becomes more or less paralyzed. The disks of *Aurelia* or of *Dactylometra*, however, begin to pulsate irregularly a few minutes after the loss of the last marginal sense-organ, but *Cassiopea* remains practically paralyzed for about 24 hours after the operation, rarely executing a pulsation unless stimulated. On the following day, however, it occasionally pulsates without apparent stimulation, and three days after the operation the disk rarely remains for a minute without pulsating. The pulsations are, however, isolated, single, and separated by irregular intervals of time, until the marginal sense-organs begin to regenerate.

Romanes showed that in Hydromedusæ the least discernible remnant of the bell-margin if left intact will maintain the rhythmical movement of the bell, but that in Scyphomedusæ the marginal sense-organs are the only parts of the rim which normally control the rhythmical pulsation. I find that if one cuts off the tip of the last remaining sense-organ of *Cassiopea*, thus removing the otoliths and

pigment spot but leaving the stalk of the sense-organ intact, the disk is instantly paralyzed. Also, when the marginal sense-organ regenerates, regular pulsation is resumed as soon as the pigment spot and a few small otoliths begin to appear. For example, figure 4 shows the appearance of the normal sense-organ, and figure 5 the condition of a regenerating sense-organ that has become capable of controlling the rhythm of the disk. Immediately after death the pigment of the sense-organ dissolves out into the sea-water; on the other hand it appears remarkably stable in the living animal, and is not faded by the most intense sunlight, nor changed by one month's confinement in absolute darkness.



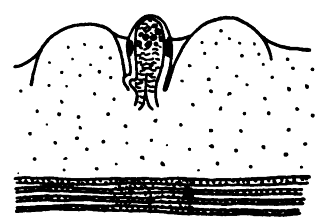


Fig. 4. Fig. 5.

Fig. 4.—Enlarged views of a sense-organ of a mature Medusa of Cassiopea. A, aboral view; B, side view; C, oral view.

Fig. 5.—Enlarged oral view of a regenerating sense-organ, showing the beginning of the formation of pigment spot and otoliths. A wide strip of new tissue (dotted) separates the sense-organ from the old muscular layer of the sub-umbrella.

If a sense-organ be cut out with the merest remnant of sub-umbrella tissue left attached to it, examination under the microscope shows that this tissue continues to pulsate rhythmically, and it is apparent that the area of the sub-umbrella tissue attached to a sense-organ may be reduced to a practical zero without any more marked effect than a not very pronounced slowing of its rate of pulsation. On the other hand, if we remove all but one of the sense-organs and then place the disk in sea-water charged with CO_2, keeping the sense-organ itself out of the fluid, the disk becomes paralyzed and can not be enervated into contraction by the sense-organ. In some of these experiments the sense-organ was also paralyzed, although it had not been in the CO_2 solution. In others the sense-organ continued to send contractions out over the adjacent tissue, but these could not extend over the parts of the sub-umbrella which were bathed by the CO_2.

All experiments serve to demonstrate that the nervous relationship between the sense-organs and the general sub-umbrella tissue is reciprocal, as has been clearly shown by Romanes, who found that if we cut a strip of tissue from the disk of *Aurelia*, leaving a sense-organ at one end, and then gently stroke the end remote from the sense-organ with a camel's hair brush, the marginal sense-organ at the other end of the strip will be stimulated into sending a contraction wave back over the strip (Romanes, 1885, pp. 74–77). This discharge is therefore of a reflex nature. Nagel (1894) supports the idea that the marginal sense-organs are reflex centers, while von Uexküll (1901), upon evidence which to me appears insufficient, concludes that the marginal sense-organs in *Rhizostoma pulmo* are merely centers for the reception of mechanical stimuli, and that each pulsation of the bell causes the sense-organs to swing to and fro, and this stimulation calls forth a new pulsation.

We will show later that any point in the sub-umbrella surface may be made to start and maintain impulses which will set the whole disk into sustained and perfectly regular rhythmical pulsation. There is, therefore, no difference of kind between the nervous activities of the marginal sense-organs and those of any other parts of the sensory surface of the sub-umbrella.

As to the function of the otocysts in Hydromedusæ, Murbach (1903) showed that in *Gonionemus* they have no static function, for if they be removed the normal movements of the Medusa will be resumed before they are regenerated. Murbach's conclusion that the seat of the static function is "muscular sensation in the velum" requires confirmation. Injury of so important a swimming organ as the velum may readily cause irregularities in movements by abnormally deflecting the water currents passing through the opening of the velum at each contraction. Moreover, Yerkes (1902), in his study of the sensory reactions of *Gonionemus*, found that the velum is unaffected by stimuli of any sort.

Romanes (1885) found that the ocelli of *Sarsia* and *Tiaropsis* are sensitive to light, and Yerkes (1902) demonstrated that the tentacles of *Gonionemus* are very sensitive to chemical, mechanical, and photic stimuli.

The rates at which waves of contraction travel over the disk in *Cassiopea* range from 150 to 1200 mm. per second, each individual displaying a characteristic and constant rate. Apparently there is no relationship between the size of the Medusa and the rate of transmission of waves over its sub-umbrella tissue. These rates were

determined by cutting spiral strips reaching from the margin inward, in the manner of Romanes. It was observed that when the spiral was made 5 mm. or less in width only powerful stimuli would travel from one end of the strip to the other, and if under these conditions a single sense-organ was left at the outer end of the strip, waves of contraction which started from this sense-organ might or might not reach the central part of the disk. If, however, the end containing this sense-organ were touched with a crystal of K_2SO_4, or any other potassium salt, a powerful wave of contraction immediately ensued and always traveled completely through the spiral. But if the inner end of the spiral were touched with the crystal of potassium salt, not only did the wave not always reach the sense-organ, but it traveled only three-quarters as fast as did the waves from the sense-organ. When the sense-organ was cut off, however, the waves traveled at the same rate from *either* end of the spiral strip, and this rate was the slower of the two mentioned above. Evidently the sense-organ reinforced the stimulus given by the potassium salt.

In this connection Romanes showed that in *Aurelia* strong stimuli may initiate waves that may travel over the disk at twice the rate of weak ones.

Peripheral parts of the disk transmit stimuli at a faster rate than do parts near the center of the disk. This was shown by Romanes to be the case in *Aurelia*. Altogether the outer parts of the sub-umbrella are more sensitive than the inner.

As Romanes showed, there must be an appreciable interval of rest between two successive responses to stimuli, and rhythmical waves can not follow one after another faster than a certain frequency. Waves traveling in opposite directions through the same strip of tissue meet and reinforce, but do not pass each other, for a stimulus can not produce a contraction over the tissue that has been in contraction only the instant before.

The sensory field of the Medusa is confined to the sub-umbrella and the mouth-arms. The ex-umbrella surface exhibits no reactions to stimuli, and indeed the epithelium of the ex-umbrella may be killed by such penetrating reagents as Gilson's fluid, and, provided the poisonous liquid does not reach the sub-umbrella, the rhythmical movement will not be altered in rate. Even near the margin of the disk, close to the sub-umbrella surface, the ex-umbrella is inert to stimuli of all sorts. The action of the sucker-like concavity at the aboral center of the ex-umbrella is entirely passive, and a Medusa deprived of all marginal sense-organs will still "cling" to the bottom or side of the aquarium, although paralyzed and motionless.

VITALITY, ETC.

The fluids of the central stomach of *Cassiopea* are practically neutral to litmus test, whereas the sea-water at Tortugas is decidedly alkaline. For example, litmus paper tinged pink by HCl is changed to blue in the sea-water in from 9 to 12 minutes, whereas a portion of the same litmus paper thrust into the central stomach cavity of *Cassiopea* will not become blue until it has remained in the stomach for 6 to 9 hours. The whole surface and all of the tissues of the Medusa are almost neutral and much less alkaline than is the sea-water. The stomach cavity may be filled with sea-water charged with CO_2, or we may place crystals of K_2SO_4 within it, and yet little or no effect will be produced upon the movements of the Medusa, although, as we shall see, these substances produce a profound effect if applied to the sub-umbrella surface. Remarkably little CO_2 is given off by the Medusæ in metabolism. A large Medusa was confined for 12 hours in a small quantity of sea-water tinged pink by rosolic acid, and the decoloration of the fluid was barely perceptible.

Cassiopea pulsates regularly and at its usual daylight rate throughout the night, and even red light has no apparent effect upon its rate of movement. If long confined in absolute darkness, however, the rate of pulsation becomes slower, and the plant cells within the tissues of the Medusa become shriveled and greatly reduced in number, so that the Medusa becomes pale blue in color and translucent. Only the filaments of the mouth-arms retain their greenish color. (Pl. II, fig. B.) The whole color of the Medusa becomes lighter and more uniform than the normal, as will be seen upon comparing figures A and B of plate II. Two Medusæ of *Cassiopea xamachana* were maintained in absolute darkness and without food for one month. When first placed in the dark their diameters were 82 and 42 mm., and their rates of pulsation 40 to 50 and 51 to 54 per minute, respectively. At the end of one month the large Medusa had shrunken so as to be but 58 mm. and the small one 25 mm. in diameter, and their rates of pulsation 23 and 17 per minute, respectively. On their being removed to the diffused daylight of the laboratory, the color remained unchanged for three weeks, but the diameters of the Medusæ continued to decrease; finally, however, the plant cells in the mid-region of the sub-umbrella and ex-umbrella became dark brown and densely crowded, so that these parts of the Medusæ were dull brown in color. After being in the light for one month the large Medusa was only 29 mm. in diameter, and its rate of pulsation was less than 1 per minute.

On the other hand, when the Medusa is maintained without food in the light it becomes dark brown in color (pl. II, fig. C), as will be seen upon comparing its photograph with that of a normally colored

A. Oral view of a normal, recently captured specimen of Cassiopea xamachana.
B. A specimen which has been maintained for one month in absolute darkness, showing its pale coloration. The plant cells are much reduced in number.
C. A specimen which has been starved for one month in the light, showing its very dark brown color.

Medusa. The greenish color of the oral filaments disappears, and the plant cells become shriveled and densely crowded. A Medusa starved in light is more active and shrinks more rapidly than does one starved in darkness, and thus it appears that metabolism proceeds more rapidly in light than in darkness. For example, a Medusa starved in diffused daylight had a diameter at the beginning of the experiment of 78 mm. At the end of 2 months its diameter was 37 mm., and at the end of 3 months, 21 mm., being still vigorous and pulsating at the rate of 16 per minute.

These starved Medusæ exhibited certain phenomena of degeneration. The mouth-arms became reduced to mere stumps, most of the mouths closed over, and the oral tentacles and filaments were absorbed or cast off, so that the oral surfaces of the mouth-arms became quite smooth and rounded. The marginal lappets of the disk became blunted, and the dull-white peripheral ring of the ex-umbrella was much reduced in width. Only immature eggs were found in the gonads of starving Medusæ. It appears remarkable that the first parts to degenerate are the mouths and mouth-arms, although these are the most important to the organism if in danger of starvation. The marginal sense-organs remained normal in size and appearance.

Cassiopea xamachana lives in salt-water lagoons having but limited communication with the sea, and it is therefore not surprising that it will survive considerable alterations of salinity. Fresh water (rain-water) is quickly fatal to the Medusæ, for they shrivel rapidly; all pulsations cease, and even if removed to salt water after less than five minutes' exposure to the fresh, recovery is very slow. On the other hand, if every night and morning we decrease the salt and increase the fresh water 5 per cent, the Medusæ can be brought into a mixture of 25 per cent sea-water plus 75 per cent fresh water, and still survive. Their rates of pulsation become successively slower as the salt water is reduced.

Thus, two Medusæ in pure sea-water had rates of pulsation of 20 and 60, respectively; in 60 per cent sea-water plus 40 per cent fresh water, 18 and 18, respectively; in 50 per cent sea-water plus 50 per cent fresh water, 14 and 18, respectively; in 40 per cent sea-water plus 60 per cent fresh water, 8 and 4, respectively; in 35 per cent sea-water plus 65 per cent fresh water, 7 and 2, respectively; in 30 per cent sea-water plus 70 per cent fresh water, 3 and 2, respectively; in 25 per cent sea-water plus 75 per cent fresh water, 3.

The small Medusa ceased to pulsate in 75 per cent fresh plus 25 per cent sea-water, and its sub-umbrella surface became insensitive to the most powerful stimuli, such as a touch of a crystal of KCl or $K SO_4$; yet when transferred to 50 per cent fresh plus 50 per cent sea water it

recovered and pulsated at the rate of 11 per minute. The large Medusa, which pulsated only 3 times per minute in 25 per cent sea-water plus 75 per cent fresh water, revived quickly and pulsated 18 times per minute in 50 per cent salt plus 50 per cent fresh water.

If instead of mixing the sea-water with distilled water, we employ a solution of fresh water containing the amounts of potassium and calcium found in the sea-water, the Medusæ do not survive as well as they would in ordinary brackish water, and their rates of pulsation are much slower, as will appear from the following: Three Medusæ in pure sea-water had rates of pulsation of about 60 per minute; the same Medusæ in 55 per cent sea-water plus 45 per cent rain-water containing the same amounts of potassium and calcium as sea-water, pulsated 8 to 14 times per minute; in 45 per cent sea-water plus 55 per cent rain-water containing the same amounts of potassium and calcium as sea-water, they pulsated 2 to 9 times per minute; in 40 per cent sea-water plus 60 per cent rain-water containing the same amounts of potassium and calcium as sea-water, they pulsated 1 to 6 times per minute; in 35 per cent sea-water plus 65 per cent rain-water containing the same amounts of potassium and calcium as sea-water, they pulsated 0 to 2 times per minute; in 25 per cent sea-water plus 75 per cent rain-water containing the same amounts of potassium and calcium as sea-water, two dead, one pulsated about once every 10 minutes.

Evidently a uniform reduction of the magnesium, sodium, potassium, and calcium is less injurious than a reduction of the sodium chloride and magnesium alone. As Ringer, Loeb, and others have shown, a *balance* in the *proportions* of the constituents of the sea-water is more important than the presence of any single salt.

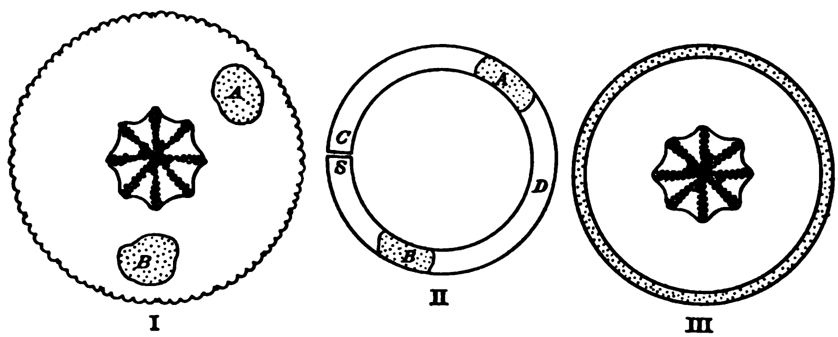
As might be expected in Medusæ living in shallow lagoons, where evaporation is great, *Cassiopea* will withstand a considerable concentration of the salt water; however, Medusæ in 100 cc. sea-water plus 1 gram NaCl will survive for 12 hours, but their pulsation becomes irregular, although on the average of about normal rate. The mouth-arms, however, are strongly contracted, and the Medusa exhibits alternate periods of rest and activity in its rhythmical movements.

Cassiopea will pulsate normally in sea-water saturated with $CaSO_4$.

As will be apparent from the above, *Cassiopea xamachana* is one of the hardiest of Scyphomedusæ. It survives for months in aquaria with but ordinary care, and exhibits wonderful recuperative powers from the effects of poisons. If subjected to constant shaking, as in a floating live-car, it does not thrive as well as in stationary aquaria where the water is not so pure.

THE NERVOUS OR EPITHELIAL NATURE OF THE STIMULUS WHICH PRODUCES CONTRACTIONS.

If the sub-umbrella be injured by scraping parts of it away, as in figure 5A, I, or if the margin be cut off as in figure 5A, III, the removed parts are soon partially regenerated, as shown in the dotted areas, but this newly regenerated tissue is at first epithelial in character, and lacks muscular elements. It therefore can not contract, yet if it be touched with a crystal of K_2SO_4, or otherwise stimulated, it transmits the stimulus across itself to the adjacent muscular tissue of the sub-umbrella, which contracts vigorously, although the newly regenerated tissue which conducted the impulse does not itself contract. This



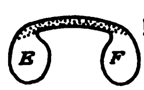
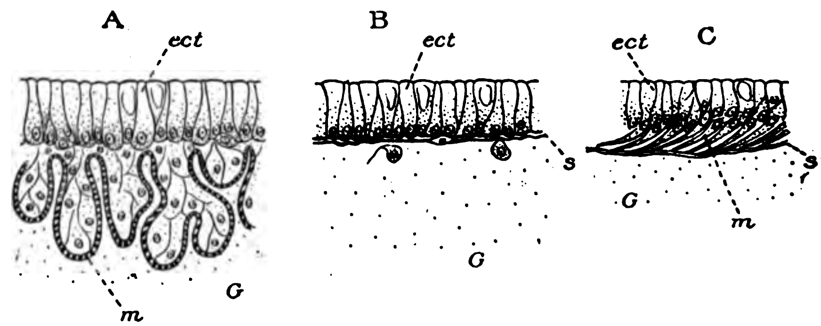
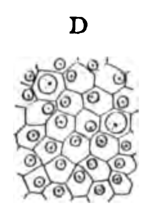


Fig. 5A.—Newly regenerated sub-umbrella tissue which lacks muscular elements, and can not itself contract, can still transmit the stimulus to pulsate to normal tissue adjacent to it. In fig. 5A, II, the stimulus crosses areas A and B, which do not contract, while C, D, and S contract in the order named. In fig. 5A, IV, the bridge of newly regenerated tissue does not itself contract, but serves nevertheless to transmit the stimulus causing contraction in E and F.

can best be demonstrated by making the newly regenerated tissue serve as a bridge connecting two pieces of uninjured sub-umbrella tissue, as is shown in figure 5A, II, or 5A, IV. Then, upon touching figure 5A, II, at *S* with a crystal of K_2SO_4 or other stimulant, a contraction wave passes from *S* through B-D-A-C; but B and A, being newly regenerated tissues without muscular elements, do not contract, although the stimulus which produces contraction passes across them. Similarly in figures 5A, IV, if E, which is normal sub-umbrella tissue, be caused to contract, every contraction is followed by F, although the bridge of newly regenerated tissue which connects them does not contract. These experiments tend to show that the stimulus which causes pulsation is transmitted by the epithelial or nervous elements to the muscular elements, and not primarily by the muscular elements themselves. I have examined many specimens of newly regenerated tissue

which did not itself contract, and yet transmitted the impulse which produced contraction in muscular tissue attached to it, and there appear to be no muscular elements in the newly regenerated tissue, although these often develop later. In these examinations I made use of intra vitem methylene blue, Retterer's method, Flemming's fluid followed by Ehrlick's acid hematoxylin, corrosive sublimate followed by aqueous carmine stain, and Hermann's fluid, but in no case could I find muscular elements in sections of the newly regenerated tissue which appeared to be a simple columnar epithelium, underlaid by a thin nervous net-work (see fig. 36). The muscle fibrillæ of the sub-umbrella are striate, and are easily demonstrated by any of the above methods.*





Figs. A-C.—Cross-sections of the sub-umbrella of Cassiopea. Fig. D.—Surface view of newly regenerated sub-umbrella tissue. *ect*, ectodermal epithelium. *G*, gelatinous substance of the disk. *M*, muscle fibers. *S*, basal membrane.

Figure A is a cross-section of the normal uninjured sub-umbrella of *Cassiopea*, cut across the trend of the circular muscle fibers; while figure B is a cross-section through regenerated sub-umbrella epithelium which has grown over an area from which all cellular elements had been cut away about 40 hours before. This newly regenerated tissue can not itself contract for, as yet, it lacks muscular elements; but it will nevertheless transmit the stimulus which produces contraction in

*Hesse (1895) finds that the nerve fibers in the sub-umbrella of *Rhizostoma pulmo* extend in all directions, but are mainly grouped in clusters extending from sense-organ to sense-organ. Bethe (1903) finds that in *Rhizostoma* and *Cotylorhiza* the epithelium of the sub-umbrella is connected with the deep-lying muscles by means of an intermediate plexus of nerve fibers.

muscular tissue adjacent to it. There are a few spindle-shaped (ganglion?) cells upon the basal membrane at the base of the regenerated epithelium, and occasionally one sees a large rounded cell in the gelatinous substance below the basal membrane. Occasionally these rounded cells have one or more delicate processes which extend into the gelatinous substance.

Figure C is a somewhat slanting section through regenerating sub-umbrella tissue about 4 days old, which is beginning to regenerate the muscle fibers and can now contract feebly. The muscle fibers appear as elongate processes of deep-lying epithelial cells, and extend parallel one with another over the basal membrane, trending circumferentially around the sub-umbrella.

Figure D is a surface view of newly regenerated sub-umbrella epithelium which transmits the pulsation-stimulus, but can not yet contract, as it still lacks muscular elements.

It is well known that Carlson has demonstrated the nervous nature of the stimulus which produces pulsation in the heart of *Limulus*. Indeed, I believe that all of the facts brought to light by Gaskell in his attempt to prove the muscular nature of the transmission of the stimulus of pulsation in the vertebrate heart will apply equally well if we assume that the impulse is transmitted by diffuse nervous elements. In the heart of the loggerhead turtle I find that the stimulus causing pulsation is transmitted entirely through the thin outer muscular part of the wall of the heart, and the thick cavernated inner part of the heart's wall may be cut away without affecting the pulsation. Also, the stimulus to pulsate is not transmitted through this cavernated tissue to the muscular tissue.

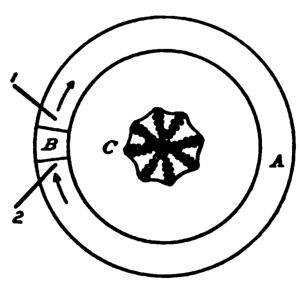


Fig. 5B.—Showing that the sub-umbrella tissue is a better electrical conductor than is the gelatinous substance of the bell. The current travels around through the long way, rather than across the shallow scratches which insulate the area B.

The sub-umbrella tissue of *Cassiopea* is a good conductor of electricity, while the gelatinous substance of the Medusa is a poor conductor. Thus in fig. 5 B, if we insulate an annulus by the shallowest possible scratch through the sub-umbrella, and then isolate a small sector, *B*, by shallow radial cuts; on touching the large sector *A* at 1 and 2

with the electrodes the contraction travels all the distance around *A*, but the sector *B* does not contract. The path of least electrical resistance is evidently through the long strip of sub-umbrella tissue, while the short path across the cuts interposes a greater resistance.

PULSATION WITHOUT MARGINAL SENSE-ORGANS.

Romanes, Eimer, von Uexküll, and others, have shown that in Scyphomedusæ the marginal sense-organs are centers which discharge the stimuli producing the rhythmical movements of the disk; and that if we remove these sense-organs, a more or less complete paralysis of the disk occurs. In some forms, such as *Aurelia* and *Dactylometra*, this paralysis lasts but a few minutes, and then more or less *irregular* contractions commence. In *Rhizostoma pulmo*, according to Hargitt, the paralysis is much more pronounced than in *Aurelia*. In *Cassiopea xamachana* the paralysis is practically complete for at least 24 hours, the disk responding only to definite stimuli, and very rarely giving a contraction without evident cause. On the second day after the operation the disk is much more sensitive to stimuli of all sorts and gives occasional isolated contractions without apparent stimulation, and at the end of a week the disk can rarely be observed for a minute without one's seeing it give a number of quick, isolated contractions. Regular rhythmical pulsation never sets in, however, unless the marginal sense-organs be regenerated.

Hitherto, disks without sense-organs have always been maintained in sustained pulsation by constant artificial stimulation, or by being placed in more or less injurious stimulating solutions. It will be recalled that Romanes obtained regular pulsation in the disks of *Aurelia* by passing through them a constant, or faradaic, current of electricity of minimal strength. He thus demonstrated that rhythmical movements might result from a constant stimulus, and he showed that one contraction could not follow another until the sub-umbrella tissue had recovered from the exhaustion caused by the previous contraction; then, and then only, can the tissue respond to the ever-present stimulus. Romanes concluded, therefore, that the ganglia of the marginal sense-organs may exert a constant stimulus, and yet give rise to periodic contractions. Romanes also found that the paralyzed bell of *Sarsia* could be set into a "flurried shivering" pulsation for one hour by a solution of 10 to 20 drops of acetic acid in 1000 cc. of sea-water, and that it would also respond by rhythmic contractions to a solution of 5 per cent glycerin in sea-water.

In 1900 Loeb found that the paralyzed disk of *Gonionemus* will pulsate rhythmically for an hour in a solution of $\frac{5}{8}$n NaCl or $\frac{5}{8}$n

NaBr, but that a small amount of calcium or potassium added to the Na solution will prevent the disk from pulsating. Loeb concluded that the calcium and potassium ions of the sea-water prevented the center of the bell of *Gonionemus* from pulsating. This is untrue for *Cassiopea*, for not only will the disk when deprived of sense-organs pulsate regularly for more than an hour in an artificial sea-water without calcium, but will also pulsate indefinitely in natural sea-water, and will contract rhythmically in solutions containing NaCl + KCl, or NaCl + $CaCl_2$, or NaCl + KCl + $CaCl_2$ in amounts and proportions found in sea-water. All solutions containing *magnesium* tend to prevent pulsation in the disk of *Cassiopea*.

As a result of his work upon the skeletal muscles in 1899 Loeb concludes that rhythmical contractions occur only in solutions of electrolytes, *i. e.*, in compounds capable of ionization, and that in solutions of non-conductors such as glycerin these rhythmical contractions are impossible. However, Romanes found that glycerin caused rhythmical pulsation in *Sarsia*. Greene (1898) and Howell (1901, p. 189) found that strips of heart muscle, after having ceased to pulsate in NaCl, will again pulsate if immersed in a pure solution of cane sugar or dextrose isotonic with the NaCl solution, and I find that the heart of *Salpa* will pulsate normally for more than half-an-hour in dextrose, isotonic with sea-water (see table 6). Thus automatic beats may occur in a solution entirely free from electrolytes, but, as Howell shows, these beats are probably dependent upon the presence of electrolytes in the tissue itself.

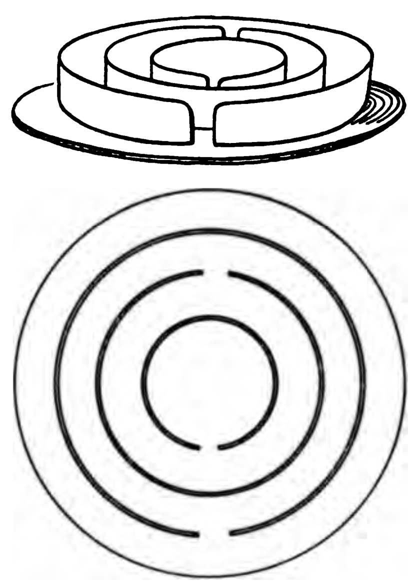


Fig. 6.—A disk of Cassiopea pressed by a concentric series of block-tin rings so as to insulate circuits of tissue. A disk so pressed may be caused to pulsate continuously.

When we come to consider the effect of ions, etc., upon *Cassiopea*, it will appear that one must be cautious of drawing general conclusions, even from the most evident effects upon any one animal. Thus I find that chemicals which produce certain perfectly definite and invariable responses upon *Cassiopea* act differently upon *Aurelia*, *Dactylometra*,

Gonionemus, *Lepas*, *Salpa*, and the loggerhead turtle. If there be marked differences between the reactions of closely related Scyphomedusæ, one may expect even greater disparity between those of vertebrates as compared with invertebrates.

Romanes, Loeb, von Uexküll, Hargitt, and others have caused disks to pulsate temporarily by subjecting them to the influence of NaCl solutions, etc., but in all cases more or less toxic effects resulted from the experiments and the sensibility of the sub-umbrella tissues became impaired or destroyed, so that further stimulation soon became impossible. We will now describe a method by which the disk of *Cassiopea* when deprived of marginal sense-organs may be made to pulsate indefinitely in sea-water with the production of effects no more injurious than those of fatigue. This may be most readily accom-

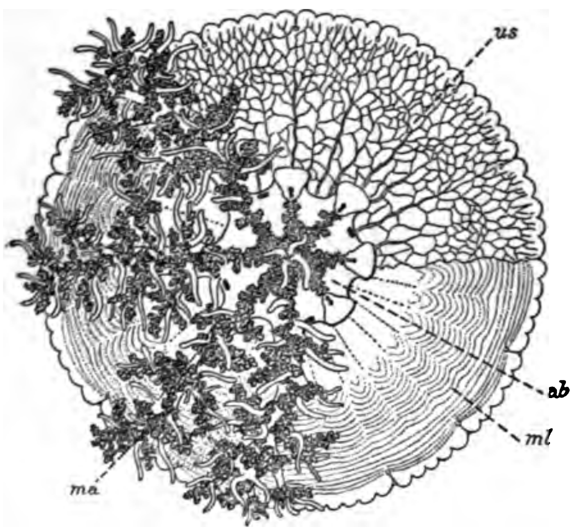


Fig. 7.—Oral view of Cassiopea xamachana. Four of the mouth-arms are cut off, and the muscle layer of the sub-umbrella in the upper right-hand quadrant removed to show the underlying vascular system. ab, Mouth-arm plate; ma, mouth-arm; ml, muscular system of the sub-umbrella; us, vascular canals of the sub-umbrella.

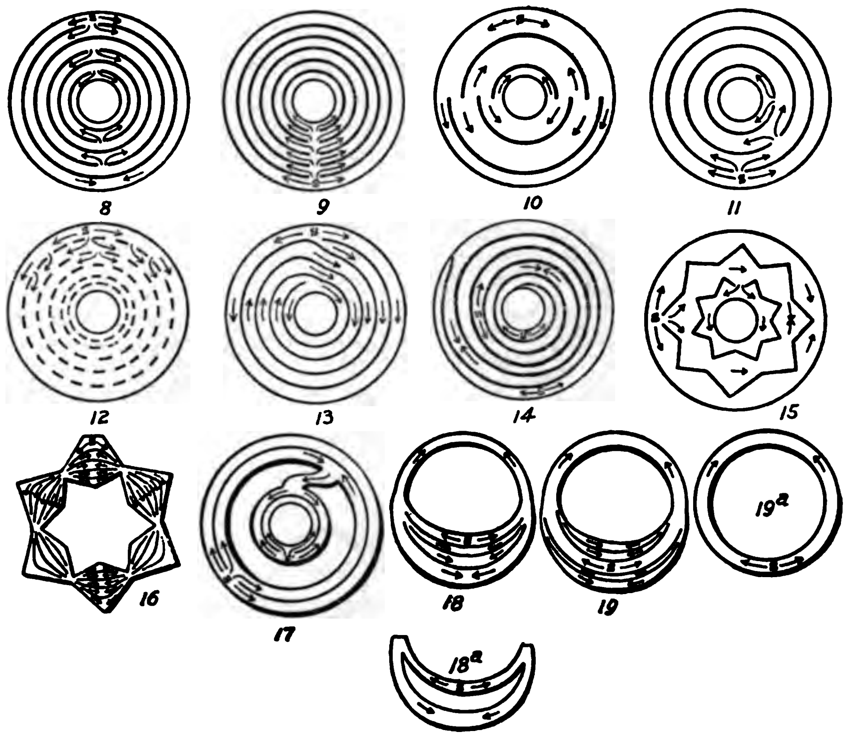
plished by cutting off all marginal sense-organs, and then making a series of concentric, discontinuous, ring-like cuts through the muscular tissue of the sub-umbrella, as is shown in figures 8 to 19.* Then upon stimulating the disk in any manner it instantly springs into rapid rhythmic pulsation, so regular and ceaseless as to remind one of the movement of clockwork. The cuts must be so made as to permit a free passage of contraction waves through sub-umbrella tissue forming a closed circuit. The simplest circuit is, of course, a single ring

* A glance at figure 7 will show that the muscular area of the sub-umbrella is a wide annulus with the mouth-arm disk and stomach in the center. In figures 8 to 33 we have represented the disk as a circle, the small concentric circle at the center being the mouth-arm disk, while the wide annulus is the sub-umbrella.

(annulus) of sub-umbrella tissue; and such a ring can readily be set into sustained pulsation.

It is not necessary, however, that cuts be made through the sub-umbrella tissue; for mere pressure prevents the transmission of contraction waves across the pressed region, and we may form circuits by pressing lightly upon the sub-umbrella with a concentric series of metallic rings, as is shown in figure 6. Then upon stimulating the disk in any manner it pulsates rhythmically.

Disks which have been cut, or pressed, as described above do not pulsate until they have been momentarily stimulated at some definite



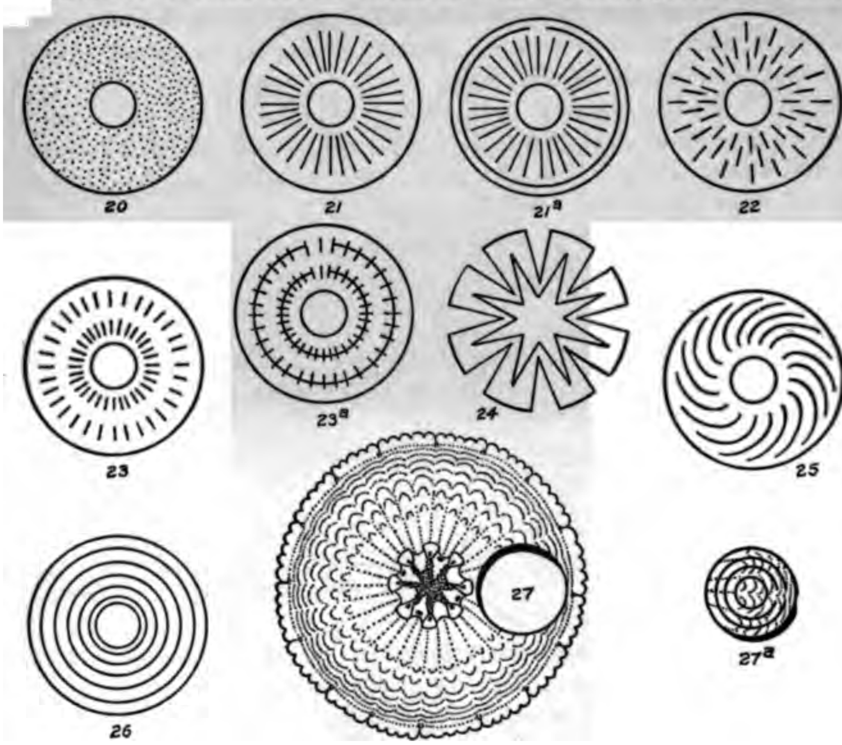
Figs. 8-19a,—Shapes cut from disks without marginal sense-organs. These will pulsate continuously in sea-water.

point by a touch of some potassium or sodium salt, a mechanical or electrical shock, or by suddenly cutting off the last remaining sense-organ immediately after it has sent out its contraction wave.

A contraction wave travels outward from the stimulated place through the circuit of sub-umbrella tissue, and when it returns to the point whence it started it is immediately reinforced, and again sent

ıgh the circuit. Thus there is normally but one contraction wave
:h proceeds from its center, travels through the labyrinth of sub-
,rella tissue, and returns to the center whence it came, only to be
n augmented and sent forth.

ιι is thus the function of the center to reinforce and maintain the
ıtraction wave. This is well shown in a long circuit such as is
wn in figure 30, I–III; where on account of the great length of
circuit the course of the wave may readily be followed by the eye.
: outer annuli of the sub-umbrella tissue are more sensitive, and
ıduct contraction waves* better than do the inner parts of the disk;



Figs. 20, 21, 22, 23, 25, 27a, disks cut so that they can not be set into continuous pulsation.
Figs. 21a, 23a, 24, 26 can be set into sustained pulsation in sea-water.

and if we touch the disk at A, figure 30, I, the greater part of the contraction wave takes the short path of *least resistance* into the interior of the labyrinth, as is shown by the full arrow, and only a very weak wave goes in the direction of the dotted arrow. The strong contraction

* The sub-umbrella tissue is a good conductor of electricity, but the gelatinous substançe of the Medusa is a poor conductor.

wave then proceeds as is shown by the sequence of arrows and numbers until it finally returns with lowered amplitude to the center, where it is instantly restimulated and again sent through the circuit with its energy restored. The same conditions apply to figures 31, II and III.

When in regular pulsation we always find that the waves of contraction start from a definite place. The position of this center tends to bear a certain relation to the geometrical figure formed by the cuts. It is marked S in figs. 8 to 19*a*, and the arrows show the observed courses of the wave of pulsation. Usually the center of pulsation lies near the periphery of the disk at a place where the tissue is widest and least interfered with by cuts, and it also tends to lie upon the axis of bilaterality of the labyrinth of tissue.

If we stimulate the disk by dropping it upon a glass plate, etc., the waves of pulsation start from the point S; and this is the place where we must touch the disk if we wish to stimulate it into sustained pulsation. *Wherever* we touch the disk with a crystal of K_2SO_4, waves of contraction immediately start out from the touched point, but it is usually impossible to establish a permanent center of

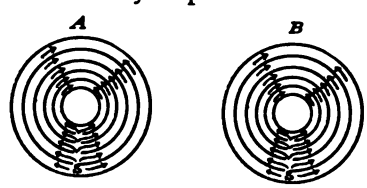


Fig. 28.

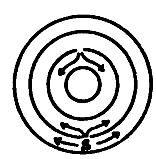


Fig. 29.

pulsation at any point other than one upon the geometrical axis of the figure. Centers at other places either cease to initiate pulsations when the effect of the initial stimulus dies out, or the center quickly shifts to the geometrical axis. Sometimes, however, when a disk is stimulated by a severe mechanical shock, two or more permanent centers of pulsation appear and waves of contraction start out from each independently and interfere where the opposing waves meet one another. Such conditions are shown in figures 14 and 17.

It will be observed that with the exception of the very elongate spiral (fig. 14) all of the labyrinths formed by the cuts are *closed circuits*, the tissue being merely a more or less complicated circuit, with the center of pulsation at the geometrical center of the figure. *After* the disk has begun to pulsate we may cut away portions of the labyrinth, and the part containing the center will still pulsate, *provided* it remains a closed circuit. Thus the crescent (figure 18*a*) is cut out from figure 18 and the ring (figure 19*a*) is made from figure 19,

by cutting them out after the more complicated circuits had been set into pulsation. Instead of simplifying the pulsating labyrinth, we may increase its complexity, but as long as the waves proceeding from the center can find a single uninterrupted circuit, the figure pulsates. Thus, a disk cut as in figure 28, A, is set into pulsation and then all of the inner rings are cut so as to be converted into "cut-off" paths as in figure 28, B; but the disk continues to pulsate until we cut across the outermost ring, when it stops instantly. Every one of the forms shown in figures 8 to 19a can be thus stopped by even the smallest cut which breaks the last circuit, although they continue to pulsate despite any cutting which does not sever the circuit. Thus, figure 16 stops at once if we cut across one of the narrow places between the rays of the star.

The center of pulsation usually establishes itself in a large uncut area, but once it be established we may greatly cut down this area and not interfere with the center. Thus, the ring shown in figure 19a may be thinned by cutting at *S*, but the center remains undisturbed.

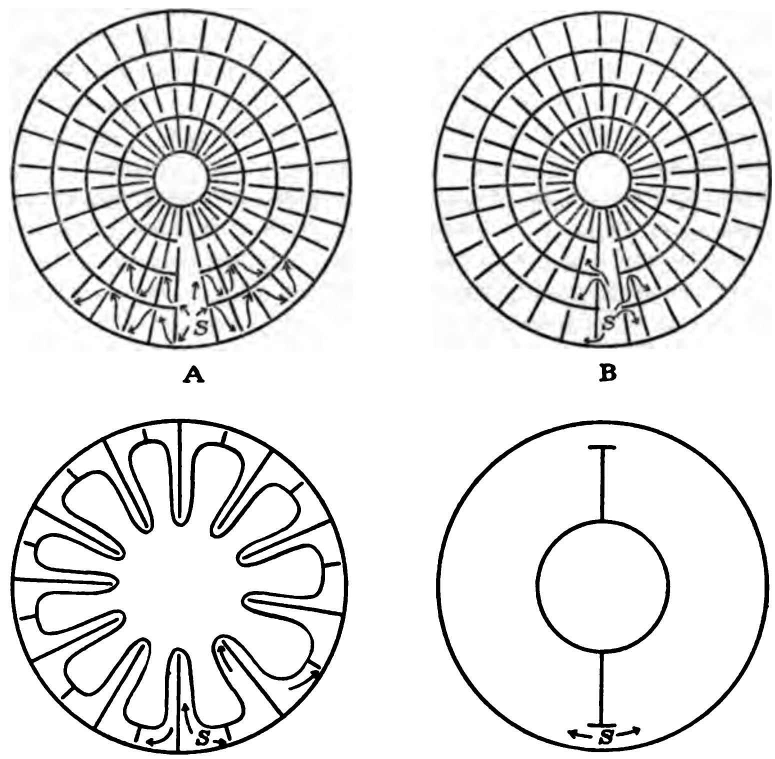
Fig. 30.—Very elongate circuits showing that the peripheral parts are better conductors of pulsation than are the inner parts of he sub-umbrella. These circuits can be caused to pulsate continuously.

Sustained pulsation without marginal sense-organs can be maintained only in tissue forming a closed circuit. These circuits may be complex and constricted at intervals to mere thread-like connectives, as in figures 31, A-C, where every annulus is crossed by radial cuts; or they may be very simple, as in figure 31, D. The circuits may either cross or trend with the muscle fibers.*

On two occasions disks were set into sustained pulsation when only the marginal sense-organs were cut away; no other cuts having been

*The statement in my preliminary paper in the Carnegie Institution Year Book for 1905 that the circuits must trend with the muscle fibers is erroneous.

made. This can rarely be accomplished, however, for the returning wave must usually be focused back upon the center in order to be sustained; and in a wide annulus it is dissipated and returns with too little force to call forth the latent ability of the center to restimulate the wave. Similarly figures 20, 21, 22, 23, and 25 represent forms which dissipate and confuse the contraction wave, setting up "eddy currents" which weaken the wave and prevent its returning definitely



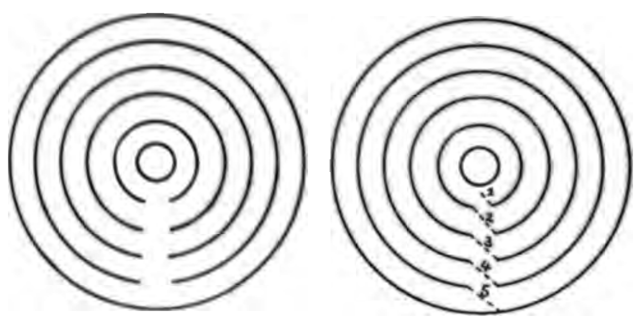
A B C D

Fig. 31.—A, B, and C, disks having every annulus crossed by radial cuts, but which may be set into sustained pulsation. D, a simple circuit which may be set into sustained pulsation.

and forcefully to the center. Hence these figures can not be set into sustained pulsation. If, however, we cut partial rings, as in figures 21*a* and 23*a*, or convert figure 21 into a shape such as is shown in figure 24, we find no difficulty in setting them into sustained pulsation. In all of these cases the figures oblige the contraction wave to return definitely and forcefully to the center. I could not obtain sustained pulsation in a disk cut out of the side of a Medusa as in figures

27, 27*a*. This, I believe, is due to the fact that the contraction wave returns so quickly to the center that an insufficient time elapses before the center is again called upon to restimulate the wave. As Romanes showed, an appreciable interval of time must elapse before tissue which has been in contraction can again contract.

Very elongate, many-whorled spirals, such as one sees in figure 14, are the only forms *not* closed circuits that we have succeeded in setting into constant pulsation. This occurs only when two or more centers arise simultaneously in the spiral, as in *S*, *S'*, and *S''*, figure 14. These centers mutually sustain one another, the contraction wave from one being restimulated and reflected back from the other. If one attempts to convert a series of partial rings (fig. 32, A) into a spiral by successive cuts, as shown in the dotted lines, 1–5, (fig. 32, B) the tissue ceases to pulsate as soon as the final cut (5) is made which breaks the last circuit.



A B

Fig. 32.—Showing that sub-umbrella tissue can not maintain itself in pulsation unless it has the shape of a closed circuit. If cuts be made as shown in the dotted lines in the order 1, 2, 3, 4, 5, the tissue ceases to pulsate as soon as cut number 5 breaks the last complete circuit.

It must be borne in mind that cuts through the sub-umbrella tissue heal over in the course of a day or two and will then transmit pulsation more or less imperfectly across the healed lines, and then a spiral will pulsate, for it is, physiologically speaking, only a series of concentric rings of readily conducting tissue with numerous more or less imperfect points of conduction between the annuli. Similarly a disk having *complete* circular cuts through the muscular tissue of the sub-umbrella, such as is shown in figure 26, can not be made to pulsate continuously as a whole until two or three days after the operation, although each annulus may be made to pulsate independently. After several days of healing the cuts will allow a more or less imperfect conduction of impulses across from one ring to another, and the con-

traction waves will be unimpeded circumferentially, but more or less hindered radially. That this is the true explanation of the matter is proven by the fact that the disk shown in figure 12, wherein the circumferential cuts are numerous and the spaces between are as wide as the cuts are long, will pulsate continuously.

Mere mutilation of a disk without sense-organs will not cause it to become capable of continuous pulsation. Thus the disk shown in figure 20, having about 800 punctures made through its sub-umbrella tissue, can not be set into a sustained rhythm.

Although I had several hundred paralyzed disks of *Cassiopea* capable of being set into pulsation by a stimulus, such as a momentary touch of a crystal of K_2SO_4, only one of these started into pulsation of its "own accord." Ordinarily they might remain for days in the aquaria awaiting the momentary stimulus which alone could call forth their latent power of rhythmical pulsation.

If disks without marginal sense-organs be set into rhythmical pulsation they move with machine-like regularity, without pauses, and without any of the irregularities shown by normal Medusæ with sense-organs intact. Their rates of pulsation are not only practically uniform, but they are much faster than are those of the uninjured normal Medusæ from which the disks were prepared, as will be shown by the following table:

TABLE 2.—*Rate at which normal Medusæ of Cassiopea pulsated and the rates of pulsation of their disks when the sense-organs were excised and circumferential cuts were made in the sub-umbrella.*

Rate of pulsation of the normal Medusa before operation.	Rate of pulsation of disk without sense-organs.	Figure showing the form of the cuts made in the disk.
25–30	77–88	8
32	63–78	9
40	183	12
51	85	13
15–20	Outermost center S .. 117 Mid-region center S′. 101 Innermost center S″. 78	14
47	Outermost center...80-82 Inner center 66-68	17

When disks without sense-organs are set into pulsation we may reduce the area of pulsating tissue by cutting parts of it away, but the rate of pulsation will remain constant, provided we do not alter the *length* of the circuit through which the wave must pass. If, however, we make cuts in such manner as to increase the length of the circuit

the rate of pulsation becomes slower. For example, twenty disks were cut as shown in figure 33, A, and after they had been set into pulsation they were cut across as shown in figure 33, B. This cut made the circuit twice as long as it was formerly, and obliged the contraction wave to travel double the distance in order to traverse the circuit.

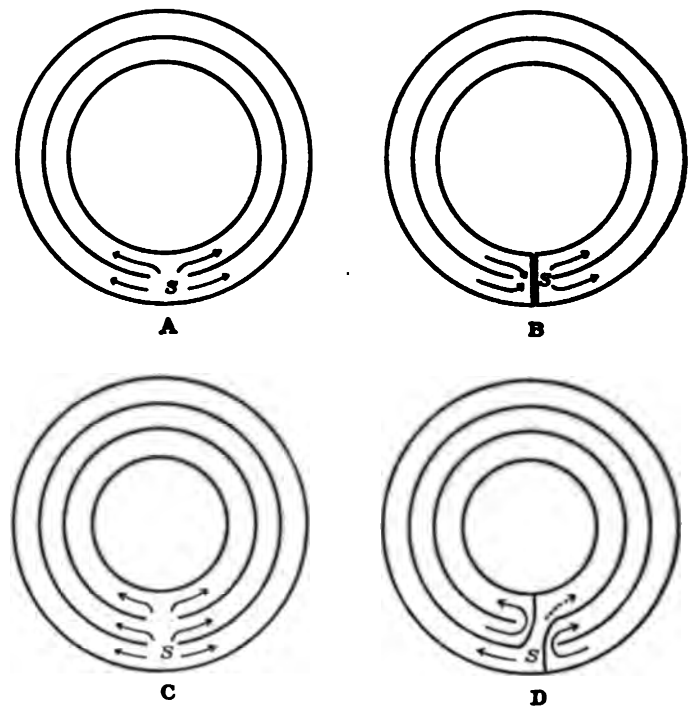


Fig. 33.—Showing how cuts may be made so as to increase the length of the pulsating circuit, thereby decreasing its rate of pulsation.

We might then expect the pulsation to be reduced to one-half its former rate, but as a matter of fact the wave traveled on an average 1.16 times as fast in the long as it did in the short circuit, so that the cut reduced the rate to but 58 per cent of its former value.

Similarly, if we set disks cut as shown in figure 33, C, into pulsation, and then make two cuts as shown in figure 33, D, making the circuit almost three times as long as it was before, the rate becomes about 0.4 of its former value, not 0.33 as we would expect. I believe that the faster rate of the contraction wave in the long circuit is due to the longer rest which the tissue enjoys, thus allowing it the more completely to recover and regain its sensibility to the stimulus which calls forth the contraction. Romanes showed that strong contraction

waves travel faster than weak ones, and that strong stimuli repeated at short intervals soon tired the tissue, so that it failed to respond.

The rate of pulsation of disks is greater than their most excited rate when the sense-organs are intact; in other words, the disk itself can maintain pulsation at a faster rate than can the marginal sense-organs. The rate of pulsation in the disks deprived of sense-organs depends simply upon the time required for the waves to traverse the circuit and restimulate the center. The wave travels faster through peripheral than through the inner annuli of the disk. When pulsating disks are suddenly seized, moved, or otherwise stimulated, the *amplitude* of their rhythmical movement suddenly increases, but the *rate* remains practically the same, and thus the presence of the marginal sense-organs is not necessary for the display of excitement. The disks of small Medusæ pulsate at a faster rate than do those of large ones, other things being equal.

These pulsating disks may continue to give regular rhythmical contractions in sea-water for 140 hours or more, but at the end of that time, if they have been deprived of their mouth-arms and central stomach, they become exhausted, and the *amplitude* of their pulsation decreases, although the *rate* remains practically constant. Suddenly the center fails to restimulate the returning wave, all movement ceases, and the disk can not be re-stimulated until after a period of rest. Indeed, the tissue appears much exhausted and responds feebly even to the strongest stimuli, such as K_2SO_4, KCl, etc. Complete recovery takes place, however, in normal sea-water, so that disks may be maintained in condition to pulsate for weeks.

While in sea-water it is almost impossible to set a Medusa, with marginal sense-organs intact, into any form of pulsation other than that controlled by the sense-organs. If, however, we cut partial rings in the sub-umbrella of a *Cassiopea*, leaving the sense-organs and margin intact, and then place the Medusa in a solution resembling sea-water but *lacking* calcium,* all pulsations will cease in from 2 to 6 minutes. Then, after the Medusa has remained motionless in the solution for one hour, if we touch the disk for an instant with a crystal of K_2SO_4 it immediately springs into a rapid rhythmical pulsation at a much faster rate than that previously maintained by the sense-organs. This pulsation, indeed, exhibits all of the features shown by disks *without* sense-organs, and therefore we see that the absence of calcium has

*965 H_2O + 26.74 NaCl + 3.75 $MgCl_2$ + 1.64 $MgSO_4$ + 0.85 K_2SO_4 + 0.07 MgBr, or Van 't Hoff's solution consisting of 100 NaCl + 2.2 KCl + 7.8 $MgCl_2$ + 3.8 $MgSO_4$, all of ⅝n concentration.

caused a paralysis of the *marginal sense-organs*, but *not* of the sub-umbrella tissue of the *disk*.

This we can prove directly, for disks without sense-organs, once they be set into pulsation, will continue to pulsate for *over three hours* in a solution resembling sea-water but lacking calcium. The amplitude of their pulsations, however, decreases steadily, but may be *restored* by adding calcium to the solution. It is evident that the central parts of the sub-umbrella of *Cassiopea* may pulsate both in normal sea-water, and for a long time in sea-water deprived of calcium, whereas the marginal sense-organs are quickly paralyzed by a deficiency of calcium in the sea-water. On the other hand, perfect Medusæ and disks deprived of sense-organs will pulsate in sea-water at 82° F. containing $CaSO_4 + CaCO_3$ to saturation, the only effect being a slight slowing of the rate of pulsation in the case of the perfect Medusæ. Hence the marginal sense-organs require calcium* to perform their function, whereas the general tissue of the sub-umbrella is relatively unaffected by the presence or absence of calcium. This is, however, a *relative* matter, for while the lack of calcium produces less effect upon the disk than upon the sense-organs, nevertheless the disk itself will *finally* cease to pulsate in the absence of calcium. It is interesting to observe that while the disk is almost unaffected by a wide range in the amount of calcium in the sea-water, it is very quickly affected by a change in the amount of the potassium. Such disks cease to pulsate in a few minutes either in a solution resembling sea-water but *lacking* potassium or in a solution of ¼ gram K_2SO_4 in 100 c.c. of natural sea-water. Indeed, the center of the disk is fully as sensitive to changes in the amount of potassium in the water as is the entire Medusa.

Under normal conditions pulsation is controlled by the marginal sense-organs, the rate being that of the fastest working sense-organ. The general sub-umbrella surface has considerable influence in sustaining the sense-organs, for if we reduce the area of the sub-umbrella enervated by the sense-organs the rate declines. Normally the pulsation is controlled by the sense-organs, not by centers of pulsation in the undifferentiated sub-umbrella tissue. Among thousands of normal Medusæ I observed only two individuals in which a center in the sub-umbrella controlled the pulsation. These two were pulsating slowly when I lifted them out of water and threw them forcibly back. They instantly began to pulsate in the rapid, uniform, clockwork-like manner characteristic of pulsation maintained by a center in the sub-umbrella, their rates being fully four times as great as the normal. I then cut off their marginal sense-organs, and the disks still continued

* The chief rôle of calcium is to counteract the anæsthetic effects of magnesium.

to pulsate without alteration in their rates. They both ceased instantly as soon as a radial cut was completed from center to margin, thus breaking the circuit of the waves of contraction.

We have seen that a center of pulsation in the undifferentiated sub-umbrella tissue sends out its stimulus only when the contraction wave returns to it through the circuit, and that therefore the rate must be constant, for it depends only upon the length of the circuit and the rapidity of the wave; and no pulsation can be maintained by a center in the sub-umbrella tissue unless the contraction wave can pass through a circuit and finally travel back to restimulate the center.

The marginal sense-organs behave differently. They send forth the stimulus, which produces contraction, at a slow, irregular rate, and they are not restimulated into immediate action by a returning wave, and can maintain tissue in pulsation even if its shape is not that of a closed circuit. They function only when calcium is present in solution in the sea-water, and if lifted out of water and dried with blotting paper they cease in a few minutes to initiate pulsations; but if then they be moistened with distilled water containing the amount of calcium found in sea-water, they recommence pulsation. Indeed, the sense-organs behave as if a slow chemical change takes place within them, the result being a contraction-stimulus; and this state of contraction in turn reducing the built-up compounds to their original condition. Calcium has the peculiar power to offset the stupefying influence of the *magnesium* of the sea-water, but calcium is of primary importance only when magnesium is present. If magnesium be absent the presence of calcium is relatively unimportant in the pulsation of *Cassiopea*. Indeed, the Medusa pulsates longer and faster in a solution containing the amounts and proportions of $NaCl + KCl$ found in sea-water than it does in $NaCl + CaCl_2$.

Before closing the account of these experiments upon disks it should be stated that the disks of *Aurelia flavidula* and *Dactylometra quinquecirra* may also be set into sustained and regular rhythm by cutting partial rings, as has been described in the case of *Cassiopea*. These Scyphomedusæ, however, soon recover to some extent from the loss of their marginal sense-organs, and the chief difference between their usual behavior after the loss of the margin and their behavior when cut by partial rings and then set into pulsation is that in the latter case the pulsation is of machine-like regularity and without pauses, whereas under normal conditions it is irregular. *Dactylometra* is more favorable for these experiments than *Aurelia*, for *Aurelia* is extremely sensitive to mechanical shocks and to chemical stimuli. It is of interest to observe that the rate at which the tissues of the disk of

Dactylometra maintain these pulsations is only a little higher than that maintained by its marginal sense-organs. For example, a *Dactylometra* which pulsated 39 times per minute when intact pulsated 46 times per minute with perfect regularity when all sense-organs were removed and partial rings were cut in its sub-umbrella.

It will be recalled that Romanes briefly mentions a specimen of the hydromedusa *Staurophora laciniata*, in which there were three centers of spontaneous contractions after the bell margin was removed. I have not succeeded in causing the disk of *Gonionemus* to pulsate continuously by cutting partial rings in its sub-umbrella after the margin had been removed. There were, however, but a few small specimens at my disposal. As Yerkes found, the central disk of *Gonionemus*, when deprived of its margin, often gives isolated contractions without external stimulation.

III. REACTIONS OF CASSIOPEA TO CHEMICAL STIMULI.

CHEMICAL STIMULATION OF PARALYZED DISKS.

As we have seen, the loss of the marginal sense-organs paralyzes the disk of *Cassiopea*, but it still reacts strongly by contractions if the surface of its sub-umbrella be touched by certain substances, while others have no effect upon it.

Strong solutions or crystals of the following produce contractions: $KAl(SO_4)_2$, KBr, KCN, K_2CO_3, KCl, $KClO_3$, K_2CrO_4, $K_2Cr_2O_7$, $K_6Fe_2C_{12}N_{12}6H_2O$, KI, $KMnO_4$, KNO_3, KOH, $KHSO_4$, K_2SO_4, $K_2S_2O_7$; also Na_2CO_3, $NaHCO_3$, NaCl, $NaClO_3$, $Na_2HPO_412H_2O$, $NaNO_3$, NaOH, $NaSO_37H_2O$, $Na_2SO_410H_2O$, and sodium oxalate; also LiCl, $BaCl_22H_2O$, $BaSO_4$, $Ba(OH)_2$, NH_4OH, glycerin, dextrose, $CuSO_4$, Fe_2Cl_6, $PtCl_2$, and iodine, etc. Contractions are also produced by very weak solutions of the following acids: Acetic, chromic, oxalic, sulphuric, hydrochloric, picric, nitric, and formic. This effect is doubtless due to hydrogen, the only element common to all of these acids.

The following substances produce no contractions, even when the crystals themselves, or their saturated solutions, are applied to the surface of the sub-umbrella: MgBr, $MgCl_2$, $MgCO_3$, $MgSO_4$; also $CaCO_3$, $CaCl_2$, CaO, $CaSO_4$, and $SrCO_3$, $SrCl_26H_2O$, $SrSO_4$, $HgCl_2$, $FeSO_47H_2O$, CH_4N_2O.

Summarizing the above, we see that all salts of potassium, sodium, lithium, barium, and platinum produce contractions, as do also weak solutions of acids, glycerin, dextrose, ammonia, and iodine. By far the strongest contractions are produced by potassium salts, while sodium salts produce much weaker effects. Nevertheless the NaCl of sea-water is a more powerful stimulant than the potassium (K_2SO_4

or KCl), owing to its far greater amount. The salts of calcium, magnesium, and strontium do not stimulate the disk and fail to produce contractions, even when in saturated solutions.

Combinations of Mg or Ca with Na or K may or may not give contractions, for the Mg always, and Ca in some cases,* tends to inhibit pulsation. Thus a series of contractions are produced by $5K_2So_4$.-Na_2SO_4, $Na_2SO_4.3K_2SO_4$, $MgCl_2.2KCl.6H_2O$, $K_2Mg(SO_4)_2$, Na_2Mg-$(SO_4)_24H_2O$, $K_2Ca(SO_4)_22H_2O.MgSO_4$, $MgCl_2K_2SO_46H_2O$, and Mg-$Cl_2.NaCl.2H_2O$; the first named giving powerful and the last weak contractions. On the other hand, $Ca_2K_2Mg(SO_4)_2$ and $CaCl_2.2MgCl_2$.-$12H_2O$ give no contractions. The salts act in accordance with their mass-effects. It is interesting that solutions of the ashes of the Medusa will not produce contractions, although Merunowicz (1875) found that an aqueous solution of the ashes of the blood will stimulate the vertebrate heart into action.

Loeb (1905) states that Ba, Li, Na, Rb, Cs, F, Cl, Br, and I are capable of bringing about contractions in skeletal muscles; whereas K, Mg, Ca, Sr, Mn, and Co give rise to no contractions or inhibit them.

It is evident that the stimulating effects of the electrolytes are generally due to their cations rather than to their anions, but contractions may also be produced by substances which can not be ionized, such as glycerin and dextrose, and weak contractions are sometimes produced by $CaBr_2$, the effect being due to the bromine. It will be recalled that Greene (1899) and Howell (1901) also found that heart muscle will pulsate in pure solutions of cane sugar and dextrose, and I find that the heart of *Salpa* and the branchial arms of *Lepas* will also pulsate in dextrose or glycerin. In his former papers Loeb maintained that rhythmic pulsation was impossible in non-ionizable solutions, but his views appear to have changed upon this point.

EFFECTS OF CALCIUM IN RESTORING PULSATION.

We have the well-known experiment of Howell (1898) and others showing that when heart muscle has ceased to beat in Ringer's solution it may be made to beat again for a short time by adding any calcium salt. This is also true for *Cassiopea*, for the Medusa will pulsate for a short time in any solution containing Na and K in amounts found in sea-water, and then after all pulsations have ceased they can be revived by adding calcium. This is illustrated in the following list of trials (table 3), wherein if the sodium chloride was replaced by any

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*Taken alone calcium inhibits or fails to stimulate pulsation, but in combination with sodium chloride and potassium, as in NaCl + KCl + $CaCl_2$, it becomes a powerful stimulant.

other salt this was made isotonic with the NaCl of sea-water. The potassium was so introduced as always to give the same amount of the element (K) as is found in sea-water.

TABLE 3.—*How calcium revives rhythmical pulsation in Cassiopea after all movement has ceased in solutions containing Na or Li, isotonic with the NaCl of sea-water, and potassium in the same amount as is found in sea-water.*

Normal Medusæ taken from sea-water and placed in—	They ceased to pulsate in—	Were restored to pulsation by the addition of any of the following calcium salts, tried separately.
$NaCl + KCl$	About 120 minutes	$CaCl_2$, $CaSO_4$.
$NaCl + K_2SO_4$	20 to 30 minutes. Very rapid pulsation at first, followed by periods of rest and activity.	$CaSO_4$, $CaCO_3$, $CaCl_2$, or CaH_2O_2.
$NaCl + K_2CO_3$	12 to 18 minutes. Pulsation not so rapid as in $NaCl + K_2SO_4$	Very active pulsation restored by $CaSO_4$, $CaCl_2$, or CaH_2O_2.
$NaCl + KClO_3$	4 to 10 minutes. Pulsation not very rapid at first. Slower than in $NaCl + K_2CO_3$	$CaSO_4$, $CaCl_2$, or CaH_2O_2.
$Na_2CO_3 + K_2CO_3$	3 to 7 minutes. Pulsation slow and weak.	$CaCO_3$ revived weakly.
$NaNO_3 + KNO_3$	Less than 1 minute. Pulsation rapid at first.	$CaCl_2$ or CaH_2O_2. Some of the Medusæ did not revive pulsation.
$LiCl + KCl$	1 to 6 minutes. Pulsated slowly at first.	$CaCl_2$. All three Medusæ revived weakly.

Table 3 shows that *Cassiopea* pulsates longer and more rapidly in a solution of $NaCl + KCl$ than in any other solution named in the above table. Also, sodium and potassium *nitrates* are more injurious than a solution in which the sodium is replaced by an isotonic amount of lithium. Evidently the *anions* as well as the *cations* of the salts have a decided influence upon the rhythmical movement. This is also shown by the fact that Medusæ pulsate longer and with greater regularity of movement in $NaCl + K_2SO_4 + CaSO_4 + CaCO_3$ than they do if we omit the $CaCO_3$ and replace it by an equivalent amount of $CaSO_4$. It will be recalled that Rogers (1905, p. 249) found that the addition of small amounts of Na_2CO_3 or $NaOH$ to solutions have a beneficial effect in maintaining the rhythm of the crab's heart, and he attributes this effect to the neutralization of small amounts of free acid in the solutions. Ammonia, KOH, or $NaOH$ in small amounts have, however, little effect upon the rhythm of *Cassiopea*, but if the sea-water be rendered almost neutral by HCl (it is normally decidedly alkaline at Tortugas) the pulsations of the Medusæ lose energy, and finally the rate declines, and movements, although regular, are feeble and slow. Thus the rates of three Medusæ declined in six hours from 37–50 to 13–17 per minute, due to the effect of a minute quantity of HCl in the sea-water, causing it to become almost neutral, but still alkaline to litmus test. It seems improbable, how-

ever, that the addition of $CaCO_3$, which improves the regularity of pulsation of Medusæ in $NaCl + K_2SO_4$, has only the effect of neutralizing acids. Distilled water and the purest obtainable salts were used in making solutions and there is no reason to suppose that there were any more free acids in the solutions than in the natural sea-water itself.

Physiologists have generally assumed (see Howell, Text-Book of Physiology, p 502) that the chief rôle of sodium chloride in pulsation is to maintain the osmotic pressure of the solution. I find, however, that *Cassiopea* pulsates more than 24 minutes in a solution of Na_2SO_4 containing the same amount and proportion of Na as is found in sea-water; whereas it will not pulsate more than 14 minutes in a solution of Na_2SO_4 isotonic with sea-water. This would lead one to believe that the sodium of the sea-water exerts a specific action, and that the salts have a specific chemical effect independent of their osmotic action. Indeed, the various salts of sodium behave very differently; for example, *Cassiopea* pulsates less than 1 minute in Na_2CO_3, 11 to 12 minutes in $NaClO_3$, and more than half an hour in NaCl, or $NaNO_3$ isotonic with sea-water.

When pulsations have ceased in 96 c.c. H_2O + 2.7 grams NaCl + 0.085 gram K_2SO_4 they may be revived temporarily by Na_2CO_3, more NaCl, KCl, K_2CO_3, or weak acids. These cause only a few irregular contractions, however, and are quite different in their effects from the long, steady revival of pulsation upon the addition of calcium. Potassium is, however, capable of reviving temporary pulsation in any solution which lacks magnesium, but if magnesium be present it can not usually revive pulsation. It is interesting to observe that after Medusæ have ceased to pulsate in the $NaCl + K_2SO_4$ and have been revived by potassium, they will not again pulsate upon the addition of calcium to the solution. On the other hand, if pulsations have ceased and have been revived by adding more sodium, they can be revived a second time by adding calcium. Potassium in excess at first stimulates the disk powerfully, but soon it poisons the tissues and inhibits the sensibility, while calcium is not a stimulant, but is necessary for pulsation in connection with sodium and potassium. The chief rôle of calcium is, however, to counteract the inhibiting effect of the magnesium.

This is shown by the fact that if we were to place *Cassiopea* in normal sea-water, and then add sufficient sodium oxalate to precipitate the calcium, pulsation ceases in less than five minutes, but is quickly restored if we place the Medusa in NaCl + KCl + sodium oxalate, or in NaCl + KCl. Pulsation is not restored, however, if we place the Medusa in NaCl + magnesium. These experiments prove that the

pulsation is inhibited by the *magnesium* of the sea-water, not merely by the loss of calcium; for pulsation may be restored in solutions which lack calcium. They also show that when calcium is present the magnesium does not inhibit pulsation.

EFFECTS OF MAGNESIUM UPON PULSATION.

The magnesium salts in sea-water retard pulsation in *Cassiopea*, and reduce its rate, amplitude, and energy. *Cassiopea* pulsates at about twice its normal rate in a solution resembling sea-water but lacking magnesium, but if we add the magnesium to this solution the Medusa immediately pulsates at normal rates. Also, an excess of magnesium added to sea-water causes the rate and energy of pulsation to decline, although Medusæ will tolerate 1.6 grams $MgCl_2$ in 100 c.c. sea-water, and will pulsate *slowly* for half an hour without the least apparent injury, their normal rate being regained in a few minutes after they are returned to pure sea-water. Magnesium acts only as a restrainer, never stimulating the disk of *Cassiopea*. When the disk, deprived of marginal sense-organs, is placed in a solution of $MgCl_2$ or $MgSO_4$ isotonic with sea-water it does not pulsate. Indeed, the rate of pulsation of normal Medusæ in natural sea-water becomes successively slower as we add more and more magnesium.

The rôle of magnesium is, however, an essential one in pulsation, for it counteracts the strongly stimulating action of the combination of NaCl, K, and Ca which occurs in Ringer's solutions, or in sea-water. For example, if we place *Cassiopea* in a solution of $NaCl+KCl+CaCl_2$ in amounts and proportions found in sea-water * the Medusa is highly stimulated and pulsates at fully twice its normal rate. If now we precipitate the magnesium in its tissues in any manner,† the stimulating effect of the sodium, potassium, and calcium is unchecked, and after a short period of violent pulsation the Medusa passes into a strong sustained tetanus and remains motionless, with its bell highly contracted.

I find also that sustained pulsation is impossible in the heart of *Salpa* or the branchial arms of *Lepas* unless magnesium be present, and that in these cases also $NaCl+KCl+CaCl_2$ is a powerful stimulant, producing rapid but not permanently sustained pulsation, but normal sustained pulsation is attained on the addition of magnesium. It appears, therefore, that a Ringer's solution is not an inorganic food for the pulsating organ, as has been commonly assumed by physiol-

* 100 NaCl + 2.2 KCl + 3 $CaCl_2$ all of $\frac{5}{8}$n concentration, as in Van 't Hoff's solution.

† The magnesium may be precipitated by a small amount of $Ba(OH)_2$, KOH, NaOH, or sodium phosphate + ammonia + ammonium chloride, etc.

ogists, but is only a stimulant which in the end produces injurious effects by the withdrawal of magnesium through osmosis. It can not sustain permanent pulsation unless a certain proportion of magnesium be present to preserve a balance.

It is interesting to see that Meltzer and Auer (1905–'06) find that magnesium affects the nervous system in such manner as to produce in mammals a deep anesthesia, with relaxation of all the voluntary muscles. It is inhibitory, never stimulating in its effects, but it does not interfere with the trigeminal reflex inhibition of respiration. Also, Carlson (1906) finds that magnesium and calcium depress the ganglionic rhythm of the heart of *Limulus* without primary stimulation. Indeed, the anesthetic effects of magnesium salts upon aquatic animals have been known since Tullberg's researches in 1892.

Macallum (1903) finds that there is about 10 per cent less magnesium in the bodies of *Cyanea* and *Aurelia* than in sea-water. Rogers (1905), however, found that the optimum solution for the continuance of rhythmic movement of the crab's heart contains fully as much magnesium as the sea-water.

Loeb (1906) finds that in *Polyorchis* the $NaCl + KCl + CaCl_2$ of sea-water produce sustained contraction without pulsation, and that magnesium is necessary in order to overcome the tetanus and permit of rhythmical pulsation. Also, this effect of magnesium can be inhibited by the addition of an equivalent amount of calcium or potassium. Also, Romanes (1885) found that the vigor of the swimming movements of *Sarsia* is impaired in a pure NaCl solution of the same strength as that of the sodium chloride in sea-water, but that this vigor of movement is *somewhat restored* by adding $MgSO_4$ to the same amount found in sea-water. In the case of *Cassiopea* all movement would cease in less than six minutes in $NaCl + MgSO_4$ in amounts found in sea-water; whereas irregular pulsation continues for half an hour in NaCl alone, although after that the Medusæ would show periods of quiescence alternating with periods of pulsation. I find also that 1 per cent magnesium added to sea-water slowly lowers the rate of the rhythmical movement of the arms of *Lepas*. It seems probable, therefore, that magnesium, while always inhibitory, plays a somewhat different rôle in the efficiency of its control over rhythmical movement in various animals.

EFFECTS OF POTASSIUM UPON PULSATION.

Potassium in small amounts temporarily stimulates and then retards pulsation. Unlike magnesium or calcium in excess, it is quite poisonous. All potassium salts, with the exception of those consisting

of combinations of potassium with magnesium and calcium, are powerful stimulants to the disk of *Cassiopea*, causing strong but temporary contractions. Repeated touches of a crystal of K_2SO_4 to any one spot on the sub-umbrella of *Cassiopea* soon renders the place insensitive to further stimulation of any sort. For example, a single spot upon a disk, deprived of sense-organs, was touched 17 times, in rapid succession, with a crystal of K_2SO_4 and each time a contraction resulted. The next 2 touches, however, gave no contractions; then followed 2 touches with contractions, 7 without contractions, 1 with, and finally 11 without contractions, etc.

If normal *Cassiopea* with sense-organs intact be placed in sea-water +0.125 to 1.55 per cent K_2SO_4, $KClO_3$, KCl, or K_2CO_3 they immediately pulsate at an abnormally high rate, but the movement soon loses force, and the disk comes to rest *expanded* with the mouth-arms strongly *contracted*. Medusæ in 0.125 per cent excess of K_2SO_4 will pulsate quickly at first and then more and more slowly, so that at the end of 13 hours their rates are only about half the normal rate in sea-water. On the other hand, Medusæ in sea-water + 1.55 per cent K_2SO_4 will pulsate with great activity for a few moments, but will cease all movement in less than 4 minutes. Also, a solution of K_2SO_4 isotonic with the NaCl of sea-water *at once reduces* the rate of pulsation of normal Medusæ and quickly brings them to rest without an initial display of excitement. It appears that a small excess of potassium acts as a temporary stimulus, whereas a large excess at once inhibits pulsation. It is possible that the initial stimulation is due to the physiological reaction of the tissues against the injurious effects of the potassium. Temporary activity is commonly called forth in animals by sudden injurious stimuli. In this connection it is interesting to see that Carlson (1906) finds that potassium is a primary stimulant for the heart of *Limulus*, but its action is quickly followed by depression.

An excess of 1 per cent potassium in the sea-water quickly lowers the rate of movement of the arms of *Lepas*, causes tetanus-like contraction, and may be fatal in 10 minutes.

The effect of potassium upon the disk without marginal sense-organs is, however, different from its effect upon the normal, perfect *Cassiopea*, for disks without sense-organs are *actively stimulated* into pulsation for a short time in all excess of potassium from sea-water + 0.25 per cent K_2SO_4 to a pure solution of K_2SO_4, or KCl, isotonic with the NaCl of sea-water. Perfect Medusæ, however, show no increase in rate of pulsation in isotonic K_2SO_4, but *steadily* decline. It seems probable, therefore, that a strong excess of potassium impairs

the activity of the marginal sense-organs sooner than it affects the disk itself. The disk without sense-organs will, however, cease to pulsate in a solution resembling sea-water but lacking potassium quite as quickly as will the perfect Medusa. It would seem, therefore, that the sense-organs and the sensory surface of the sub-umbrella are equally intolerant of a lack of potassium in the sea-water. This is interesting in view of the fact that the disk without sense-organs is relatively indifferent to calcium, or magnesium, and will pulsate either in sea-water saturated with $CaSO_4$, in normal sea-water, or for more than an hour in a solution resembling sea-water but without calcium. The Medusa with sense-organs intact, however, ceases to pulsate in a solution containing all of the elements of sea-water excepting calcium in less than six minutes, but will pulsate in sea-water saturated with $CaSO_4$. It is evident that the *accurate balance* between the proportions of calcium, potassium, and sodium insisted upon by Loeb as being *necessary* for the continuance of pulsation need not be maintained and yet pulsation may continue. As Howell has pointed out, marine animals are attuned to the sea-water in which they live, and any change in its constituents must be expected to affect them more or less adversely. Loeb's theory of the influence of ions upon pulsation, although of fundamental value, unfortunately neglects, in some measure, to consider the effects of the salts as a whole. As we shall soon see, however, *Cassiopea* will pulsate for at least 30 minutes in a pure ⅝n NaCl solution, whereas it is paralyzed in less than a minute in an isotonic solution of Na_2CO_3. Indeed, the various potassium salts stimulate in different degrees. KI, K_2SO_4, and KCl are powerful stimulants, whereas $KMnO_4$, $KAl(SO_4)_2$, and potassium metabisulphite produce weak contractions.

Matthews (1905) concludes that valence, as such, either of the anion or cation, is of secondary or no importance in determining either the toxic or antitoxic action of the salt.

Loeb (1900) concluded that the potassium and calcium ions of sea-water prevent the center of the bell of *Gonionemus* from pulsating rhythmically. His experiment, however, does not prove this point, for he found that the center of the bell of *Gonionemus* would pulsate in ⅝n NaCl, but not in sea-water; and thus he concluded that the K and Ca of sea-water inhibited pulsation,* but he neglected to consider

*While this paper was in press Loeb (1906: Journ. Biol. Chemistry, vol. 1, p. 431) concludes that *magnesium* and calcium inhibit the center of Gonionemus. In so far as the effect of magnesium is concerned his view now accords with the researches of Tullberg (1892), Meltzer and Auer (1905–06), and Mayer (1906) that magnesium is anesthetic or inhibitory.

the effects of magnesium. I find, indeed, that the center of the bell of *Gonionemus does* occasionally pulsate spontaneously in sea-water, and always pulsates actively whenever one touches it with a crystal of KCl or K_2SO_4. It is not stimulated by the sea-water, but the inhibitory effect of the sea-water is probably due to magnesium, not to potassium or calcium. The center of *Gonionemus* is strongly *stimulated* by Na salts, and the reason it pulsates in $\frac{5}{8}$n NaCl is that *magnesium*, as well as calcium and potassium, is withdrawn from the tissues by osmosis by the pure NaCl solution, thus giving a preponderating influence to the Na, which acts as a stimulant. Indeed, Loeb himself found that the center of *Gonionemus* pulsates *slowly* in 96 c.c. $\frac{5}{8}$n NaCl + 2 c.c. $\frac{5}{8}$n KCl + 2 c.c. $\frac{10}{8}$n $CaCl_2$. I also find that *Gonionemus* pulsates slowly but *without pauses* in a solution resembling sea-water* but lacking *magnesium* salts. The characteristic pauses which occur periodically in the normal pulsation of *Gonionemus* are thus due to magnesium. Magnesium fails to stimulate the center of *Gonionemus*, and, indeed, if the center be touched with $MgSO_4$ or $MgCl_2$ it deadens the part touched, so that it responds weakly or not at all to such powerful stimuli as the touch of a crystal of NaCl or K_2SO_4. The disk of *Cassiopea* deprived of sense-organs behaves exactly as does *Gonionemus*, for it does not pulsate spontaneously in sea-water but does so in $\frac{5}{8}$n NaCl, or in any solution containing NaCl + K or Ca, but lacking *magnesium*. If, however, we stimulate it with KCl or K_2SO_4 it gives some active pulsations in sea-water; or better still, if we cut partial rings in its sub-umbrella and then stimulate it mechanically by a shock, it pulsates indefinitely in sea-water.

It is significant that the disks of *Aurelia* and *Dactylometra*, when deprived of marginal sense-organs, still pulsate irregularly in sea-water; and the disks of both of these Scyphomedusæ sometimes respond by weak contractions to $MgSO_4$ and $MgCl_2$.† They therefore pulsate in sea-water as soon as they recover from the shock-effects resulting from loss of their marginal sense-organs, because their disks are *stimulated* by everything (Na, K, Mg) in the sea-water, except the calcium, which, taken singly, exerts only a slight inhibitory action. In the case of *Cassiopea*, *Gonionemus*, and *Polyorchis* the sea-water is a balanced fluid. Na stimulates while Mg inhibits pulsation. Ca in *connection* with Na and K is necessary to, and stimulates, pulsation.

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* 96 c.c. H_2O + 2.7 grams NaCl + 0.124 $CaSO_4$ + 0.01 $CaCO_3$ + 0.065 K_2SO_4.

† These reactions are so irregular and the Medusæ so extremely sensitive to mechanical effects that I am in doubt concerning the validity of this statement. It may be that the occasional response is due to some chemical shock-effect.

The disk of *Cassiopea* does not pulsate in sea-water, because the sea-water as a whole does not stimulate it. Disks of *Aurelia* and *Dactylometra* behave in sea-water as if they were weakly stimulated.

Howell (1901, pp. 200, 204) concludes as a result of his own work and a review of the labors of others that potassium acts somewhat as an inhibitory agent upon the rhythmical pulsation of the heart muscle of the ventricle of the terrapin, for it lengthens the period of diastole, causing the rate to become slower,* but at the same time the heart muscle pulsates longer when potassium is present than it does when only sodium and calcium are present. A small excess of potassium in physiological doses is not toxic in its effects, yet it inhibits the pulsation of the heart muscle; but the muscle will beat again in solutions containing less potassium or more calcium. Other physiologists conclude that small amounts of potassium stimulate "the vertebrate heart." (See Carlson, 1906, p. 397.)

It is interesting to observe that Macallum (1903) finds that the bodies of *Cyanea* and *Aurelia* contain considerably more potassium than does sea-water. He found the various elements to exist in the following proportions:

	Na.	Ca.	K.	Mg.
Sea-water........	100	3.84	3.66	11.99
Cyanea arctica ...	100	3.86	7.67	11.31
Aurelia flavidula..	100	4.13	5.18	11.43

GENERAL INFLUENCE OF CALCIUM UPON PULSATION.

Calcium is essential for pulsation on account of its power to counteract the inhibiting influence of magnesium. Its importance in connection with sodium and potassium in maintaining pulsation has been known since Ringer's important experiments in 1883.

If we place perfect Medusæ of *Cassiopea*, with marginal sense-organs intact, in a solution resembling sea-water but merely lacking calcium,† the Medusæ pulsate more and more weakly, and all movement ceases in less than 6 minutes. The Medusæ are not poisoned, however, for if, after remaining motionless for fully an hour we add calcium to the solution, or restore the Medusæ to sea-water, pulsation is resumed almost at once, beginning feebly at first but rapidly regaining its normal vigor in a few minutes.

*I find that in the embryo loggerhead turtle, 14 days old, the heart pulsates *faster* in NaCl + KCl than it does in pure NaCl.

†96 c.c. CH_2O + 2.7 grams NaCl + 03.7 gram $MgCl_2$ + 0.16 gram $MgSO_4$ + 0.085 gram K_2SO_4, or 100 NaCl + 2.2 KCl + 7.8 $MgCl_2$ + 3.8 $MgSo_4$, all of 5/8n concentration.

The Medusæ are, however, inhibited from pulsating by the *presence* of magnesium, not by the mere *absence* of calcium; for if magnesium be absent, calcium may also be absent and the Medusæ will pulsate fully two hours.

A large excess of calcium lowers the rate of pulsation of *Cassiopea*, after a momentary increase. The inhibitory effect of calcium is, however, far less marked than that of magnesium, or than the final toxic effect of potassium. For example, if we add $CaSO_4$ + $CaCO_3$ to sea-water at 82° F., to saturation, normal perfect Medusæ of *Cassiopea* pulsate at about two-thirds their normal rate after being in this solution 12½ hours. One gram of $CaCl_2$ in 100 c.c. sea-water also slightly reduces the rate of pulsation without injurious effects, recovery being almost immediate in normal sea-water. Perfect *Cassiopea* with sense-organs intact when placed in a pure solution of $CaCl_2$ isotonic with the NaCl of sea-water ceases to pulsate in 10 seconds, and can not be restored to pulsation by being placed in NaCl + K_2SO_4 in amounts found in sea-water. A strong solution of K_2SO_4 in NaCl, however, revives them into active pulsation. Evidently their sensibility to stimuli is impaired but not destroyed.

Calcium salts never stimulate the disk of *Cassiopea* into pulsation, even when placed upon it in concentrated solutions.

We see that calcium, while not of itself a stimulant, is *necessary* to pulsation and is a stimulant *in connection* with sodium and potassium. An excess of calcium tends to retard pulsation, but even a saturated solution of $CaSO_4$ in sea-water exerts no appreciable toxic influence. It is far more important to pulsation than potassium; for *Cassiopea* will pulsate for more than an hour with irregular periods of rest and activity in the absence of potassium, but in the absence of calcium pulsation ceases in less than 6 minutes. This importance is due solely to the remarkable ability which calcium has to counteract the inhibiting effect of magnesium.

EFFECTS OF SODIUM UPON PULSATION.

All of the sodium salts are *weak* stimulants to the disk of *Cassiopea* deprived of its marginal sense-organs, producing not very powerful contractions. The sodium salts, however, vary considerably in their stimulating power, NaCl or NaOH giving strong and Na_2CO_3 or Na_2SO_4 weak contractions.

The disk of *Cassiopea* deprived of marginal sense-organs pulsates for about 20 minutes in a pure ⅝n NaCl solution, and also in NaCl + K_2SO_4 or NaCl+K_2SO_4+$CaSO_4$ or NaCl+$CaSO_4$.*

* The proportions of Na, Ca, and K were such as are found in sea-water.

It will not pulsate, however, in $NaCl + MgSO_4$ or $MgCl_2$ or both, and it is evident that the magnesium salts contained in sea-water counteract the stimulating effect of the sodium. Disks that have ceased to pulsate in 5/8n NaCl will revive a few pulsations if supplied with calcium, or with a strong *excess* of potassium, or both, but no revival results when magnesium is added to the NaCl solution. Indeed, it may be said of the sea-water that the chief stimulant, owing to its large amount, is sodium chloride, and the chief inhibitor of pulsation is the magnesium. As is well known, however, pure sodium chloride solutions can not sustain pulsation, for in all known cases of rhythmical movement from that of Medusæ to that of the vertebrate heart, calcium and potassium must be associated with the sodium, and I find that *magnesium* must also be present to *restrain* the highly stimulating influence of the combination of sodium, calcium, and potassium. Indeed, in order to pulsate rhythmically an organ must be in that delicately balanced state known to physiologists as being upon the threshold of stimulation. When in this condition a constantly accumulating internal stimulus, which is reduced at each contraction, will maintain rhythmical pulsation.

Normal Medusæ of *Cassiopea* with marginal sense-organs intact will pulsate for a short time with abnormal rapidity in a pure 5/8n NaCl solution, but their rate quickly declines so as to become abnormally slow, and in about 10 minutes they begin to pulsate only at intervals with longer and longer periods of rest between periods of pulsation. Practically all movement ceases at the end of about 30 minutes. Little or no toxic effect is produced, however, for recovery is almost instantaneous in sea-water, and pulsation can be revived, even after several hours, by the addition of any calcium salt to the NaCl solution.

Pulsation of normal *Cassiopea* ceases in 1 to 6 minutes in a solution containing the amounts of NaCl and $MgSO_4 + MgCl_2$ found in sea-water, but it can sometimes be revived temporarily by adding potassium, or always by the amount of calcium found in sea-water.

Normal Medusæ of *Cassiopea* are but little affected by an excess of NaCl in the sea-water, and will pulsate for more than 18 hours in sea-water + 1 per cent excess of NaCl. Their pulsation, however, becomes somewhat irregular, although of practically normal average rate, but the mouth-arms are strongly and abnormally contracted. Recovery in sea-water is, however, very rapid and no apparent toxic effects are produced. A Medusa in sea-water + 1.55 per cent excess of NaCl pulsates with abnormal rapidity for half an hour, and although shriveled, recovers quickly on being replaced in normal sea-water.

When we proportionately reduce the sodium chloride and magnesium, but at the same time maintain the amounts of calcium and potassium of the sea-water, the rate of pulsation and general energy of the Medusæ steadily decline. This was done by diluting sea-water with distilled water containing the amounts of calcium and potassium found in sea-water, as is described on page 18. If we simply dilute the sea-water with distilled water the rate of pulsation does not decline so rapidly, and the injurious effects are not so pronounced.

These experiments show that a relative excess of Ca and K retards pulsation, even when the *actual amounts* of Ca and K are such as are found in sea-water.

Cassiopea will pulsate longer in $LiCl + K_2SO_4 + CaSO_4$ than in a solution wherein the NaCl is replaced by Na_2CO_3. In these solutions the LiCl and Na_2CO_3 were isotonic with the NaCl of sea-water, while the amounts of K and Ca were the same as are found in sea-water. The Medusæ ceased pulsating in about 6 minutes in the LiCl solution, but it seems somewhat remarkable, in illustrating the effects of *salts as a whole*, that LiCl should replace the NaCl with less injury than Na_2CO_3.

We have seen that NaCl in excess or in pure solutions has very little toxic effect upon *Cassiopea*. This appears remarkable, for its marked toxic effects have been made known by Loeb, Lingle, Cushing, and others upon a number of animals, and I find that pure solutions of NaCl have a very rapidly injurious effect upon the movement of the branchial arms of *Lepas*. We must remember, however, that *Cassiopea* normally lives in semi-stagnant salt-water lagoons where considerable range in density must take place through evaporation and rainfall. It is also one of the most hardy of marine animals and will survive without serious effects several minutes' immersion in sea-water containing such poisons as 0.1 per cent KCN.

It will be recalled that Macallum (1903) found that while the amount of NaCl in brackish estuaries might change greatly with the condition of the tide, the amount of NaCl in the bodies of the *Aurelia* and *Cyanea* remained practically constant. It is therefore possible that *Cassiopea* may resist osmosis of NaCl to some extent and thus avoid its possibly toxic influences.

We conclude that NaCl is a stimulant and is counteracted in this respect by the magnesium of sea-water so as to produce a balanced solution. It can not maintain pulsation except in *connection* with calcium and potassium, in combination with which it forms a powerful stimulant which produces a rapid but only temporary pulsation, magnesium being necessary to reduce and sustain its action.

ARTIFICIAL SEA-WATER AND THE EFFECTS OF THE SALTS OF SEA-WATER, AS A WHOLE, UPON PULSATION.

In the experiments upon *Cassiopea* the solutions containing some or all of the chief constituents of sea-water were made up in accordance with the formula given by Dittmar (1884)*, and also according to Van 't Hoff's formula (100 $NaCl + 2.2\ KCl + 7.8\ MgCl_2 + 3.8\ MgSO_4 + 3\ CaCl_2$, all of ⅝n concentration.

Medusæ pulsate normally in an artificial sea-water made according to Van 't Hoff's formula, but pulsation is somewhat irregular in a sea-water made according to Dittmar's formula. Table 4 shows the results of experiments with Dittmar's formula, and table 5 gives the results obtained by using Van 't Hoff's formula.

Tables 4 and 5 show the effects upon *Cassiopea* of various solutions containing one or more of the constituents of sea-water. It will be apparent that *magnesium* is the chief restrainer of pulsation, and that it prevents the spontaneous contraction of disks deprived of marginal sense-organs and retards pulsation in perfect Medusæ. When magnesium is present the absence of calcium quickly stops pulsation, but when magnesium is *absent* we may have calcium also absent and the Medusæ will pulsate for a considerable time. It is apparent, therefore, that calcium assists the NaCl to counteract the retarding influence of magnesium. This is also shown by the fact that Medusæ pulsate for a long time in Na + Mg + Ca, whereas all movement ceases very soon in Na + Mg.

Potassium, however, does not assist the NaCl to resist the stupefying influence of magnesium, for Medusæ cease to pulsate almost as soon in Na + Mg + K as they do in Na + Mg. Potassium serves mainly to stimulate movement in *connection with both* calcium and sodium; thus Na + K and Na + Ca give temporary pulsations at about normal rate; whereas Na + Ca + K gives strong pulsations at fully twice the normal rate, but these can not be sustained indefinitely unless magnesium be present to counteract the too powerful stimulating effects of the Na + Ca + K. A Ringer's solution is only a powerful stimulant, and can not sustain pulsation indefinitely unless tempered by magnesium. Potassium has little power to revive pulsation, whereas calcium possesses this power to a marked degree; thus, when pulsations have ceased in NaCl they can always be revived by calcium, but at best only a very few isolated contractions can be revived by potassium in the amount and proportion found in sea-water.

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*Reports of voyage of H. M. S. Challenger, Chemistry, vol. 1, p. 204.

TABLE 4.—*Effects upon the rhythmical pulsation of Cassiopea exerted by solutions containing Na, Ca, K, and Mg in proportions and amounts found in average sea-water.*

[1,000 grams of sea-water is supposed to contain 965.6 H_2O + 26.74 NaCl + 1.24 $CaSO_4$ + 0.12 $CaCO_3$ + 0.85 K_2SO_4 + 3.74 $MgCl_2$ + 1.64 $MgSO_4$ + 0.07 MgBr (see W. Dittmar (1884): Composition of Ocean Water, Reports H. M. S. "Challenger," Chemistry, vol. 1, p. 204).]

Composition of the solution.	Pulsation of perfect Medusæ taken from sea-water and placed in the solution.	Paralysed, quiescent disks without sense-organs, taken from sea-water and placed in the solution, behave as follows:	Effect produced upon perfect Medusæ by adding the salts to make sea-water, after the Medusæ have come to rest.
NaCl	For a few moments it pulsates at about double the normal rate. Then the rate declines, and soon alternating periods of rest and pulsation set in. The periods of rest become longer, and all movement practically dies out in 30 minutes.	In newly cut disks 15 to 30 irregular contractions set in, but these cease in about three minutes and the disk remains motionless. Disks which have regenerated for two or more days may pulsate for hours in an irregular manner.	Pulsation is always revived by adding $CaSO_4$ or $CaSO_4$ + $CaCO_3$ or $CaSO_4$ + K_2SO_4. Revival ensues from adding Ca, or Ca + K, never from Mg.
NaCl + $CaSO_4$	Effect similar to that of NaCl, but the rhythmical movement is usually more prolonged. Practically all movement dies out in about half an hour.	In newly cut disks a few pulsations during not more then 2 minutes. Regenerating disks pulsate longer.	Occasionally a few pulsations are revived by adding K_2SO_4, $CaSO_4$, or $CaCO_3$. Mg never revives pulsations.
NaCl + $CaSO_4$ + $CaCO_3$	Behavior similar to that in NaCl + $CaSO_4$.	In new disks a few weak contractions. Regenerating disks pulsate longer in an irregular manner.	K_2SO_4 occasionally revives a few pulsations; Mg never revives pulsation.
NaCl + K_2SO_4	Rapid at first, then slow, then periods of quiescence alternating with pulsation. All movement dies out in about 20 minutes.	Newly cut disks give a few very strong pulsations lasting about 3 minutes. Regenerating disks pulsate longer, but in an irregular manner.	Pulsation is *always* revived by any calcium salt, *never* by magnesium.
NaCl + K_2SO_4 + $CaSO_4$	Active, sustained, *irregular* pulsation at fully twice the normal rate. Pulsation at an abnormally rapid rate continues for more than one hour.	Newly cut disks give about 18 pulsations, then quiescence. Disks two or more days old may pulsate for hours in an irregular manner.	Pulsation is rendered more regular by $CaCO_3$. It is rendered slower, or stopped, by $MgSO_4$ or $MgCl_2$.

$NaCl + K_2SO_4 + CaCO_3$	Active, *irregular* pulsation at about double the normal rate. Periods of rest alternating with periods of active pulsation soon set in. These continue for more than an hour	Newly made disks give a few pulsations then, quiescence. Regenerating disks, two or more days old pulsate irregularly, sometimes for hours.	Pulsation is made regular, without periods of rest, by adding $CaSO_4$. Mg salts, in the absence of $CaSO_4$, reduce or stop pulsation.
$NaCl + K_2SO_4 + CaSO_4 + CaCO_3$.	Violent sustained pulsation at about twice the normal rate. Pulsation continues more than an hour without periods of rest.	Recently cut disks give a few pulsations, then quiescence. Disks two or more days old are strongly stimulated and pulsate irregularly for many hours.	Pulsation is reduced, or stopped, by Mg salts.
$NaCl + MgSO_4 + MgCl_2$	All movement ceases in less than 6 minutes.	No pulsations.	A few pulsations are sometimes revived by K_2SO_4, but prolonged active pulsation is revived by calcium salts.
$NaCl + MgCl_2 + CaSO_4$	Periods of quiescence alternating with activity. Practically all movement dies out in about 50 minutes. Pulsation not rapid.	Not tried.	Not tried.
$NaCl + MgSO_4 + MgCl_2 + CaSO_4$.	Periods of quiescence alternating with activity. Whenever the Medusa comes to rest its mouth-arms contract, but they expand soon after movement is resumed. Pulsates thus for more than an hour. The rate is normal.	No pulsations; no effect.	K_2SO_4 renders movement slightly more regular.
$NaCl + MgSO_4 + MgCl_2 + K_2SO_4$	Rapid pulsation for a moment, then more and more slow. All movement ceases in less than 6 minutes.	No contractions; no effect.	Pulsation is always revived, even after 1½ hours quiescence by calcium.
$NaCl + MgSO_4 + MgCl_2 + CaSO_4 + CaCO_3$	Periods of activity alternating with resting periods. For the first half hour the periods of rest are about equal to those of activity. The rate of pulsation is about normal. Pulsation ceases at the end of about 1½ hours.	No contractions	Adding K_2SO_4 renders the periods of rest shorter, and the movement more uniform. Recovery very rapid in sea-water.
$NaCl + MgSO_4 + MgCl_2 + CaSO_4 + K_2SO_4$	Long periods of normal pulsation alternating with short resting periods. Pulsation usually continues for several hours.	No contractions	Adding $CaOC_3$ renders pulsation more regular. Recovery is instantaneous in natural sea-water.

TABLE 4.—*Effects upon the rhythmical pulsation of Cassiopea exerted by solutions, etc.*—Continued.

Composition of the solution.	Pulsation of perfect Medusæ taken from sea-water and placed in the solution.	Paralyzed quiescent disks without sense-organs, taken from sea-water and placed in the solution, behave as follows:	Effect produced upon perfect Medusæ by adding the salts to make sea-water, after the Medusæ have come to rest.
$NaCl + MgSO_4 + MgCl_2 + CaCO_3 + K_2SO_4$.	Short periods of pulsation, alternating with increasingly long periods of quiescence. Practically all movement ceases in about 10 minutes.	No contractions	Pulsation is rendered nearly normal by adding $CaSO_4$, or perfectly normal, at once, by replacing the Medusa in sea-water.
K_2SO_4 isotonic with the NaCl of sea-water.	Pulsation rapidly declines without even momentary acceleration. All movement dies out in about 3 minutes.	Active contractions for about 3 minutes, followed by tetanus.	Pulsation can not be revived by calcium.
$CaCl_2$ isotonic with the NaCl of sea-water.	Medusæ give only 5 to 8 slow pulsations, then all movement ceases.	No effect; no contractions...........	
$MgSO_4$ or $MgCl_2$ isotonic with the NaCl of sea-water.	Medusæ cease to pulsate almost instantly.	No effect; no contractions............	Pulsation can not be revived by calcium.
Na_2CO_3 isotonic with the NaCl of sea-water.	Almost instant cessation of all movement.	....	Pulsation can not be revived by calcium.

TABLE 5.—*Effects upon the rhythmical pulsation of Cassiopea, exerted by solutions containing Na, Ca, K, and Mg in amounts and proportions found in sea-water according to Van 't Hoff's formula, wherein sea-water is supposed to contain 100 $NaCl + 2.2\ KCl + 7.8\ MgCl_2 + 3.8\ MgSO_4 + 3CaCl_2$, all of $\frac{5}{8}n$ concentration.*

Composition of the solution.	Normal Medusæ taken from sea-water and placed in the solution pulsate as follows:	Disks made by cutting off the marginal sense-organs of *Cassiopea* behave as follows:
$NaCl$	Pulsation is abnormally rapid at first, but in from 7 to 10 minutes periods of rest appear and these increase in duration and frequency while the periods of active pulsation decrease. All movement dies out before the end of 45 minutes.	Newly cut disks give a few contractions, then subside into quiescence. Disks two or more days old usually pulsate slowly and irregularly for hours.
$NaCl+CaCl_2$	At first the rate is about normal. Periods of rest begin to appear after being about 15 minutes in the solution. These periods of rest increase in duration and frequency so that all movement ceases before the end of 1½ hours.	Disks behave very much as they do in NaCl, but are somewhat more actively stimulated.
$NaCl+KCl$	Pulsation is abnormally rapid at first, but at the end of about half an hour pauses set in, and these periods of rest gradually increase in length and frequency. All movement ceases after the Medusa has pulsated somewhat more than 2 hours.	Disks behave as they do in $NaCl+CaCl_2$, but are more strongly stimulated.
$NaCl+KCl+CaCl_2$	Pulsation is maintained without pauses at fully twice the normal rate for more than 2 hours; then periods of rest are apt to commence. The Medusa pulsates over 4 hours.	Disks are powerfully stimulated, and even newly made disks may commence spontaneous, irreglar pulsation which may continue for several hours.
$NaCl+MgCl_2$	Pulsation declines steadily and ceases before the end of 6 minutes.	No pulsation.
$NaCl+MgSO_4$	Pulsation declines at once and all movement ceases before end of 10 minutes.	No pulsation.
$NaCl+MgCl_2+MgSO_4$	Pulsation declines rapidly and usually ceases before the end of 30 seconds. Some Medusæ may pulsate longer than 5 but less than 6 minutes. It is restored if we add $CaCl_2$, but not usually by KCl.	No pulsation.
$NaCl+MgCl_2+MgSO_4+KCl$.	Pulsation steadily declines and ceases before the end of 6 minutes. It is effectually restored by adding $CaCl_2$.	No pulsation occurs.
$NaCl+MgCl_2+MgSO_4+CaCl_2$.	Rate normal at first but declines slowly with pauses, so that all movement ceases before the end of 40 minutes. Pulsation is restored by adding KCl.	No pulsation.
$NaCl+MgCl_2+MgSO_4+KCl+CaCl_2$.	Pulsation is normal as in natural sea-water.	Newly made disks do not pulsate. Disks 2 or more days old pulsate irregularly.

IV. PULSATION OF THE BRANCHIAL ARMS OF LEPAS, THE HEART OF SALPA, AND THE HEART OF THE LOGGERHEAD TURTLE.

The Medusæ are the most primitive of the metazoans which display rhythmical pulsation, and therefore a study of the laws which control their movement is important, for it is practically certain that pulsation began to attain physiological importance in primitive marine animals, and that the vertebrate heart developed in creatures living in salt water. In the most primitive forms the body pulsates as a whole, but finally pulsation is assumed by or restricted to special organs. It is therefore interesting to consider various sorts of pulsating organs in order to see whether some fundamental conditions may not apply to all of them.

Accordingly studies were made of the pulsation of the heart of the solitary asexual form of *Salpa democratica*, the rhythmical movement of the branchial arms of *Lepas*, and the pulsation of the heart of the embryo loggerhead turtle, *Thalassochelys caretta*, and these varied sorts of pulsation were compared with that of the jellyfish *Cassiopea*.

The results are presented in condensed form in table 6 (p. 60) which shows the number of minutes that pulsation endures in various solutions consisting of one or all of the ingredients NaCl, KCl, $CaCl_2$, $MgSO_4$, and $MgCl_2$. In the experiments upon *Cassiopea*, *Lepas*, and *Salpa* Van t' Hoff's sea-water solution was employed. This consists of 100 NaCl + 2.2 KCl + 7.8 $MgCl_2$ + 3.8 $MgSO_4$ + 3$CaCl_2$, all of $\frac{5}{8}$n concentration. In experiments upon the heart of the loggerhead turtle the proportions of the above-named salts were changed so as to be 0.7 per cent NaCl + 0.03 per cent KCl + 0.1 per cent $MgCl_2$ + 0.025 per cent $CaCl_2$. The various animals were placed in solutions containing one or all of these salts in the amounts and proportions stated above. Where + follows a number it means that pulsation occasionally lasts a few more minutes than is here recorded, and on the other hand, — following a number means that the pulsation does not usually last as long as is recorded.

An inspection of table 6 (p. 60) will show that pulsation in all of these forms (jellyfish, barnacle, tunicate, and reptile) is most powerfully stimulated by solutions composed of sodium chloride, potassium, and calcium, and that all are depressed by magnesium. Nevertheless sustained pulsation can only take place in a solution containing sodium, potassium, calcium, and *magnesium*, the last-named element being necessary to "tone down" and restrain the strong stimulation caused by the first three, thus giving a slower but indefinitely sustained pulsation. This important rôle of magnesium has hitherto

been unsuspected, and we see that Ringer's solutions, which consist of combinations of sodium, potassium, and calcium chlorides, are only stimulants, and must be partially inhibited and restrained by magnesium in order that they may sustain pulsation indefinitely.

In simple marine animals such as Medusæ, barnacles, and *Salpa* the optimum solution for pulsation is the sea-water itself, but in the higher terrestrial forms the proportions and amounts of the ingredients of the optimum solution have changed, although still composed of sodium chloride, potassium, calcium, and *magnesium*. In *Cassiopea*, *Lepas*, and *Salpa* it is the special rôle of calcium to assist the sodium chloride to overcome the anesthetic effect of magnesium, whereas potassium practically lacks this power.

A further inspection of table 6 shows that there are considerable differences in the effects of various elements upon different animals. For example, pulsation is sustained fairly well in *Cassiopea*, the heart of *Salpa democratica*, and the loggerhead turtle embryo by a pure NaCl solution, but this quickly stops the movement of the branchial arms of *Lepas*. Also, the addition of KCl to NaCl greatly improves the solution in its ability to sustain the pulsation of *Cassiopea*, whereas it has but little beneficial effect in the case of the arms of *Lepas*. Calcium, on the other hand, has but little power to sustain pulsation in connection with NaCl in *Cassiopea*, but in the case of the arms of *Lepas* it is very efficient. In *Cassiopea* pulsation ceases almost instantly in such non-ionizable solutions as urea, dextrose, and glycerin, but the heart of *Salpa democratica* will pulsate for a considerable time in these solutions, and the heart of the embryo loggerhead turtle pulsates as long in dextrose as it does in NaCl. These differences in the effects of the several salts upon pulsation in different animals are so considerable that we must be cautious of drawing general conclusions from the behavior of any one animal and applying them to related forms. For example, *Cassiopea* can not pulsate for 6 minutes in a solution resembling sea-water but simply lacking calcium, whereas another Scyphomedusa, *Linerges mercurius*, will pulsate for 45 minutes in the same solution. Both *Linerges* and *Cassiopea* are, however, restored to normal pulsation by the addition of calcium, and the difference in their behavior is one of degree, not of kind. The papers of physiologists abound in general conclusions concerning the action of "the vertebrate heart" when only the heart of the terrapin or the dog has been studied, and undoubtedly these sweeping conclusions are often misleading. For example, when the loggerhead turtle embryo is 11 to 14 days old its heart ceases to pulsate in less than 22 minutes in the

albumen of its own egg, but when it is 41 days old it pulsates from 3 to 7 hours in the albumen of its egg, which then sustains it better than can a Ringer's solution, or any solution I could devise. The albumen contains Na, K, Ca, and Mg.

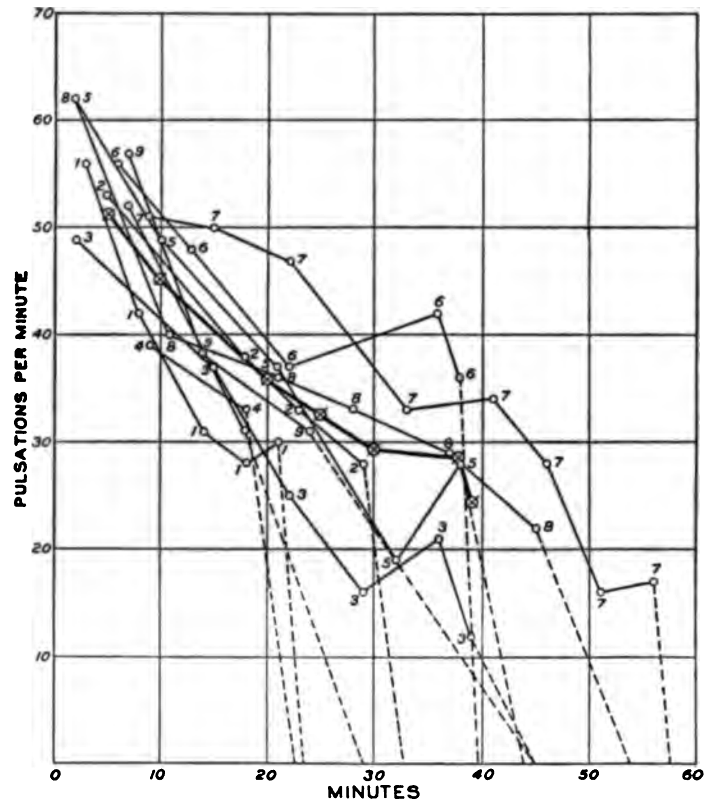

Fig. 34.—Showing the decline in rate and also the length of time that pulsation endured in the hearts of 10 loggerhead turtle embryos (11 to 14 days old) placed in 0.7 per cent NaCl + 0.1 per cent $MgCl_2$. The heavy dark line shows the average condition, and the fine full lines show the behavior of individual embryos. The dotted lines cover periods from the last observation to the time when the heart ceased to beat.

The " all or none " principle in pulsation does not apply to the pulsation of the heart of the embryo loggerhead turtle, for the ventricle ceases first, then after a long time the auricles cease to pulsate, but the sinus still pulsates. Normally, as is well known, the heart-beat originates in the sinus; then after an interval the auricles respond, and finally the ventricle contracts. After the heart which has been removed from the body has ceased to pulsate, however, we may stim-

ulate the ventricle by an induction current, and after the current has been removed the heart may pulsate for several minutes in a reverse manner, each contraction originating at the stimulated place in the ventricle, then after a pause the auricles, and finally the sinus contracting.

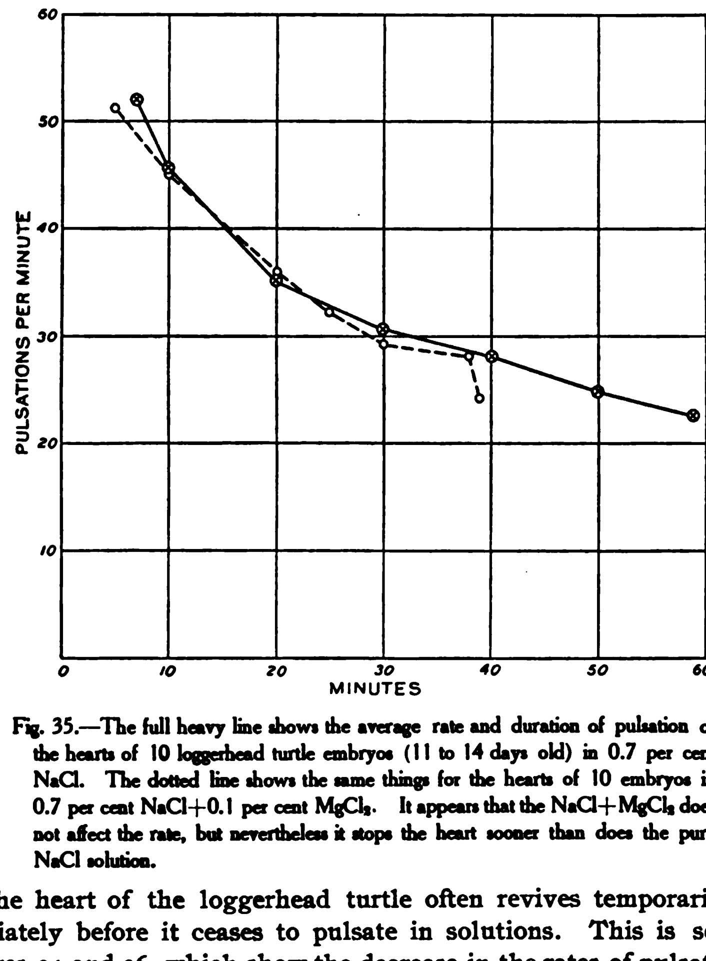

Fig. 35.—The full heavy line shows the average rate and duration of pulsation of the hearts of 10 loggerhead turtle embryos (11 to 14 days old) in 0.7 per cent NaCl. The dotted line shows the same things for the hearts of 10 embryos in 0.7 per cent NaCl+0.1 per cent $MgCl_2$. It appears that the NaCl+$MgCl_2$ does not affect the rate, but nevertheless it stops the heart sooner than does the pure NaCl solution.

The heart of the loggerhead turtle often revives temporarily immediately before it ceases to pulsate in solutions. This is seen in figures 34 and 36, which show the decrease in the rates of pulsation of the hearts of 20 loggerhead turtle embryos 11 to 14 days old. Ten of these (fig. 36) were placed in 0.7 per cent NaCl, and 10 others whose pulsation is shown in figure 34 were placed in 0.7 per cent NaCl+0.1 per cent $MgCl_2$. The $MgCl_2$ has no effect upon the *rate*, but it stops the heart sooner than does the pure NaCl. (See fig. 35.)

After the heart of the loggerhead turtle has ceased to pulsate in NaCl it may be revived temporarily by $CaCl_2$. KCl will also revive

it, but not so powerfully, and even *distilled water* or $MgCl_2$ will often give rise to a few final, weak pulsations. In other words, the heart responds to any osmotic change, be it beneficial or injurious. It is worthy of note, however, that if the heart ceases to beat in $NaCl + MgCl_2$ it is usually impossible to revive it, even by $CaCl_2$.

The heart of the loggerhead turtle embryo, 14 days old, pulsates more rapidly, and usually longer, in 0.7 per cent NaCl + 0.03 per cent KCl than it does in 0.7 per cent NaCl. Thus the addition of a small amount of KCl acts as a stimulus. Physiologists are in dispute concerning the action of potassium upon the "vertebrate heart," the general opinion being that potassium depresses the heart. The literature of this subject is reviewed by Carlson (1906, Amer. Journ. Physiology, vol. 16, p. 397). Much of the discrepancy in results arises from the sweeping conclusions which physiologists have drawn in applying to all vertebrates the results achieved from experiments upon a few forms. Moreover, in some papers experiments are conducted upon each salt separately, and the assumption is made that the effect of a mixture of these salts is merely the summation of their individual effects. Nothing could be more erroneous. For example, calcium alone never stimulates, but even inhibits pulsation in *Cassiopea*, but in *connection with* sodium and potassium chlorides it forms a most powerful stimulant.

In closing we will state that the heart of the embryo loggerhead turtle behaves quite differently from that of the animal after hatching, but we will leave the discussion of this and other points to a future paper, wherein we hope to treat of the general effects of different salts upon the hearts of various vertebrates and invertebrates.

In conclusion it may be said that rhythmical pulsation can be sustained only when an external stimulant is counteracted by an inhibitor, so that the pulsating organism is in a state bordering upon the threshold of stimulation. This allows the weakest internal stimuli to produce periodic contractions. Each contraction either produces a chemical change which periodically reduces the internal stimulus, or the tissue can not again respond to the ever-present, *constant* stimulus until after a period of rest.

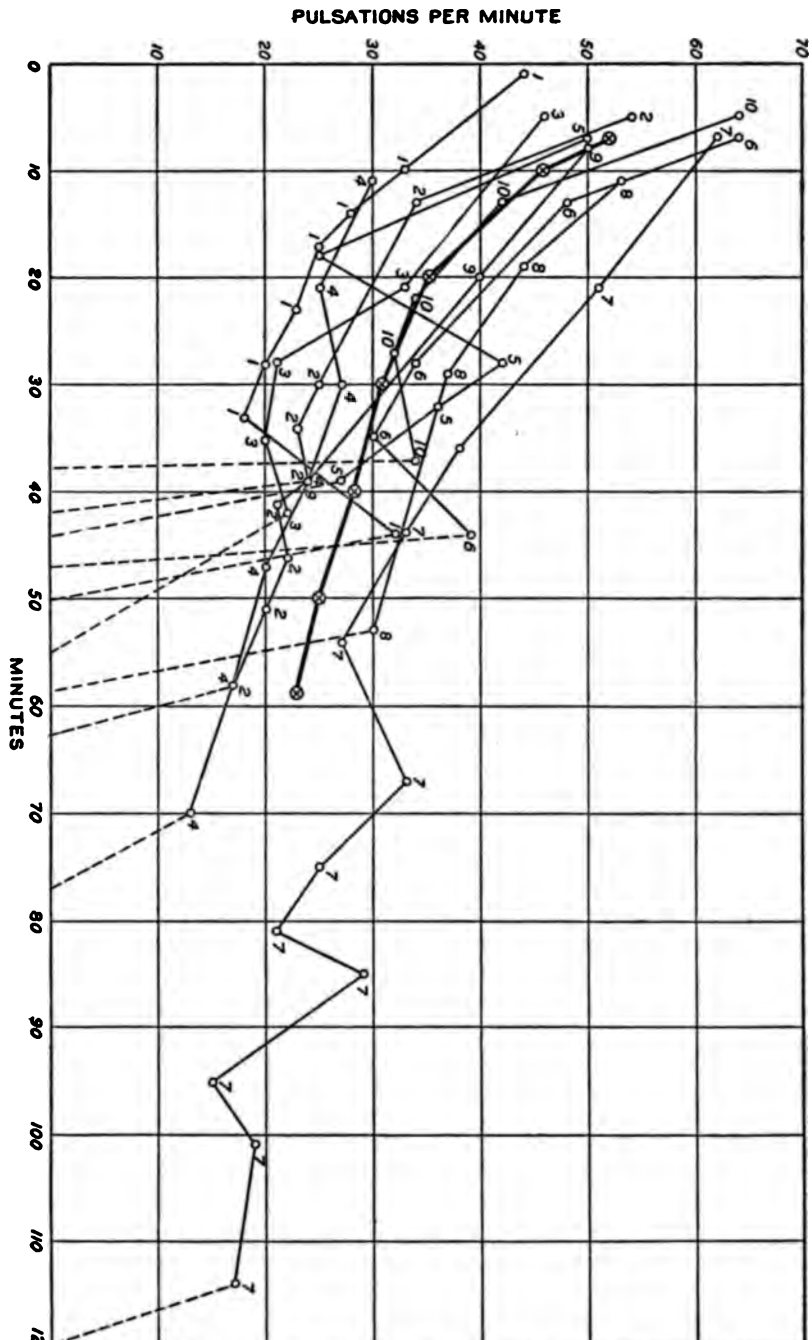

Fig. 36.—Showing the decline in rate and also the length of time that pulsation lasted in the hearts of 10 loggerhead turtle embryos (11 to 14 days old) placed in 0.7 per cent NaCl. The heavy line shows the average condition, and the fine unbroken lines show behavior of individual embryos numbered from 1 to 10. The dotted lines cover periods from the last observation to the time when the heart ceased to beat.

TABLE 6.—*Showing the time that pulsation endures in various solutions containing Na, Ca, K, and Mg.*

[*Cassiopea*, *Salpa*, and *Lepas* were placed in solutions containing some or all of the constituents of sea-water according to Van 't Hoff's formula: $\frac{5}{8}n\,(100\,NaCl + 2.2\,KCl + 7.8\,MgCl_2 + 3.8\,MgSO_4 + 3CaCl_2)$. The embryos of the loggerhead turtle were placed in solutions containing some or all of the constituents of the following: 0.7 per cent NaCl + 0.03 per cent KCl + 0.025 per cent $CaCl_2$ + 0.1 per cent $MgCl_2$.]

Composition of the solution.	*Cassiopea*, normal Medusæ.	Branchial arms of *Lepas anatifera*.	Branchial arms of *Lepas fascicularis*.	Heart of *Salpa democratica* (duration of normal pulsation).	Heart of embryo loggerhead turtle (*T. caretta*) embryos in eggs 11 to 14 days after being laid.
$NaCl$	45— minutes	1 to 9 minutes	1 to 12— minutes	45 to 90— minutes	37¼ to 109½ minutes. Average of ten, 59.1 minutes.
$NaCl + CaCl_2$	90— minutes	9 to 75 minutes	15 to 71 minutes		58 to 59 minutes.
$NaCl + KCl$	120 minutes	8 to 32 minutes	3 to 22— minutes		50 to 82 minutes.
$NaCl + CaCl_2 + KCl$	More than 180 minutes at more than double the normal rate.	180+ minutes. Still pulsating normally at the end of 2 hours.	180+ minutes. Still pulsating normally at the end of 2 hours.	180+ minutes. Still pulsating normally at the end of 2 hours.	60 to 80¼ minutes.
$NaCl + MgSO_4$	7½ to 10 minutes		4 to 9 minutes		
$NaCl + MgCl_2$	5 to 6 minutes		4— to 13 minutes		23 to 57⅝ minutes. Average of ten, 39.3 minutes
$NaCl + MgSO_4 + MgCl_2$	¼ to 6 minutes	15— minutes	3½ to 12— minutes	45 to 90+ minutes	
$NaCl + MgCl_2 + CaCl_2$					39 to 41¾ minutes.
$NaCl + MgSO_4 + MgCl_2 + CaCl_2$	40— minutes	100 to 133 minutes	170 to 220 minutes		
$NaCl + MgSO_4 + MgCl_2 + KCl$	6— minutes	20 minutes	30— minutes	45— minutes	
$\frac{5}{8}n\ MgCl_2$	Stops almost instantly.		2— to 3 minutes	12 minutes	
$\frac{5}{8}n\ MgSO_4$	do.			4 minutes	
$\frac{5}{8}n\ KCl$	do.		0 minutes	3 minutes	
$\frac{5}{8}n\ CaCl_2$	do.		0 to 1 minute	8 minutes	
Urea isotonic with $\frac{5}{8}n$ NaCl	do.		2— to 3 minutes	5 to 10 minutes	
Dextrose isotonic with $\frac{5}{8}n$ NaCl.	do.	11 to 13 minutes	1½— to 4 minutes	37 to 51 minutes	37 to 49 minutes. This dextrose solution was isotonic with 0.7 per cent NaCl.
Glycerin isotonic with $\frac{5}{8}n$ NaCl.	do.		4 to 16 minutes	14 to 21— minutes	22— min. in embryos 11 to 14 days old, 180+ to 410+ min. in embryos 41 days old.
Clear albumen of loggerhead turtle egg.	1 minute				

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